PHYS 1443 – Section 004
Lecture #13

Tuesday, Oct. 7, 2014
Dr. Jaehoon Yu

- Conservation of Mechanical Energy
- Work Done By a Non-Conservative Force
- Energy Diagram
- Universal Gravitational Field
- General Gravitational Potential Energy
- Power

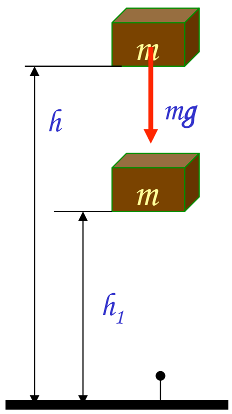
Today's homework is homework #7, due 11pm, Tuesday, Oct. 14!!

Announcements

- Results of the 1st non-comprehensive exam:
  - Class average: 73.7/106
    - Equivalent to 69.5/100
  - Top score: 102/106
  - Will take the better of the two non-comprehensive exam after normalizing to the class average between the two exams
- Quiz this Thursday, Oct. 9
  - Beginning of the class
  - Covers from CH6.2 to what we finish today
  - Bring your own formula sheet
- Mid-term comprehensive exam on Tuesday, Oct. 21
- Colloquium Wednesday at 4pm in SH101

Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies $E \equiv K + U$



Let's consider a brick of mass m at the height h from the ground

What is the brick's potential energy?

$$U_g = mgh$$

What happens to the energy as the brick falls to the ground?

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

The brick gains speed *By how much?* $v = gt$

So what? *The brick's kinetic energy increases to* $K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$

And? *The lost potential energy is converted to the kinetic energy!!*

What does this mean?

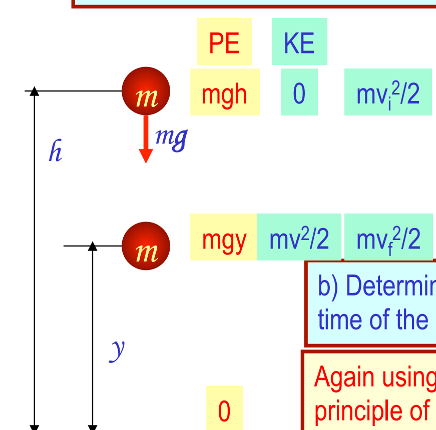
The total mechanical energy of a system remains constant in any isolated system of objects that interacts only through conservative forces: ***Principle of mechanical energy conservation***

$$E_i = E_f$$

$$K_i + \sum U_i = K_f + \sum U_f$$

Example

A ball of mass m at rest is dropped from the height h above the ground. a) Neglecting air resistance determine the speed of the ball when it is at any given height y above the ground.



Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f \quad 0 + mgh = \frac{1}{2}mv^2 + mgy$$

$$\frac{1}{2}mv^2 = mg(h-y)$$

$$\therefore v = \sqrt{2g(h-y)}$$

b) Determine the speed of the ball at y if it had initial speed v_i at the time of the release at the original height h.

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mgy$$

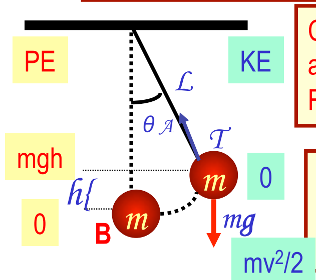
$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) = mg(h-y)$$

$$\therefore v_f = \sqrt{v_i^2 + 2g(h-y)}$$

This result look very similar to a kinematic expression, doesn't it? Which one is it?

Example

A ball of mass m is attached to a light cord of length L, making up a pendulum. The ball is released from rest when the cord makes an initial angle θ_A with the vertical, and the pivoting point P is frictionless. Find the speed of the ball when it is at the lowest point, B.



Compute the potential energy at the maximum height, h. Remember where the 0 is.

$$h = L - L\cos\theta_A = L(1-\cos\theta_A)$$

$$U_i = mgh = mgL(1-\cos\theta_A)$$

Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = mgL(1-\cos\theta_A) = \frac{1}{2}mv^2$$

$$v^2 = 2gL(1-\cos\theta_A) \quad \therefore v = \sqrt{2gL(1-\cos\theta_A)}$$

b) Determine tension T at the point B.

Using Newton's 2nd law of motion and recalling the centripetal acceleration of a circular motion

$$\sum F_r = T - mg = ma_r = m\frac{v^2}{r} = m\frac{v^2}{L}$$

$$T = mg + m\frac{v^2}{L} = m\left(g + \frac{v^2}{L}\right) = m\left(g + \frac{2gL(1-\cos\theta_A)}{L}\right)$$

$$= m\frac{gL + 2gL(1-\cos\theta_A)}{L}$$

$$\therefore T = mg(3 - 2\cos\theta_A)$$

Cross check the result in a simple situation. What happens when the initial angle θ_A is 0? $T = mg$

Work Done by Non-conservative Forces

Mechanical energy of a system is not conserved when any one of the forces in the system is a non-conservative (dissipative) force.

Two kinds of non-conservative forces:

*Applied forces: Forces that are **external** to the system. These forces can take away or add energy to the system. So the **mechanical energy of the system is no longer conserved.***

If you were to hit a free falling ball, the force you apply to the ball is external to the system of the ball and the Earth. Therefore, you add kinetic energy to the ball-Earth system.

$$W_{you} + W_g = \Delta K; \quad W_g = -\Delta U$$

$$W_{you} = W_{applied} = \Delta K + \Delta U$$

*Kinetic Friction: **Internal** non-conservative force that causes irreversible transformation of energy. The friction force causes the kinetic and potential energy to transfer to internal energy*

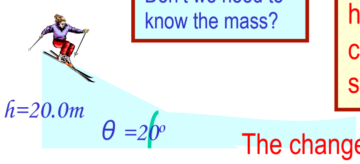
$$W_{friction} = \Delta K_{friction} = -f_k d$$

$$\Delta E = E_f - E_i = \Delta K + \Delta U = -f_k d$$

Example of Non-Conservative Force

A skier starts from rest at the top of frictionless hill whose vertical height is 20.0m and the inclination angle is 20°. Determine how far the skier can get on the snow at the bottom of the hill when the coefficient of kinetic friction between the ski and the snow is 0.210.

Don't we need to know the mass?



Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom

$$ME = mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.8 \times 20.0} = 19.8m/s$$

The change of kinetic energy is the same as the work done by the kinetic friction.

What does this mean in this problem?

Since we are interested in the distance the skier can get to before stopping, the friction must do as much work as the available kinetic energy to take it all away.

$$\Delta K = K_f - K_i = -f_k d$$

Since $K_f = 0$

$$-K_i = -f_k d; \quad f_k d = K_i$$

$$f_k = \mu_k n = \mu_k mg$$

$$d = \frac{K_i}{\mu_k mg} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g} = \frac{(19.8)^2}{2 \times 0.210 \times 9.80} = 95.2m$$

Well, it turns out we don't need to know the mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten starting from the same height.

How is the conservative force related to the potential energy?

Work done by a force component on an object through the displacement Δx is

$$W = F_x \Delta x = -\Delta U$$

For an infinitesimal displacement Δx

$$\lim_{\Delta x \to 0} \Delta U = -\lim_{\Delta x \to 0} F_x \Delta x$$

$$dU = -F_x dx$$

Results in the conservative force-potential E relationship

$$F_x = -\frac{dU}{dx}$$

This relationship says that any conservative force acting on an object within a given system is the same as the negative derivative of the potential energy of the system with respect to the position.

Does this statement make sense?

1. spring-ball system:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

2. Earth-ball system:

$$F_g = -\frac{dU_g}{dy} = -\frac{d}{dy}(mgy) = -mg$$

The relationship works in both the conservative force cases we have learned!!!

Energy Diagram and the Equilibrium of a System

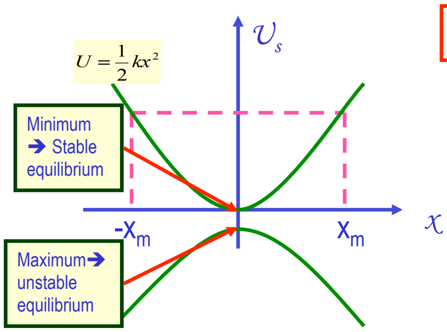
One can draw potential energy as a function of position ➔ Energy Diagram

Let's consider potential energy of a spring-ball system

$$U_s = \frac{1}{2}kx^2$$

What shape is this diagram?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 at the position.*
3. *x=0 is the stable equilibrium position of this system where the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new forms of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

*In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one to another. **The total energy of universe is constant as a function of time!!** **The total energy of the universe is conserved!***

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$E_R = mc^2$$

How many joules does your body correspond to?

The Gravitational Field

The gravitational force is a field force. The force exists everywhere in the universe.

If one were to place a test object of mass m at any point in the space in the existence of another object of mass M, the test object will feel the gravitational force exerted by M, $\vec{F}_g = m\vec{g}$.

Therefore the gravitational field g is defined as $\vec{g} = \frac{\vec{F}_g}{m}$

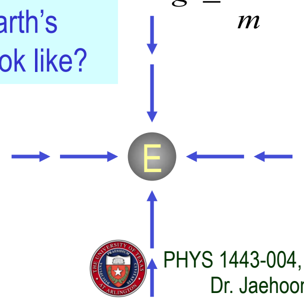
In other words, the gravitational field at a point in the space is the gravitational force experienced by a test particle placed at the point divided by the mass of the test particle.

$$\vec{g} = \frac{\vec{F}_g}{m} = -\frac{GM_E}{R_E^2}\hat{r}$$

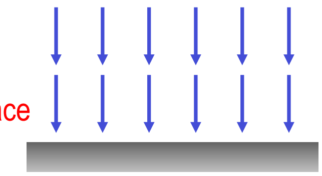
Where $\hat{r}$ is the unit vector pointing outward from the center of the Earth

So how does the Earth's gravitational field look like?

Far away from the Earth's surface



Close to the Earth's surface



The Gravitational Potential Energy

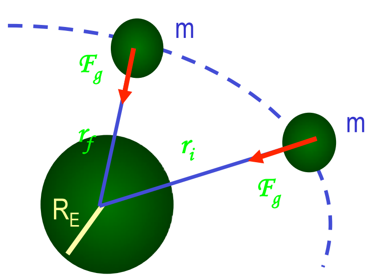
What is the potential energy of an object at the height y from the surface of the Earth?

$$U = mgy$$

Do you think this would work in general cases? No, it would not.

Why not? *Because this formula is only valid for the case where the gravitational force is constant, near the surface of the Earth, and the generalized gravitational force is inversely proportional to the square of the distance.*

OK. Then how would we generalize the potential energy in the gravitational field?



Since the gravitational force is a central force, and a central force is a conservative force, the work done by the gravitational force is independent of the path.

The path can be considered as consisting of many tangential and radial motions. Tangential motions do not contribute to work!!!

More on The Gravitational Potential Energy

Since the gravitational force is a radial force, it performs work only when the path has component in radial direction. Therefore, the work performed by the gravitational force that depends on the position becomes:

$$dW = \vec{F} \cdot d\vec{r} = F(r)dr \quad \text{For the whole path} \quad W = \int_{r_i}^{r_f} F(r)dr$$

Potential energy is the negative change of the work done through the path $\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$

Since the Earth's gravitational force is $F(r) = -\frac{GM_E m}{r^2}$

Thus the potential energy function becomes $U_f - U_i = \int_{r_i}^{r_f} \frac{GM_E m}{r^2} dr = -GM_E m\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$

Since only the difference of potential energy matters, by taking the infinite distance as the initial point of the potential energy, we obtain $U = -\frac{GM_E m}{r}$

For any two objects? $U = -\frac{Gm_1 m_2}{r}$ The energy needed to take the particles infinitely apart.

For many objects? $U = \sum_{i,j} U_{i,j}$

Example of Gravitational Potential Energy

A particle of mass m is displaced through a small vertical distance Δy near the Earth's surface. Show that in this situation the general expression for the change in gravitational potential energy is reduced to the $\Delta U = -mg\Delta y$.

Taking the general expression of gravitational potential energy

$$\Delta U = -GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Reorganizing the terms w/ the common denominator

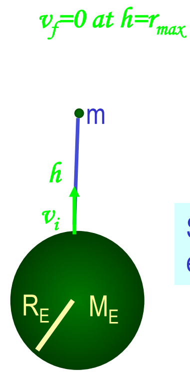
$$= -GM_E m\frac{(r_f - r_i)}{r_f r_i} = -GM_E m\frac{\Delta y}{r_f r_i}$$

Since the situation is close to the surface of the Earth $r_i \approx R_E$ and $r_f \approx R_E$

Therefore, ΔU becomes $\Delta U = -GM_E m\frac{\Delta y}{R_E^2}$

Since on the surface of the Earth the gravitational field is $g = \frac{GM_E}{R_E^2}$ The potential energy becomes $\Delta U = -mg\Delta y$

The Escape Speed



Consider an object of mass m is projected vertically from the surface of the Earth with an initial speed v_i and eventually comes to stop v_f=0 at the distance r_{max}.

Since the total mechanical energy is conserved

$$ME = K + U = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{\max}}$$

Solving the above equation for v_i, one obtains

$$v_i = \sqrt{2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{\max}}\right)}$$

Therefore if the initial speed v_i is known, one can use this formula to compute the final altitude h of the object.

$$h = r_{\max} - R_E = \frac{v_i^2 R_E^2}{2GM_E - v_i^2 R_E}$$

In order for an object to escape Earth's gravitational field completely without an additional acceleration, the initial speed needs to be

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 5.98\times 10^{24}}{6.37\times 10^{6}}}$$

$$= 1.12\times 10^{4}\, m/s = 11.2\, km/s$$

This is called the escape speed. This formula is valid for any planet or large mass objects.

How does this depend on the mass of the escaping object?

Independent of the mass of the escaping object