PHYS 1443 – Section 004
Lecture #12

Thursday, Oct. 2, 2014
Dr. Jaehoon Yu

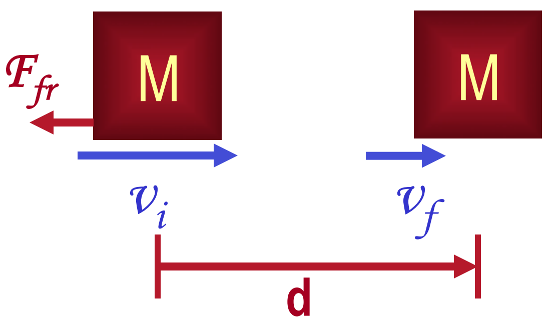
- Work-Kinetic Energy Theorem
- Work under friction
- Potential Energy and the Conservative Force
  - Gravitational Potential Energy
  - Elastic Potential Energy
- Conservation of Energy
- Energy Diagram

Reminder for Special Project #4

- Two protons are separated by 1m.
  - Compute the gravitational force (F_G) between the two protons (10 points)
  - Compute the electric force (F_E) between the two protons (10 points)
  - Compute the ratio of FG/FE (5 points) and explain what this tells you (5 point)
- You must specify the formulae for each of the forces and the values of necessary quantities, such as mass, charge, constants, etc, in your report
- Due: Beginning of the class, Tuesday, Oct. 7

Work and Energy Involving Kinetic Friction

- What do you think the work looks like if there is friction?
  - Static friction does not matter! Why? **It isn't there when the object is moving.**
  - Then which friction matters? **Kinetic Friction**



Friction force $\mathcal{F}_{fr}$ works on the object to slow down

The work on the object by the friction $\mathcal{F}_{fr}$ is

$$W_{fr} = \vec{F}_{fr} \bullet \vec{d} = F_{fr} d \cos(180) = -F_{fr} d \quad \Delta KE = -F_{fr} d$$

The negative sign means that the work is done on the friction!

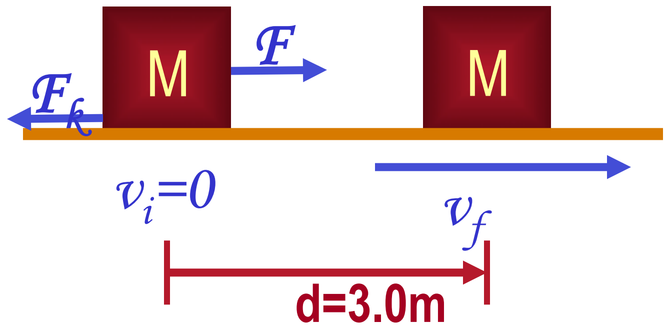
The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and all other sources of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

t=0, KE_i **Friction, Engine work** **t=T, KE_f**

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36(J)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k \cdot \vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k mg\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26(J)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

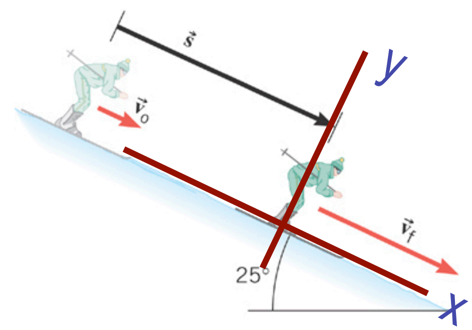
$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$

Ex. Downhill Skiing

A 58kg skier is coasting down a 25° slope. A kinetic frictional force of magnitude f_k=70N opposes her motion. At the top of the slope, the skier's speed is v_0=3.6m/s. Ignoring air resistance, determine the speed v_f at the point that is displaced 57m downhill.



(a)

What are the forces in this motion?

Gravitational force: F_g Normal force: F_N Kinetic frictional force: f_k

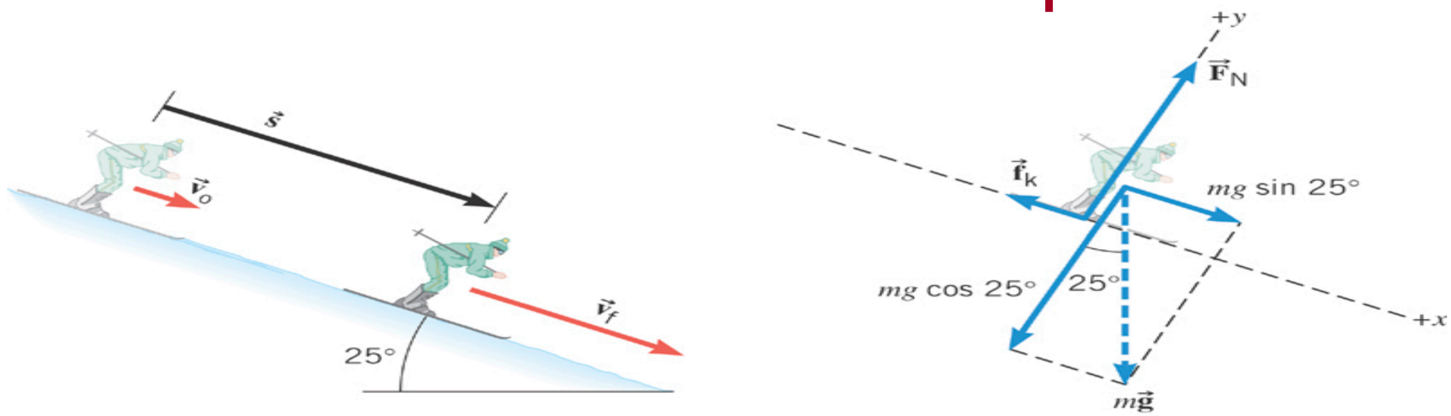
What are the X and Y component of the net force in this motion?

Y component $\sum F_y = F_{gy} + F_N = -mg\cos 25^o + F_N = 0$

From this we obtain $F_N = mg\cos 25^o = 58 \cdot 9.8 \cdot \cos 25^o = 515N$

What is the coefficient of kinetic friction? $f_k = \mu_k F_N \Rightarrow \mu_k = \frac{f_k}{F_N} = \frac{70}{515} = 0.14$

Ex. Now with the X component



(a) (b) Free-body diagram for the skier

X component $$\sum F_x = F_{gx} - f_k = mg\sin 25^\circ - f_k = \left(58\cdot 9.8\cdot \sin 25^\circ - 70\right) = 170N = ma$$

Total work by this force $$W = \left(\sum F_x\right)\cdot s = \left(mg\sin 25^\circ - f_k\right)\cdot s = \left(58\cdot 9.8\cdot \sin 25^\circ - 70\right)\cdot 57 = 9700J$$

From work-kinetic energy theorem $$W = KE_f - KE_i \Rightarrow KE_f = \frac{1}{2}mv_f^2 = W + KE_i = W + \frac{1}{2}mv_0^2$$

Solving for v_f $$v_f^2 = \frac{2W + mv_0^2}{m} \Rightarrow v_f = \sqrt{\frac{2W + mv_0^2}{m}} = \sqrt{\frac{2\cdot 9700 + 58\cdot (3.6)^2}{58}} = 19\,m/s$$

What is her acceleration? $$\sum F_x = ma \Rightarrow a = \frac{\sum F_x}{m} = \frac{170}{58} = 2.93\,m/s^2$$

Potential Energy & Conservation of Mechanical Energy

Energy associated with a system of objects → *Stored energy which has the potential or the possibility to work or to convert to kinetic energy*

What does this mean?

In order to describe potential energy, U, a system must be defined.

The concept of potential energy can only be used under the special class of forces called the ***conservative force*** *which results in the principle of* ***conservation of mechanical energy.***

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

What are other forms of energies in the universe?

Mechanical Energy *Chemical Energy* *Biological Energy*

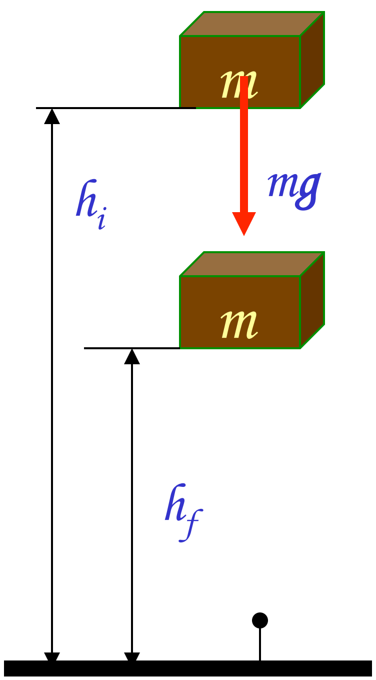
Electromagnetic Energy *Nuclear Energy*

These different types of energies are stored in the universe in many different forms!!!

If one takes into account ALL forms of energy, the total energy in the entire universe is conserved. It just transforms from one form to another.

Gravitational Potential Energy

This potential energy is given to an object by the gravitational field in the system of Earth by virtue of the object's height from an arbitrary zero level



When an object is falling, the gravitational force, Mg, performs the work on the object, increasing the object's kinetic energy. So the potential energy of an object at height h, the potential to do work, is expressed as

$$PE = \vec{F}_g \cdot \vec{y} = \left|\vec{F}_g\right|\left|\vec{y}\right|\cos\theta = \left|\vec{F}_g\right|\left|\vec{y}\right| = mgh \qquad PE \equiv mgh$$

The work done on the object by the gravitational force as the brick drops from h_i to h_f is:

$$W_g = PE_i - PE_f$$
$$= mgh_i - mgh_f = -\Delta PE$$

$$\left(since \ \Delta PE = PE_f - PE_i\right)$$

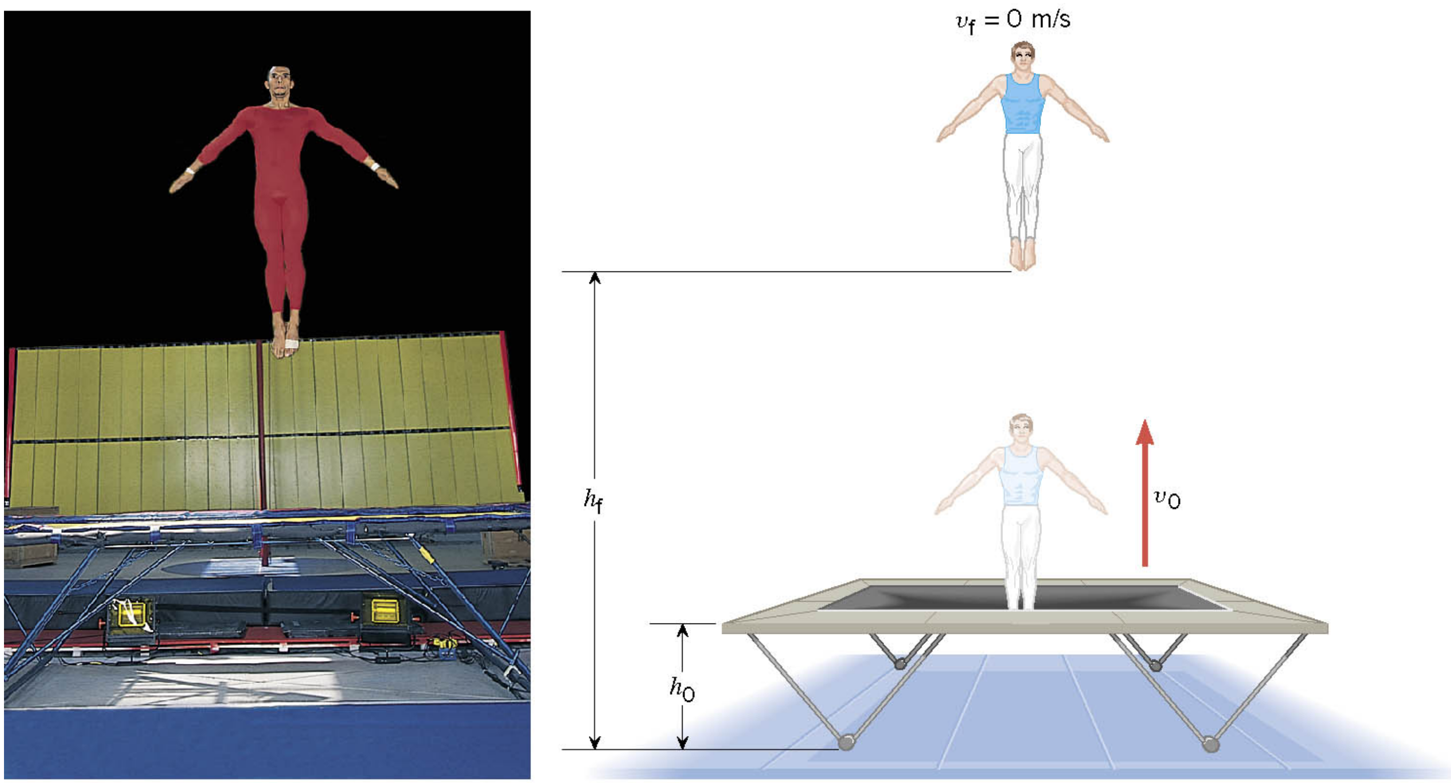
What does this mean?

Work by the gravitational force as the brick drops from y_i to y_f is the negative change of the system's potential energy

➔ Potential energy was spent in order for the gravitational force to increase the brick's kinetic energy.

Ex. A Gymnast on a Trampoline

The gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?

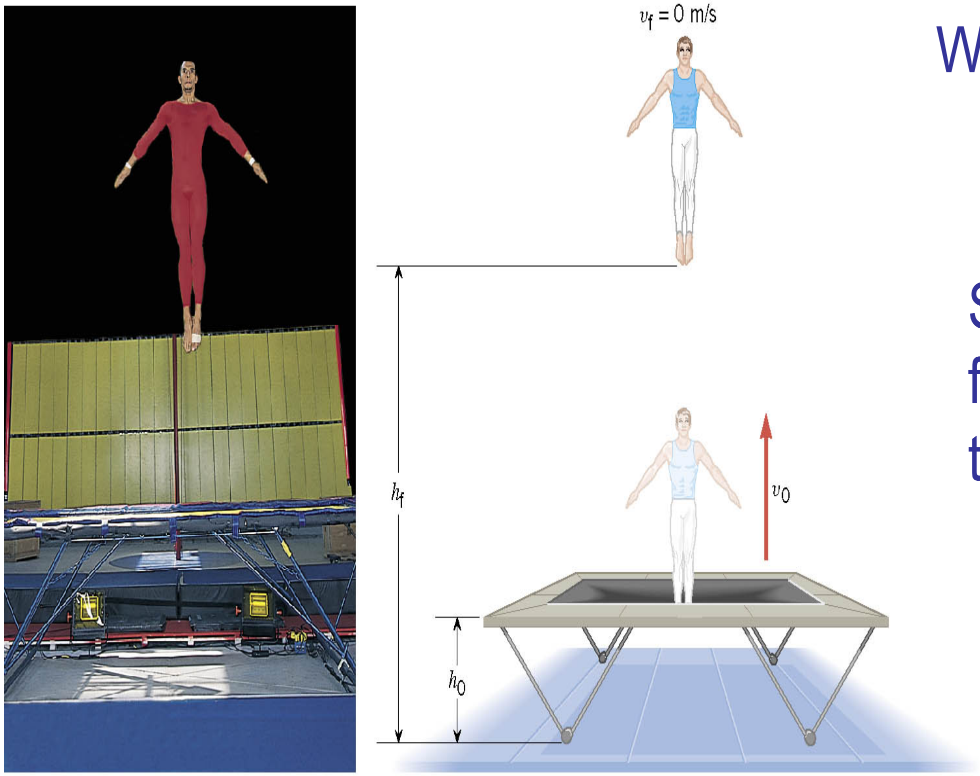


(a) (b)

Ex. Continued

From the work-kinetic energy theorem

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$



(a) (b)

Work done by the gravitational force

$$W_{\text{gravity}} = mg\left(h_o - h_f\right)$$

Since at the maximum height, the final speed is 0. Using work-KE theorem, we obtain

$$\cancel{m}g\left(h_o - h_f\right) = -\tfrac{1}{2}\cancel{m}v_o^2$$

$$v_o = \sqrt{-2g\left(h_o - h_f\right)}$$

$$\therefore v_o = \sqrt{-2\left(9.80\,\mathrm{m/s^2}\right)\left(1.20\text{ m} - 4.80\text{ m}\right)} = 8.40\,\mathrm{m/s}$$

Conservative Forces and Potential Energy

The work done on an object by a conservative force is equal to the decrease in the potential energy of the system

$$W_c = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

What does this statement tell you?

The work done by a conservative force is equal to the negative change of the potential energy associated with that force.

Only the changes in potential energy of a system is physically meaningful!!

We can rewrite the above equation in terms of the potential energy U

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

So the potential energy associated with a conservative force at any given position becomes

$$U_f(x) = -\int_{x_i}^{x_f} F_x dx + U_i$$

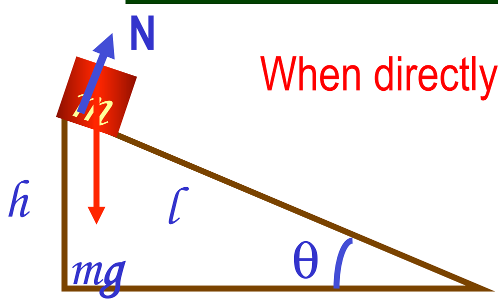
Potential energy function

What can you tell from the potential energy function above?

Since U_i is a constant, it only shifts the resulting $U_f(x)$ by a constant amount. One can always change the initial potential so that U_i can be 0.

More Conservative and Non-conservative Forces

The work done on an object by the gravitational force does not depend on the object's path in the absence of a retardation force.



When directly falls, the work done on the object by the gravitation force is $W_g = mgh$

When sliding down the hill of length l, the work is

$$W_g = F_{g-incline} \times l = mg\sin\theta \times l$$
$$= mg(l\sin\theta) = mgh$$

How about if we lengthen the incline by a factor of 2, keeping the height the same??

Still the same amount of work☺

$$\boxed{W_g = mgh}$$

So the work done by the gravitational force on an object is independent of the path of the object's movements. It only depends on the difference of the object's initial and final position in the direction of the force.

Forces like gravitational and elastic forces are called the conservative force

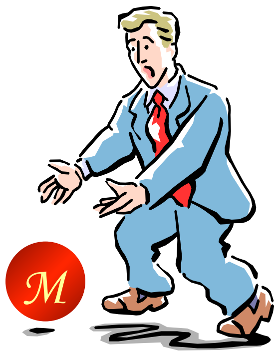
1. If the work performed by the force does not depend on the path.
2. If the net work performed on a closed path is 0.

Total mechanical energy is conserved!!

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

Example for Potential Energy

A bowler drops bowling ball of mass 7kg on his toe. Choosing the floor level as y=0, estimate the total work done on the ball by the gravitational force as the ball falls on the toe.



Let's assume the top of the toe is 0.03m from the floor and the hand was 0.5m above the floor.

$$U_i = mgy_i = 7\times 9.8\times 0.5 = 34.3J \quad U_f = mgy_f = 7\times 9.8\times 0.03 = 2.06J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.24J \cong 30J$$

b) Perform the same calculation using the top of the bowler's head as the origin.

What has to change?

First we must re-compute the positions of the ball in his hand and on his toe.

Assuming the bowler's height is 1.8m, the ball's original position is –1.3m, and the toe is at –1.77m.

$$U_i = mgy_i = 7\times 9.8\times\left(-1.3\right) = -89.2J \quad U_f = mgy_f = 7\times 9.8\times\left(-1.77\right) = -121.4J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.2J \cong 30J$$

Elastic Potential Energy

Potential energy given to an object by a spring or an object with elasticity in the system that consists of an object and the spring.

The force spring exerts on an object when it is distorted from its equilibrium by a distance x is

$$F_s = -kx$$

Hooke's Law

The work performed on the object by the spring is

$$W_s = \int_{x_i}^{x_f} (-kx)dx = \left[-\frac{1}{2}kx^2\right]_{x_i}^{x_f} = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

The potential energy of this system is

$$U_s \equiv \frac{1}{2}kx^2$$

What do you see from the above equations?

The work done on the object by the spring depends only on the initial and final position of the distorted spring.

Where else did you see this trend?

The gravitational potential energy, U_g

So what does this tell you about the elastic force?

A conservative force!!!