PHYS 1443 – Section 004
Lecture #18

Tuesday, Oct. 28, 2014

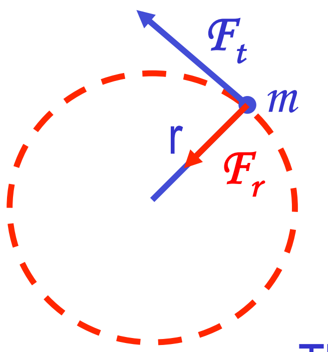
Dr. Jaehoon Yu

- Torque and Angular Acceleration
- Rolling Motion and Rotational Kinetic Energy
- Work, Power and Energy in Rotation
- Angular Momentum
- Angular Momentum Conservation
- Similarities Between Linear and Rotational Quantities

Announcements

- Mid-term exam results
  - Class average: 57.7/105
    - Equivalent to 55/100
    - Previous exam: 69.5/100
  - Top score: 99/105
- Grading scheme
  - Homework: 25%
  - Midterm and Final non-comprehensive exams: 19% each
  - One better of non comprehensive exam: 12%
  - Lab: 15%
  - Pop quizzes: 10%
  - Extra credit: 10%
- Mid-term grade discussion today
- Quiz coming this Thursday, Oct. 30; covers from CH10.2 to today's
  - Prepare your own formula sheet

Torque & Angular Acceleration



Let's consider a point object with mass m rotating on a horizontal circle.

What forces do you see in this motion?

The tangential force $\mathcal{F}_t$ and the radial force $\mathcal{F}_r$

The tangential force $\mathcal{F}_t$ is $F_t = ma_t = mr\alpha$

The torque due to tangential force $\mathcal{F}_t$ is $\tau = F_t r = ma_t r = mr^2\alpha = I\alpha$

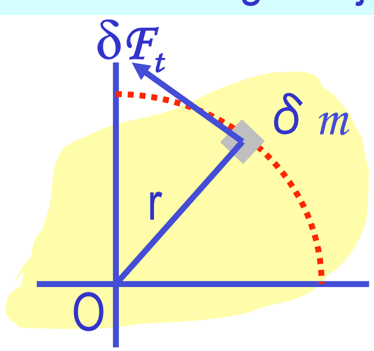
What do you see from the above relationship? $\tau = I\alpha$

What does this mean? The torque acting on a particle is proportional to the angular acceleration.

What law do you see from this relationship? Analogs to Newton's 2nd law of motion in rotation.

How about a rigid object?

The external tangential force $\delta\mathcal{F}_t$ is $\delta F_t = \delta m a_t = \delta m r\alpha$



The torque due to tangential force $\mathcal{F}_t$ is $\delta\tau = \delta F_t r = \left(r^2\delta m\right)\alpha$

The total torque is $\tau = \lim_{\delta\tau\to 0}\sum\delta\tau = \int d\tau = \alpha\lim_{\delta m\to 0}\sum r^2\delta m = \alpha\int r^2\,dm = I\alpha$

What is the contribution due to radial force and why?

Contribution from radial force is 0, because its line of action passes through the pivoting point, making the moment arm 0.

Rolling Motion of a Rigid Body

What is a rolling motion?

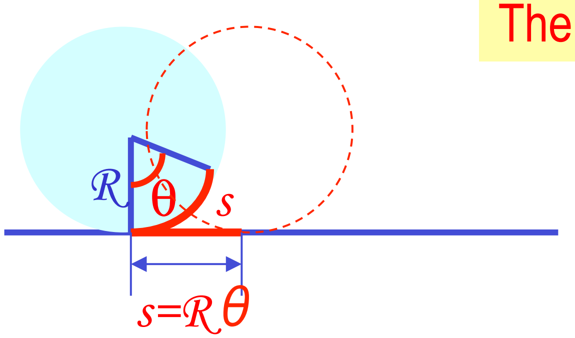
A more generalized case of a motion where the rotational axis moves together with an object

A rotational motion about a moving axis

To simplify the discussion, let's make a few assumptions

1. Limit our discussion on very symmetric objects, such as cylinders, spheres, etc
2. The object rolls on a flat surface

Let's consider a cylinder rolling on a flat surface, without slipping.



Under what condition does this "Pure Rolling" happen?

The total linear distance the CM of the cylinder moved is $s = R\theta$

Thus the linear speed of the CM is

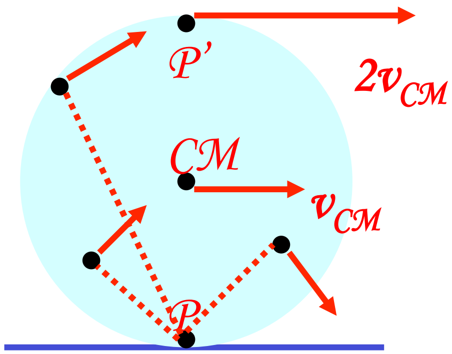
$$\bar{v}_{CM} = \frac{\Delta s}{\Delta t} = R\frac{\Delta\theta}{\Delta t} = R\omega$$

The condition for a "Pure Rolling motion"

More Rolling Motion of a Rigid Body

The magnitude of the linear acceleration of the CM is

$$a_{CM} = \frac{dv_{CM}}{dt} = R\frac{d\omega}{dt} = R\alpha$$



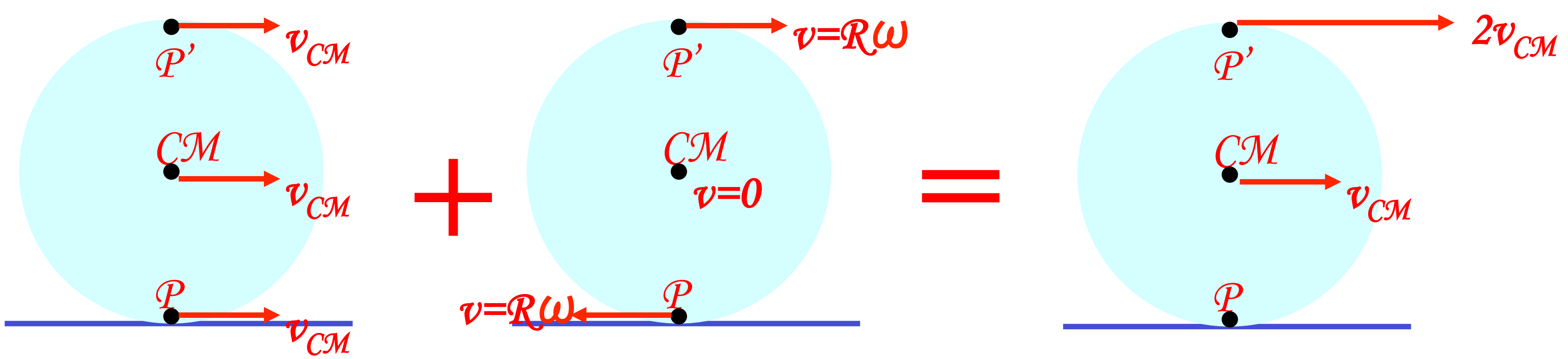
As we learned in rotational motion, all points in a rigid body moves at the same angular speed but at different linear speeds.

CM is moving at the same speed at all times.

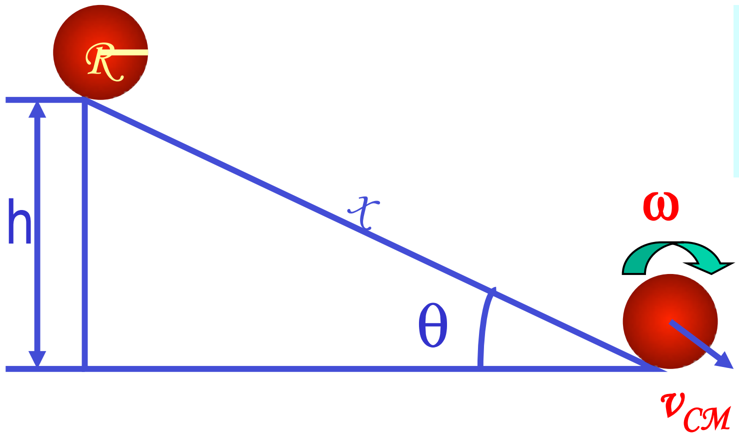
At any given time, the point that comes to P has 0 linear speed while the point at P' has twice the speed of CM

Why??

A rolling motion can be interpreted as the sum of Translation and Rotation



Kinetic Energy of a Rolling Sphere



Let's consider a sphere with radius R rolling down the hill without slipping.

$$K = \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}MR^2\omega^2$$

$$= \frac{1}{2}I_{CM}\left(\frac{v_{CM}}{R}\right)^2 + \frac{1}{2}Mv_{CM}^2$$

$$= \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right)v_{CM}^2$$

Since $v_{CM} = R\omega$

What is the speed of the CM in terms of known quantities and how do you find this out?

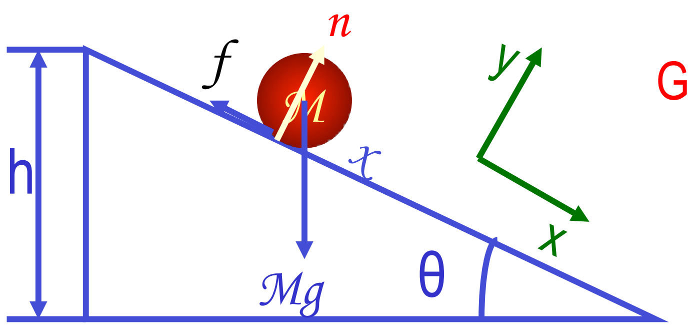
Since the kinetic energy at the bottom of the hill must be equal to the potential energy at the top of the hill

$$K = \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right)v_{CM}^2 = Mgh$$

$$v_{CM} = \sqrt{\frac{2gh}{1 + I_{CM}/MR^2}}$$

Ex: Rolling Kinetic Energy

For a solid sphere as shown in the figure, calculate the linear speed of the CM at the bottom of the hill and the magnitude of linear acceleration of the CM. Solve this problem using Newton's second law, the dynamic method.



What are the forces involved in this motion?

Gravitational Force, Frictional Force, Normal Force

Newton's second law applied to the CM gives

$$\sum F_x = Mg\sin\theta - f = Ma_{CM}$$

$$\sum F_y = n - Mg\cos\theta = 0$$

Since the forces Mg and **n** go through the CM, their moment arm is 0 and do not contribute to torque, while the static friction f causes torque

$$\tau_{CM} = fR = I_{CM}\alpha$$

We know that

$$I_{CM} = \frac{2}{5}MR^2$$

$$a_{CM} = R\alpha$$

We obtain

$$f = \frac{I_{CM}\alpha}{R} = \frac{\frac{2}{5}MR^2}{R}\left(\frac{a_{CM}}{R}\right) = \frac{2}{5}Ma_{CM}$$

Substituting f in dynamic equations

$$Mg\sin\theta = \frac{7}{5}Ma_{CM} \Rightarrow a_{CM} = \frac{5}{7}g\sin\theta$$