PHYS 1443 – Section 004
Lecture #24

Tuesday, Nov. 25, 2014

Dr. Jaehoon Yu

- Refresher: Simple Block-Spring system
- Energy of the Simple Harmonic Oscillator
- Pendulum
    - Simple Pendulum
    - Physical Pendulum
    - Torsion Pendulum
- SHM & Uniform Circular Motion
- Superposition and Interference

Today's homework is homework #13, due 11pm, Tuesday, Dec. 2!!

Announcements

- Final comprehensive exam
  - In this room, 9:00 – 10:30am, Thursday, Dec. 11 in this room
  - Covers CH 1.1 through what we finish Tuesday, Dec. 2 plus the math refresher
  - Mixture of multiple choice and free response problems
  - Bring your calculator but DO NOT input formula into it!
    - Your phones or portable computers are NOT allowed as a replacement!
  - You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
    - None of the parts of the solutions of any problems
    - No derived formulae, derivations of equations or word definitions!
  - Do NOT Miss the exam!
- Happy Thanksgiving!

Refresher: Simple Block-Spring System

A block attached at the end of a spring on a frictionless surface experiences acceleration when the spring is displaced from an equilibrium position.

$$F_{spring} = ma$$
$$= -kx$$
$$a = -\frac{k}{m}x$$

This becomes a second order differential equation $\frac{d^2x}{dt^2} = -\frac{k}{m}x$ If we denote $\omega^2 = \frac{k}{m}$

The resulting differential equation becomes $$\frac{d^2x}{dt^2} = -\omega^2 x$$

Since this satisfies condition for simple harmonic motion, we can take the solution $$\boxed{x = A\cos(\omega t + \varphi)}$$

Does this solution satisfy the differential equation?

Let's take derivatives with respect to time $\frac{dx}{dt} = A\frac{d}{dt}(\cos(\omega t + \varphi)) = -\omega A\sin(\omega t + \varphi)$

Now the second order derivative becomes

$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}(\sin(\omega t + \varphi)) = -\omega^2 A\cos(\omega t + \varphi) = -\omega^2 x$$

Whenever the force acting on an object is linearly proportional to the displacement from some equilibrium position and is in the opposite direction, the particle moves in simple harmonic motion.

Refresher: Simple Block-Spring System

How do the period and frequency of this harmonic motion look?

Since the angular frequency ϖ is $\omega = \sqrt{\frac{k}{m}}$

The period, T, becomes $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

So the frequency is $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

What can we learn from these?

- Frequency and period do not depend on amplitude
- Period is inversely proportional to spring constant and proportional to mass

Special case #1

Let's consider that the spring is stretched to a distance A, and the block is let go from rest, giving 0 initial speed; x_i=A, v_i=0,

$$x = A\cos\omega t \quad v = \frac{dx}{dt} = -\omega A\sin\omega t \quad a = \frac{d^2x}{dt^2} = -\omega^2 A\cos\omega t \quad a_i = -\omega^2 A = -kA/m$$

This equation of motion satisfies all the SHM conditions. So it is the solution for this motion.

Special case #2

Suppose a block is given non-zero initial velocity v_i to positive x at the instant it is at the equilibrium, x_i=0

$$\varphi = \tan^{-1}\left(-\frac{v_i}{\omega x_i}\right) = \tan^{-1}(-\infty) = -\frac{\pi}{2} \quad x = A\cos\left(\omega t - \frac{\pi}{2}\right) = A\sin(\omega t)$$

Is this a good solution?

Example for Spring Block System

A car with a mass of 1300kg is constructed so that its frame is supported by four springs. Each spring has a force constant of 20,000N/m. If two people riding in the car have a combined mass of 160kg, find the frequency of vibration of the car after it is driven over a pothole in the road.

Let's assume that mass is evenly distributed to all four springs.

The total mass of the system is 1460kg.

Therefore each spring supports 365kg each.

From the frequency relationship based on Hook's law

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Thus the frequency for vibration of each spring is

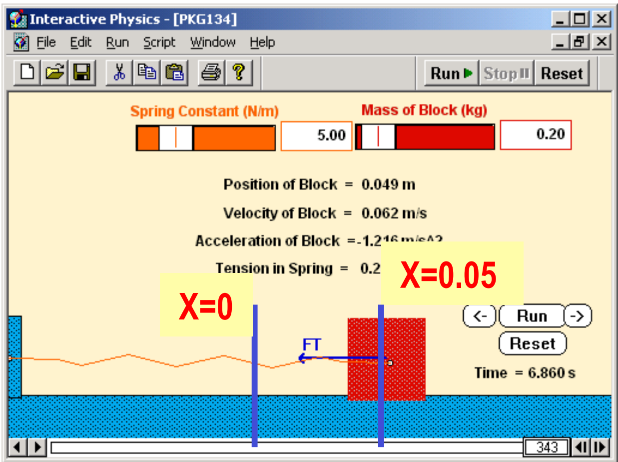
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{20000}{365}} = 1.18s^{-1} = 1.18Hz$$

How long does it take for the car to complete two full vibrations?

The period is $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} = 0.849s$ For two cycles $2T = 1.70s$

Example for Spring Block System

A block with a mass of 200g is connected to a light spring for which the force constant is 5.00 N/m and is free to oscillate on a horizontal, frictionless surface. The block is displaced 5.00 cm from equilibrium and released from reset. Find the period of its motion.



From the Hook’s law, we obtain

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00}{0.20}} = 5.00 s^{-1}$$

As we know, period does not depend on the amplitude or phase constant of the oscillation, therefore the period, T, is simply

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = 1.26 s$$

Determine the maximum speed of the block.

From the general expression of the simple harmonic motion, the speed is

$$v_{\max} = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi)$$

$$= \omega A = 5.00 \times 0.05 = 0.25 m/s$$

Energy of the Simple Harmonic Oscillator

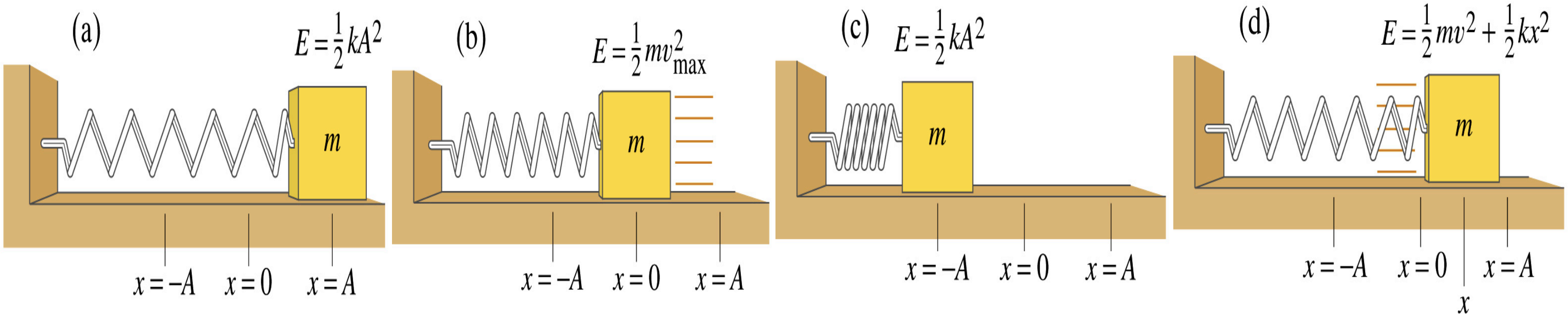
What do you think the mechanical energy of the harmonic oscillator look without friction?

Kinetic energy of a harmonic oscillator is $$KE=\frac{1}{2}mv^2=\frac{1}{2}m\omega^2A^2\sin^2(\omega t+\varphi)$$

The elastic potential energy stored in the spring $$PE=\frac{1}{2}kx^2=\frac{1}{2}kA^2\cos^2(\omega t+\varphi)$$

Therefore the total mechanical energy of the harmonic oscillator is $$E=KE+PE=\frac{1}{2}\left[m\omega^2A^2\sin^2(\omega t+\varphi)+kA^2\cos^2(\omega t+\varphi)\right]$$

Since $\omega=\sqrt{k/m}$ $$E=KE+PE=\frac{1}{2}\left[kA^2\sin^2(\omega t+\varphi)+kA^2\cos^2(\omega t+\varphi)\right]=\frac{1}{2}kA^2$$



Total mechanical energy of a simple harmonic oscillator is proportional to the square of the amplitude.

Energy of the Simple Harmonic Oscillator cont'd

Maximum KE is when PE=0

$$KE_{\max} = \frac{1}{2} m v_{\max}^2 = \frac{1}{2} k A^2$$

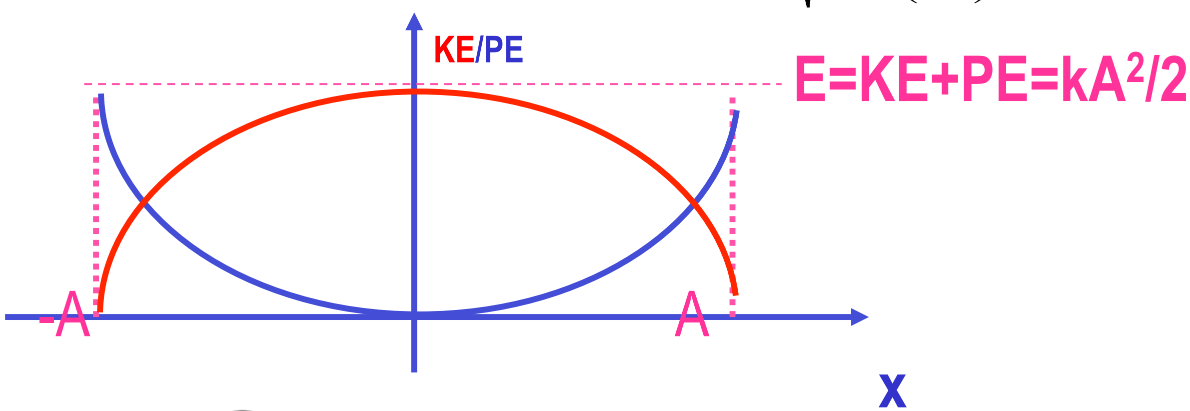
Maximum speed

$$v_{\max} = \sqrt{\frac{k}{m}} A$$

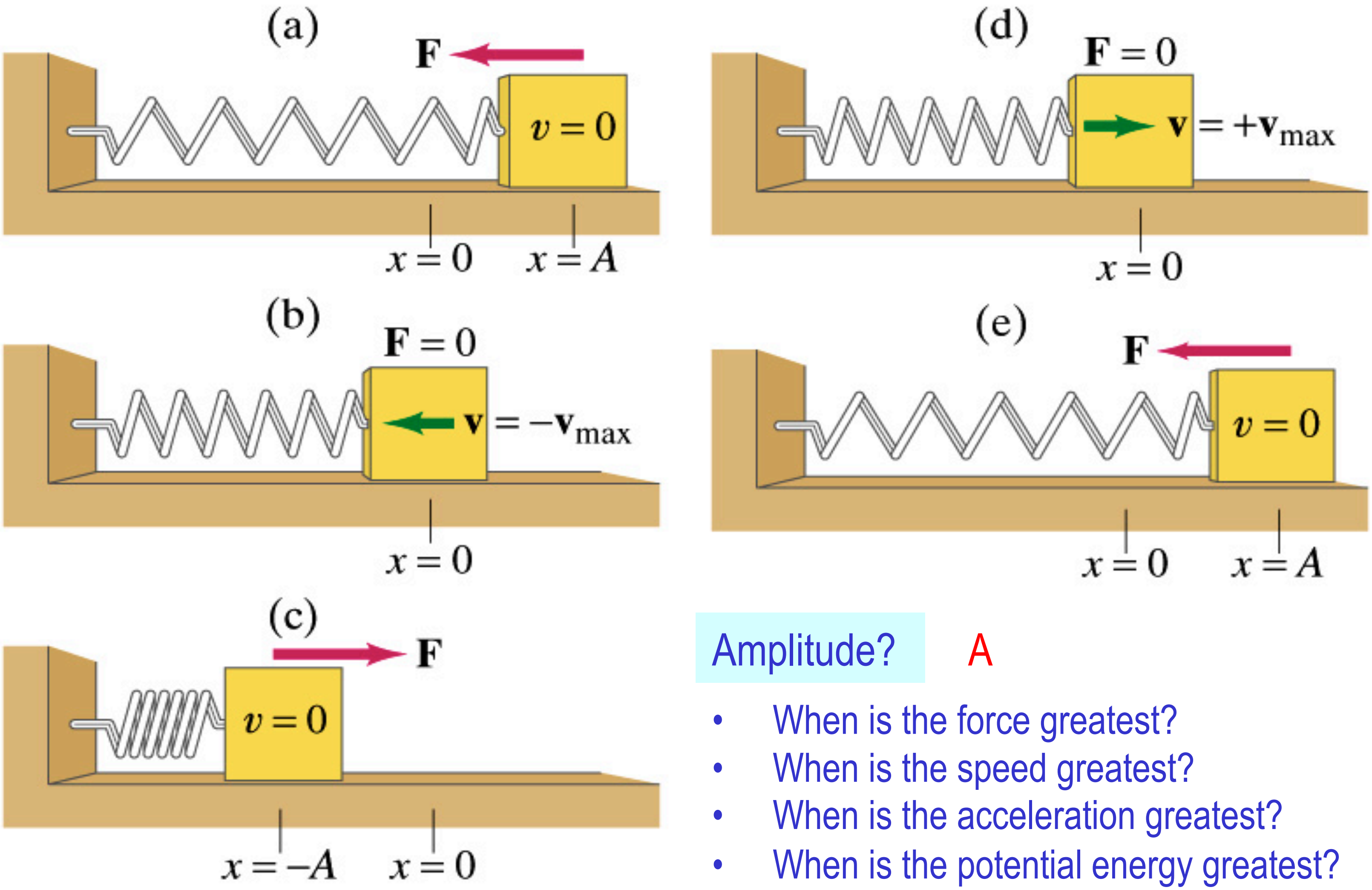
The speed at any given point of the oscillation

$$E = KE + PE = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$v = \sqrt{k/m\left(A^2 - x^2\right)} = v_{\max}\sqrt{1 - \left(\frac{x}{A}\right)^2}$$



Oscillation Properties



Amplitude? A

- When is the force greatest?
- When is the speed greatest?
- When is the acceleration greatest?
- When is the potential energy greatest?
- When is the kinetic energy greatest?

Example for Energy of Simple Harmonic Oscillator

A 0.500kg cube connected to a light spring for which the force constant is 20.0 N/m oscillates on a horizontal, frictionless track. a) Calculate the total energy of the system and the maximum speed of the cube if the amplitude of the motion is 3.00 cm.

From the problem statement, A and k are

$$k = 20.0 N/m$$
$$A = 3.00 cm = 0.03 m$$

The total energy of the cube is

$$E = KE + PE = \frac{1}{2}kA^2 = \frac{1}{2}(20.0)\times(0.03)^2 = 9.00\times 10^{-3} J$$

Maximum speed occurs when kinetic energy is the same as the total energy

$$KE_{\max} = \frac{1}{2}mv_{\max}^2 = E = \frac{1}{2}kA^2$$
$$v_{\max} = A\sqrt{\frac{k}{m}} = 0.03\sqrt{\frac{20.0}{0.500}} = 0.190 m/s$$

b) What is the velocity of the cube when the displacement is 2.00 cm.

velocity at any given displacement is

$$v = \sqrt{k/m\left(A^2 - x^2\right)} = \sqrt{20.0\cdot\left(0.03^2 - 0.02^2\right)/0.500} = 0.141 m/s$$

c) Compute the kinetic and potential energies of the system when the displacement is 2.00 cm.

Kinetic energy, KE

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}0.500\times(0.141)^2 = 4.97\times 10^{-3} J$$

Potential energy, PE

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}20.0\times(0.02)^2 = 4.00\times 10^{-3} J$$