PHYS 3446 – Lecture #3

Wednesday, Sept 7, 2016

Dr. Jae Yu

1. Rutherford Scattering
2. Rutherford Scattering with Coulomb force
3. Scattering Cross Section
4. Measurement of Cross Sections

Announcements

- 1st colloquium at 4pm today
  - UTA Physics faculty expo
- Physics department picnic
  - 12 – 3pm, Sat. Sept. 10
  - 1st Floor MAC

Reminder: Homework Assignment #1

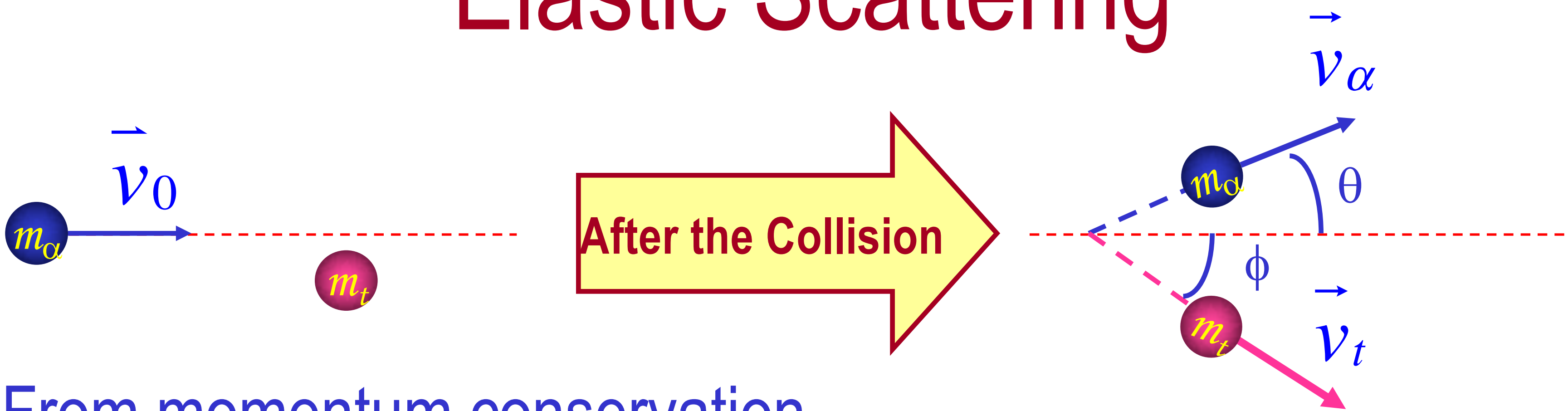
1. Compute the masses of electron, proton and alpha particles in MeV/c^2, using $E=mc^2$. (9 points)
   - Need to look up and specify the masses of electrons, protons and alpha particles in kg on your paper.
2. Compute the gravitational and the Coulomb forces between two protons separated by 10^{-10}m and compare their strengths (15)
3. Derive the following equations in your book:
   - Eq. # 1.3 (5 pts) , 1.17 (8 pts), 1.32 (12 pts)
   - Must show detailed work and accompany explanations
   - Copying the book or your friend will result in no credit for both of you!

- These assignments are due coming Monday, Sept. 12.

Rutherford Scattering

- A fixed target experiment with alpha particle as projectile shot on thin gold foil
  - Alpha particle's energy is low ➔ Speed is well below 0.1c (non-relativistic)
- An elastic scattering of the particles
- What are the conserved quantities in an elastic scattering?
  - Momentum
  - Kinetic Energy

Elastic Scattering



- From momentum conservation

$$\vec{v}_0 = \frac{m_\alpha \vec{v}_\alpha + m_t \vec{v}_t}{m_\alpha} = \vec{v}_\alpha + \frac{m_t \vec{v}_t}{m_\alpha}$$

- From the kinetic energy conservation

$$\frac{1}{2} m_\alpha v_0^2 = \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_t v_t^2 \Rightarrow \boxed{v_0^2 = v_\alpha^2 + \frac{m_t}{m_\alpha} v_t^2}$$

- From these two, we obtain

$$\boxed{v_t^2 \left(1 - \frac{m_t}{m_\alpha}\right) = 2 \vec{v}_\alpha \cdot \vec{v}_t}$$

Analysis Case I

$$v_t^2\left(1-\frac{m_t}{m_\alpha}\right)=2\vec{v}_\alpha\cdot\vec{v}_t$$

- If $m_t \ll m_\alpha$,
  - left-hand side becomes positive
  - v_α and v_t must be in the same hemisphere
  - Using the actual masses
  - $m_e = 0.5\,MeV/c^2$ and $m_\alpha = 4\times10^3\,MeV/c^2$
  - We obtain $v_e = v_t \le 2v_\alpha$
  - If $m_t=m_e$, then $m_t/m_\alpha \sim 10^{-4}$. ➔ $v_\alpha \approx v_0$
  - ➔ $m_e v_e = m_\alpha\left(m_e/m_\alpha\right)v_e \le 2\times10^{-4}\,m_\alpha v_\alpha = 2\times10^{-4}\,m_\alpha v_0$
  - Thus, momentum transfer to target is $p_e/p_{\alpha0}<10^{-4}$.
  - Change of momentum of alpha particle is negligible

Analysis Case II

$$v_t^2\left(1-\frac{m_t}{m_\alpha}\right)=2\vec{v}_\alpha\cdot\vec{v}_t$$

- If $m_t >> m_\alpha$,
  - left-hand side of the above becomes negative
  - v_α and v_t are in opposite hemisphere
  - Using the actual masses
  - $m_t \approx m_{Au} \approx 2\times10^5\ MeV/c^2$ and $m_\alpha = 4\times10^3\ MeV/c^2$
  - We obtain $v_t \le 2m_\alpha v_\alpha/m_t$ so v_t is small
  - If $m_t=m_{Au}$, then m_t/m_α~50. ➔ $v_\alpha \approx v_0$
  - ➔ $m_t v_t \le 2m_\alpha v_\alpha \approx 2m_\alpha v_0$
  - Thus, $p_e/p_{\alpha0}$ could be as large as $2p_{\alpha0}$.
  - Change of momentum of alpha particle is large
    - α particle can even recoil

Rutherford Scattering with EM Force 1

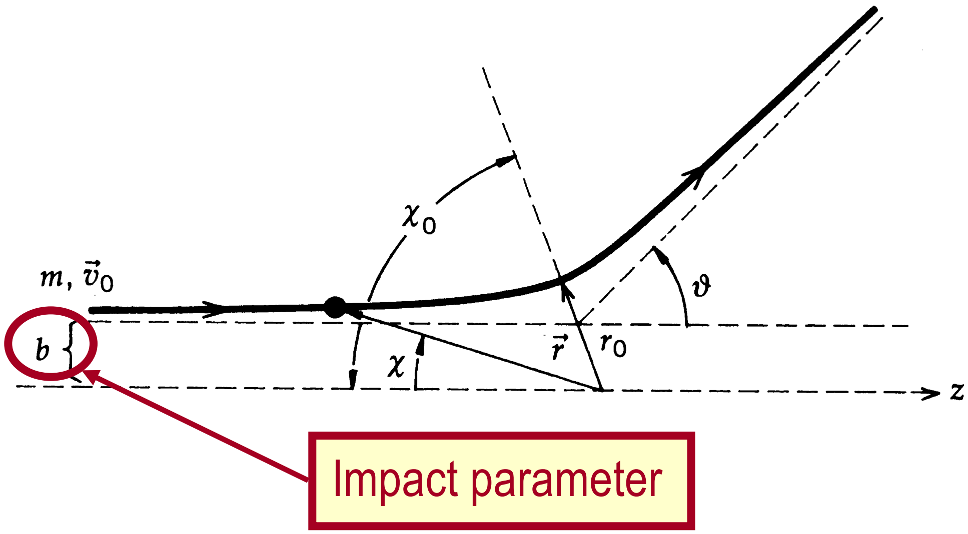
- Let's take into account only the EM force between the α and the atom
- Coulomb force is a central force, so a conservative force
- Coulomb potential between particles with Ze and Z'e electrical charge separated by distance r is

$$V(r) = \frac{ZZ'e^2}{r}$$

- Since the total mechanical energy is conserved,

$$E = \frac{1}{2}mv_0^2 = \text{constant} > 0 \Rightarrow \mathrm{v}_0 = \sqrt{\frac{2E}{m}}$$

Rutherford Scattering with EM Force 2



- The distance vector **r** is always the same direction as the force throughout the entire motion, so the net torque (**r**x**F**) is 0.
- Since there is no net torque, the angular momentum (**l=rxp**) is conserved. ➔ The magnitude of the angular momentum is ***l=mv₀b***.

- From the energy relation, we obtain

$$l = m\sqrt{2E/m}\,b = b\sqrt{2mE} \Rightarrow b^2 = l^2/2mE$$

- From the definition of angular momentum, we obtain an equation of motion $d\chi/dt = l/mr^2$
- From energy conservation, we obtain another equation of motion

$$E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}mr^2\left(\frac{d\chi}{dt}\right)^2 + V(r) \Rightarrow \frac{dr}{dt} = \pm\sqrt{\frac{2}{m}\left(E - V(r) - \frac{l^2}{2mr^2}\right)}$$

Centrifugal barrier: $\frac{l^2}{2mr^2}$

Effective potential: $V(r) + \frac{l^2}{2mr^2}$

Rutherford Scattering with EM Force 3

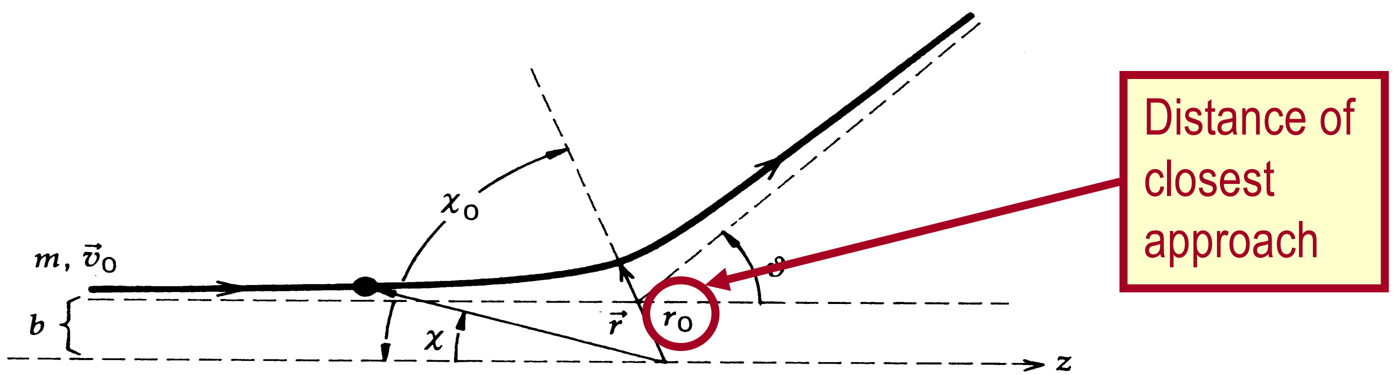
- Rearranging the terms for approach, we obtain

$$\frac{dr}{dt} = -\frac{l}{mrb}\sqrt{r^2\left(1-\frac{V(r)}{E}\right)-b^2}$$

- and

$$d\chi = -\frac{b\,dr}{r\left[r^2\left(1-\frac{V(r)}{E}\right)-b^2\right]^{1/2}}$$

- Integrating this from r_0 to infinity gives the angular distribution of the outgoing alpha particle



Rutherford Scattering with EM Force 4

- What happens at the DCA?
  - Kinetic energy reduces to 0. $$\left.\frac{dr}{dt}\right|_{r=r_0} = 0$$
  - The incident alpha could turn around and accelerate
  - We can obtain $$r^2\left(1-\frac{V(r)}{E}\right)-b^2=0$$
  - This allows us to determine DCA for a given potential and χ_0.
- Define scattering angle θ as the changes in the asymptotic angles of the trajectory, we obtain

$$\theta = \pi - 2\chi_0 = \pi - 2b\int_{r_0}^{\infty}\frac{dr}{r\left[r^2\left(1-\frac{V(r)}{E}\right)-b^2\right]^{1/2}}$$

Rutherford Scattering with EM Force 5

- For a Coulomb potential

$$V(r)=\frac{ZZ'e^2}{r}$$

- DCA can be obtained for the given impact parameter b,

$$r_0=\frac{ZZ'e^2/E}{2}\left(1+\sqrt{1+4b^2E^2/\left(ZZ'e^2\right)^2}\right)$$

- And the angular distribution becomes

$$\theta=\pi-2b\int_{r_0}^{+\infty}\frac{dr}{r\left[r^2\left(1-\frac{V(r)}{E}\right)-b^2\right]^{1/2}}$$

Rutherford Scattering with EM Force 6

- Replace the variable 1/r=x, and performing the integration, we obtain

$$\theta = \pi + 2\cos^{-1}\left(\frac{1}{\sqrt{1 + 4b^2E^2 / \left(ZZ'e^2\right)^2}}\right)$$

- This can be rewritten

$$\frac{1}{\sqrt{1 + 4b^2E^2 / \left(ZZ'e^2\right)^2}} = \cos\left(\frac{\theta - \pi}{2}\right)$$

- Solving this for b, we obtain

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

Rutherford Scattering with EM Force 7

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2} \Rightarrow \tan\frac{\theta}{2} = \frac{ZZ'e^2}{2bE}$$

- From the solution for b, we can learn the following
  1. For fixed b, E and Z'
     - The scattering angle is larger for a larger value of Z.
       - Makes perfect sense since Coulomb potential is stronger with larger Z.
       - Results in larger deflection.
  2. For fixed b, Z and Z'
     - The scattering angle is larger when E is smaller.
       - If particle has low energy, its velocity is smaller
       - Spends more time in the potential, suffering greater deflection
  3. For fixed Z, Z', and E
     - The scattering angle is larger for smaller impact parameter b
       - Makes perfect sense also, since as the incident particle is closer to the nucleus, it feels stronger Coulomb force.

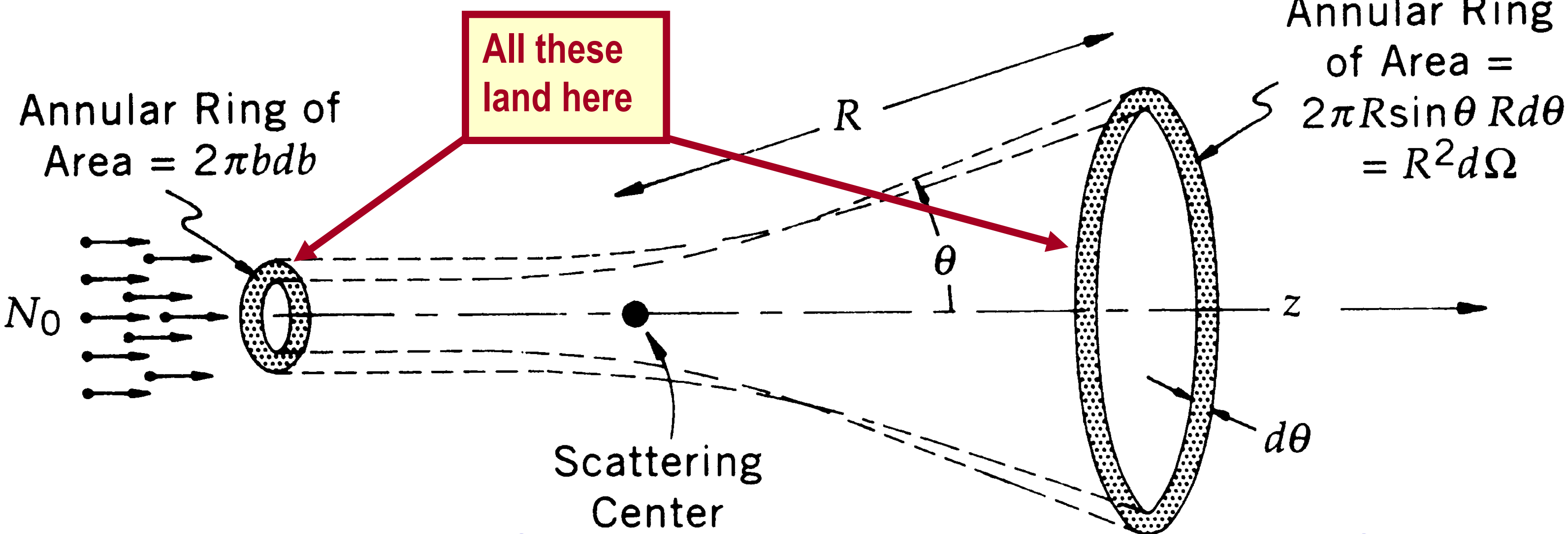
What do we learn from scattering?

- Scattering of a particle in a potential is completely determined when we know both
  - The impact parameter, b, and
  - The energy of the incident particle, E

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- For a fixed energy, the deflection is defined by
  - The impact parameter, b.
- What do we need to perform a scattering experiment?
  - Incident flux of beam particles with known E
  - A device that can measure the number of scattered particles at various scattering angle, θ.
  - Measurements of the number of scattered particles reflect
    - Impact parameters of the incident particles
    - The effective size of the scattering center
- By measuring the scattering angle θ, we can learn about the potential or the force between the target and the projectile

Scattering Cross Section



- N_0: The number of particles incident on the target foil per unit area per unit time.
- Any incident particles entering with impact parameter b and b+db will scatter to the angle θ and θ-dθ.
- In other words, they scatter into the solid angle dΩ ($=2\pi\sin\theta d\theta$).
- So the number of particles scattered into the solid angle dΩ per unit time is $2\pi N_0 bdb$.

Scattering Cross Section

- For a central potential
  - Such as Coulomb potential
  - Which has spherical symmetry
- The scattering center presents an effective transverse x-sectional area of

$$\Delta\sigma = 2\pi b db$$

- For the particles to scatter into θ and $\theta+d\theta$

Scattering Cross Section

- In more generalized cases, $\Delta\sigma$ depends on both θ & ϕ.

$$\Delta\sigma(\theta,\phi) = bdbd\phi = -\frac{d\sigma}{d\Omega}(\theta,\phi)d\Omega = -\frac{d\sigma}{d\Omega}(\theta,\phi)\sin\theta d\theta d\phi$$

Why negative?

Since the deflection and the change of b are in opposite direction!!

- With a spherical symmetry, ϕ can be integrated out:

$$\Delta\sigma(\theta) = \frac{d\sigma}{d\Omega}(\theta)2\pi\sin\theta d\theta = 2\pi bdb$$

reorganize

$$\frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin\theta}\frac{db}{d\theta}$$

Differential Cross Section

What is the dimension of the differential cross section?

Area!!

Scattering Cross Section

- For a central potential, measuring the yield as a function of θ, or the differential cross section, is equivalent to measuring the entire effect of the scattering
- So what is the physical meaning of the differential cross section?

$\Rightarrow$ Measurement of yield as a function of specific experimental variable

$\Rightarrow$ This is equivalent to measuring the probability of occurrence of a physical process in a specific kinematic phase space

- Cross sections are measured in the unit of barns:

$$1\,\text{barn} = 10^{-24}\,\text{cm}^2$$

Where does this come from?

Cross sectional area of a uranium nucleus!

couldn't hit the broad side of a barn!!