PHYS 3446 – Lecture #4

Monday, Sept 12, 2016
*Dr. **Jae** Yu*

1. Scattering Cross Section
2. Total Cross Section
3. Measurement of Cross Sections

Announcements

- Colloquium at 4pm this Wednesday
  - UTA Physics faculty expo II
- A special lecture this Wednesday
  - Dr. Ben Jones

Homework Assignment #2

1. Plot the differential cross section of the Rutherford scattering as a function of the scattering angle θ with some sensible lower limit of the angle + express your opinion on the sensibility of the cross section, along with good physical reasons (10points)
2. Compute the total cross section of the Rutherford scattering in unit of barns to the cut-off angle of your choice above (5 points)
3. Make a list of tasks, goals and milestones to accomplish each of the projects
   – This list should be written up and presented by each group at the beginning of the class on Monday Sept. 19
   – Send me power point slides no later than 10pm Sunday, Sept. 18

• Due for this homework is Monday Sept. 19

Scattering Cross Section

- For a central potential, measuring the yield as a function of θ, the differential cross section, is equivalent to measuring the entire effect of the scattering
- So what is the physical meaning of the differential cross section?

$\Rightarrow$ Measurement of yield as a function of specific experimental variable

$\Rightarrow$ This is equivalent to measuring the probability of occurrence of a physical process in a specific kinematic phase space

- Cross sections are measured in the unit of barns:

$$1\,\mathrm{barn} = 10^{-24}\,\mathrm{cm}^2$$

Where does this come from?

Cross sectional area of a uranium nucleus!

Total Cross Section

- Total cross section is the integration of the differential cross section over the entire solid angle, Ω:

$$\sigma_{Total} = \int_0^{4\pi} \frac{d\sigma}{d\Omega}(\theta,\phi) d\Omega = 2\pi \int_0^{2\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}(\theta)$$

- Total cross section represents the effective size of the scattering center at all possible impact parameter

Cross Section of Rutherford Scattering

- The impact parameter in Rutherford scattering is

$$b = \frac{ZZ'e^2}{2E}\cot\frac{\theta}{2}$$

- Thus,

$$\frac{db}{d\theta} = -\frac{1}{2}\frac{ZZ'e^2}{2E}\operatorname{cosec}^2\frac{\theta}{2}$$

- Differential cross section of Rutherford scattering is

$$\frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin\theta}\frac{db}{d\theta} = \frac{1}{4}\left(\frac{ZZ'e^2}{2E}\right)^2\operatorname{cosec}^4\frac{\theta}{2} = \frac{1}{4}\left(\frac{ZZ'e^2}{2E}\right)^2\frac{1}{\sin^4\frac{\theta}{2}}$$

Rutherford Scattering Cross Section

- Let's plug in the numbers
  - $Z_{Au}=79$
  - $Z_{He}=2$
  - For E=10keV
- Differential cross section of Rutherford scattering

$$\frac{d\sigma}{d\Omega}(\theta)=\frac{1}{4}\left(\frac{ZZ'e^2}{2E}\right)^2\frac{1}{\sin^4\frac{\theta}{2}}=\left(\frac{79\cdot 2\cdot\left(1.6\times 10^{-19}\right)^2}{2\cdot 10\times 10^3\times 1.6\times 10^{-19}}\right)^2\frac{1}{\sin^4\frac{\theta}{2}}=$$

$$=\frac{4.0\times 10^{-43}}{\sin^4\frac{\theta}{2}}\left(m^2\right)=\frac{4.0\times 10^{-39}}{\sin^4\frac{\theta}{2}}\left(cm^2\right)=\frac{4.0\times 10^{-15}}{\sin^4\frac{\theta}{2}}\left(barns\right)$$

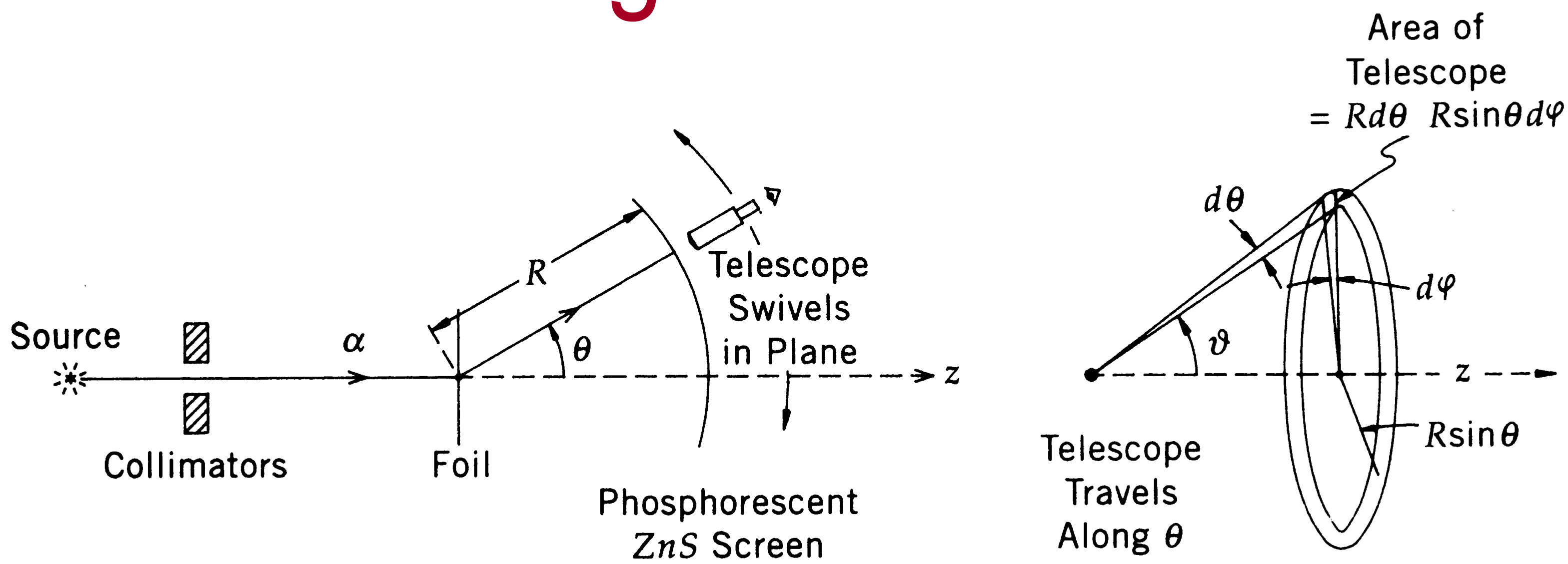
Total X-Section of Rutherford Scattering

- To obtain the total cross section of Rutherford scattering, one integrates the differential cross section over all θ:

$$\sigma_{Total} = 2\pi \int_0^{\pi} \frac{d\sigma}{d\Omega}(\theta) \sin\theta d\theta = 8\pi \left(\frac{ZZ'e^2}{2E} \right)^2 \int_0^1 d\left(\sin\frac{\theta}{2} \right) \frac{1}{\sin^3\frac{\theta}{2}}$$

- What is the result of this integration?
  - Infinity!!
- Does this make sense?
  - Yes
- Why?
  - Since the Coulomb force's range is infinite.
- Is this physically meaningful?
  - No
- What would be the sensible thing to do?
  - Integrate to a cut-off angle since after certain distance the force is too weak to impact the scattering. ($\theta=\theta_0>0$)

Measuring Cross Sections



- Rutherford scattering experiment
  - Used a collimated beam of α particles emitted from Radon
  - A thin Au foil target
  - A scintillating glass screen with ZnS phosphor deposit
  - Telescope to view limited area of solid angle
  - Telescope only need to move along θ not ϕ. Why?
    - Due to the spherical symmetry, scattering only depends on θ not ϕ.

Measuring Cross Sections

- With the incident flux of N_0 per unit area per second
- Any α particles in b and b+db will be scattered into θ and θ-dθ
- The telescope aperture limits the measurable area to

$$A_{Tele} = Rd\theta \cdot R\sin\theta d\phi = R^2 d\Omega$$

- How could they have increased the rate of measurement?
  - By constructing an annular telescope
  - By how much would it increase?

$\boxed{2\pi/d\phi}$

Measuring Cross Sections

- Fraction of incident particles approaching the target in the small area $\Delta\sigma = b^2 d\phi d\theta$ at the impact parameter b is $-dn/N_0$.
  - dn particles scatter into $R^2 d\Omega$, the aperture of the telescope
- This ratio is the same as
  - The sum of $\Delta\sigma$ over all N nuclear centers in the entire foil divided by the total area (S) of the foil.
  - Probability for incident particles to enter within the N areas of the annular rings and subsequently scatter into the telescope aperture
- So this ratio can be expressed as

$$-\frac{dn}{N_0} = \frac{N}{S}\Delta\sigma(\theta,\phi) = \frac{Nb^2 d\phi d\theta}{S}$$

Measuring Cross Sections

- For a foil with thickness t, mass density ρ, atomic weight A:

$$N = \frac{\rho t S}{A} A_0$$

A_0: Avogadro's number of atoms per mol

- Since from what we have learned previously

$$-\frac{dn}{N_0} = \frac{N b^2 d\phi d\theta}{S}$$

- The number of α scattered into the detector angle (θ,ϕ) is

$$dn = \frac{N_0 \rho t A_0}{A} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega = N_0 \frac{N}{S} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega$$

Measuring Cross Sections

Detector acceptance

$$dn = \frac{N_0 \rho t A_0}{A}\frac{d\sigma(\theta,\phi)}{d\Omega}\,d\Omega = N_0 \frac{N}{S}\frac{d\sigma(\theta,\phi)}{d\Omega}\,d\Omega$$

Number of detected particles

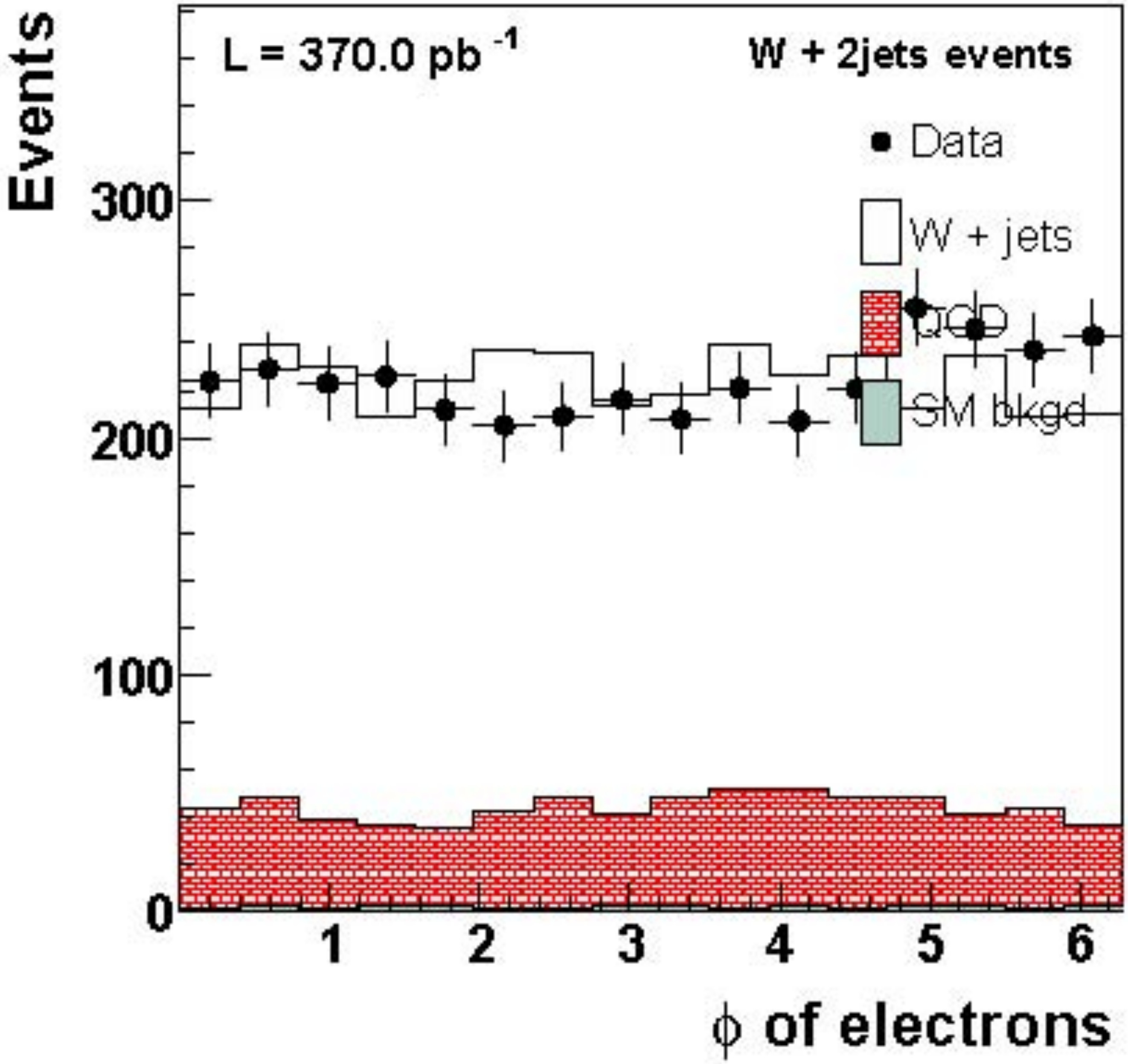
Projectile particle flux

Density of the target particles

Scattering cross section

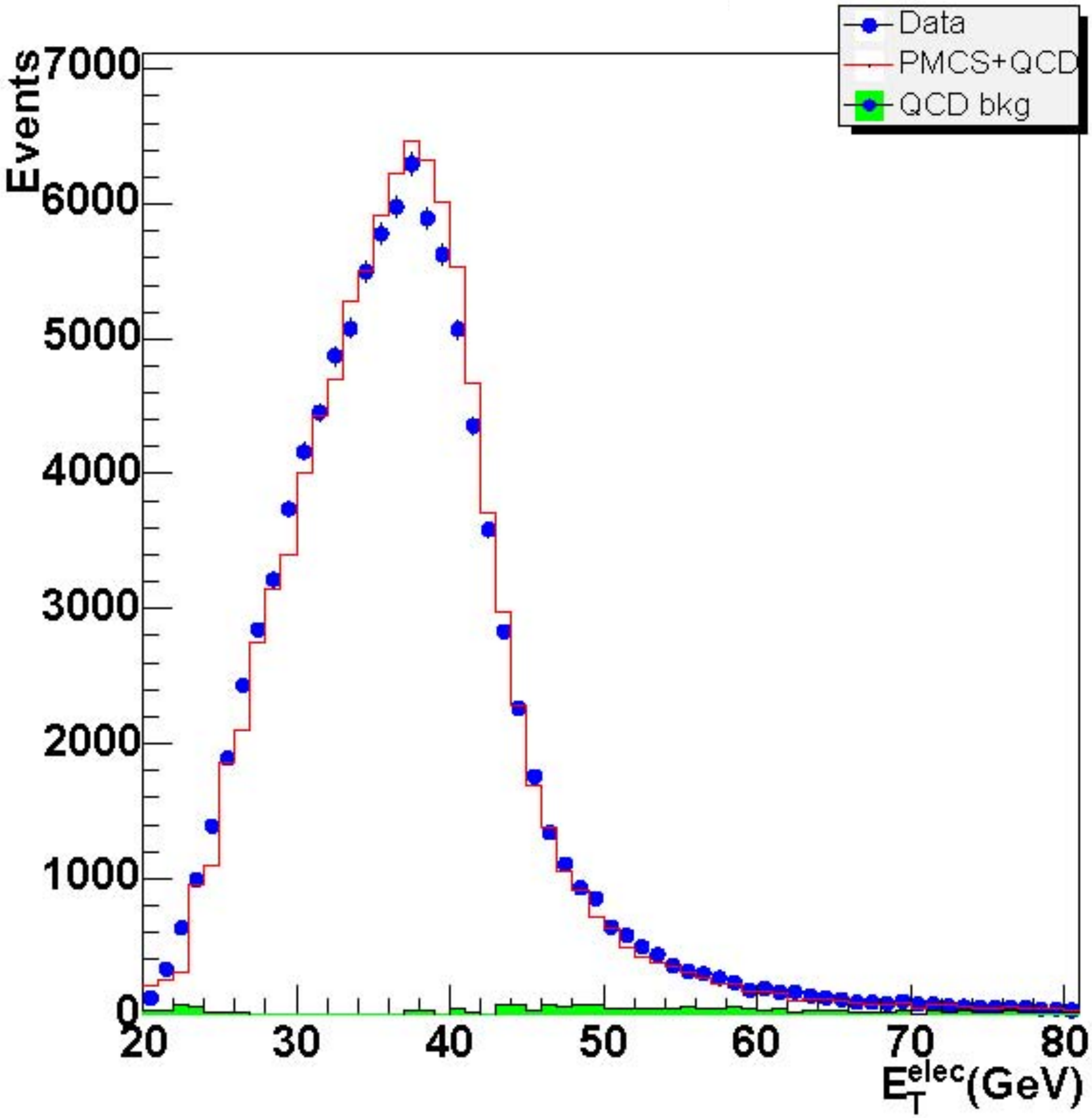
- This is the general expression for any scattering process, independent of the existence of theory
- This gives an observed counts per second

Some Example Cross Section Measurements



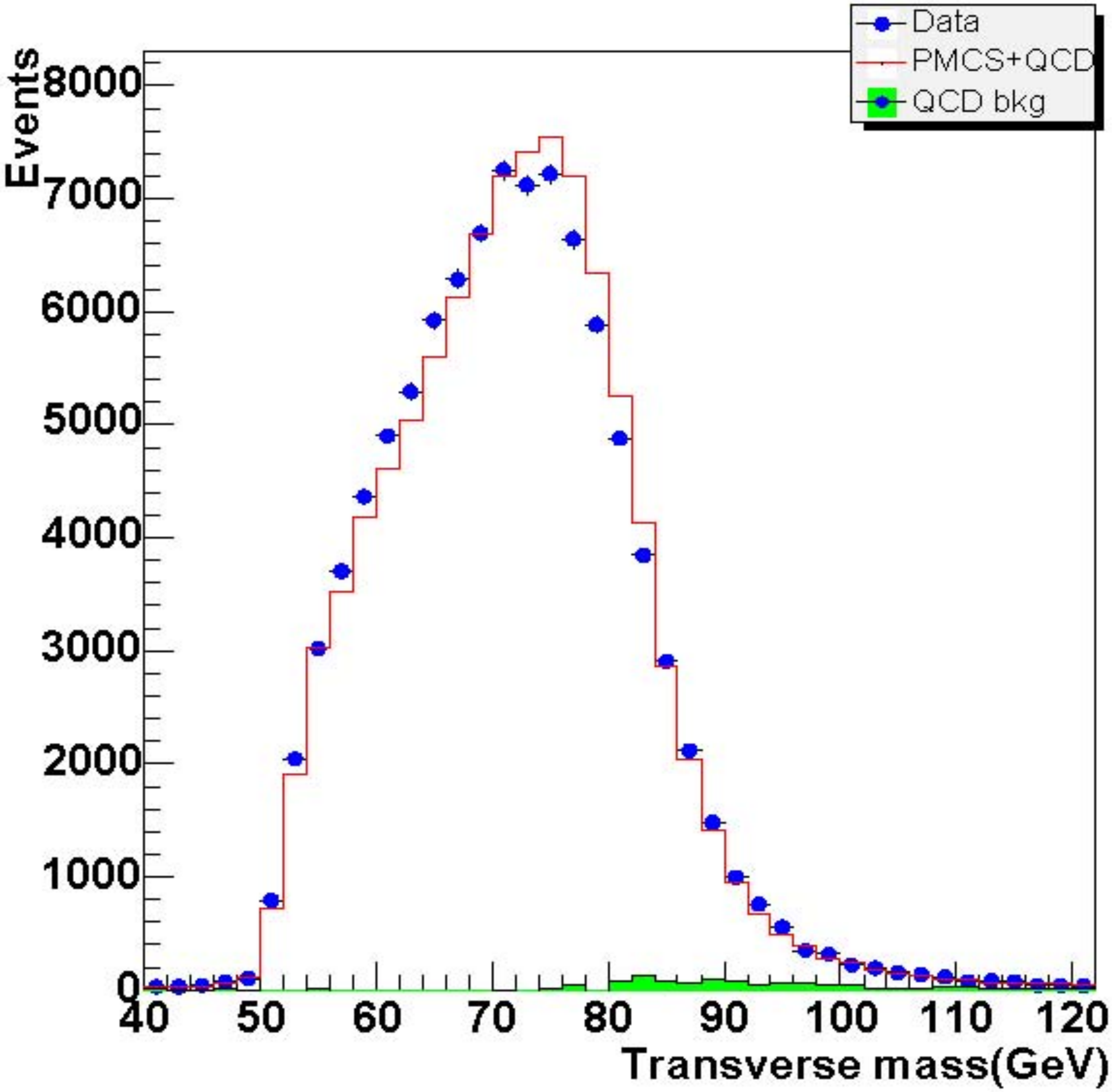
- Azimuthal angle distribution of electrons in W+2jet events

Example Cross Section: W($\rightarrow e\nu$) +X



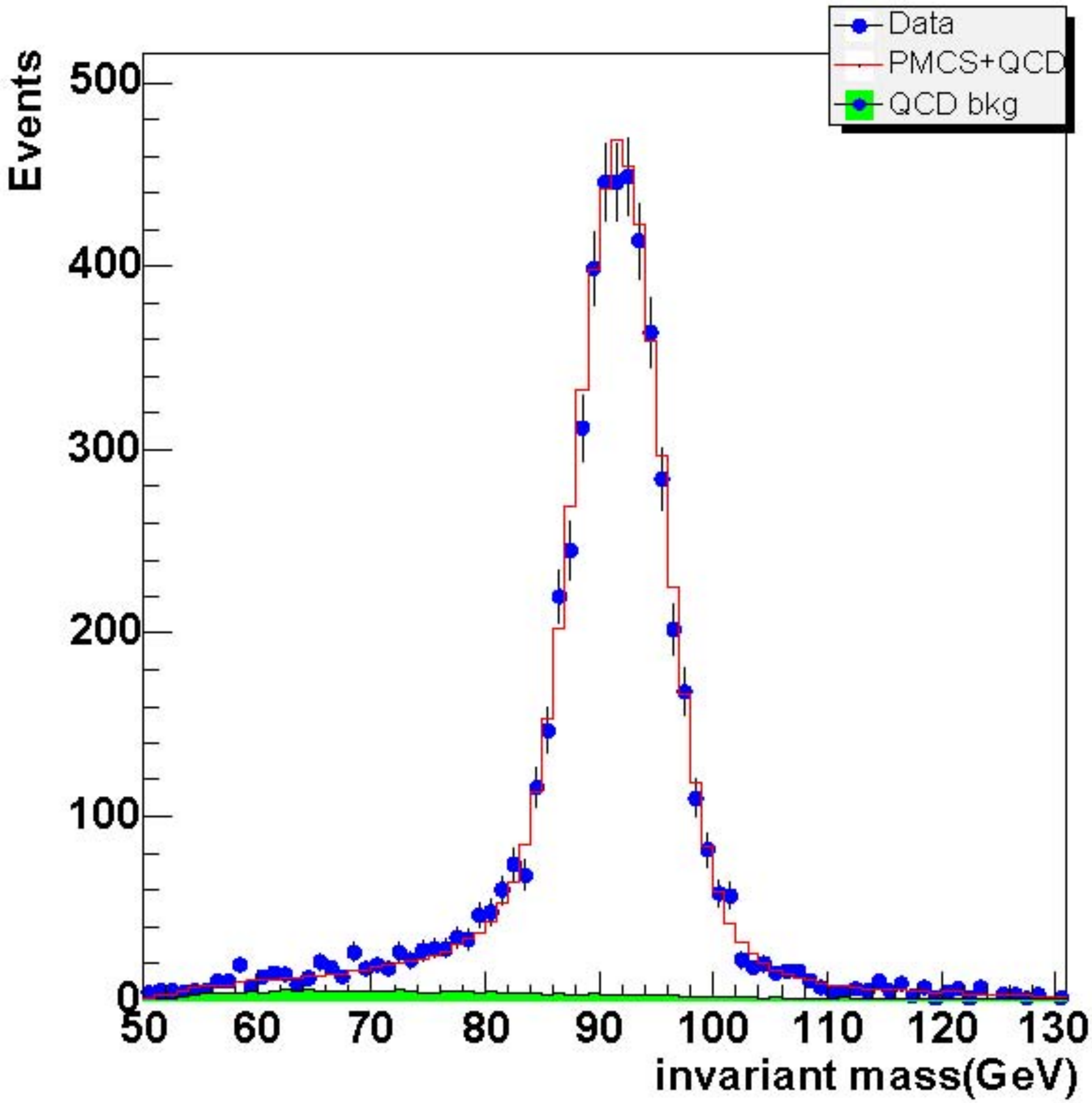
- Transverse momentum distribution of electrons in W+X events
- Mass of the W boson is 80GeV

Example Cross Section: W(→eν) +X



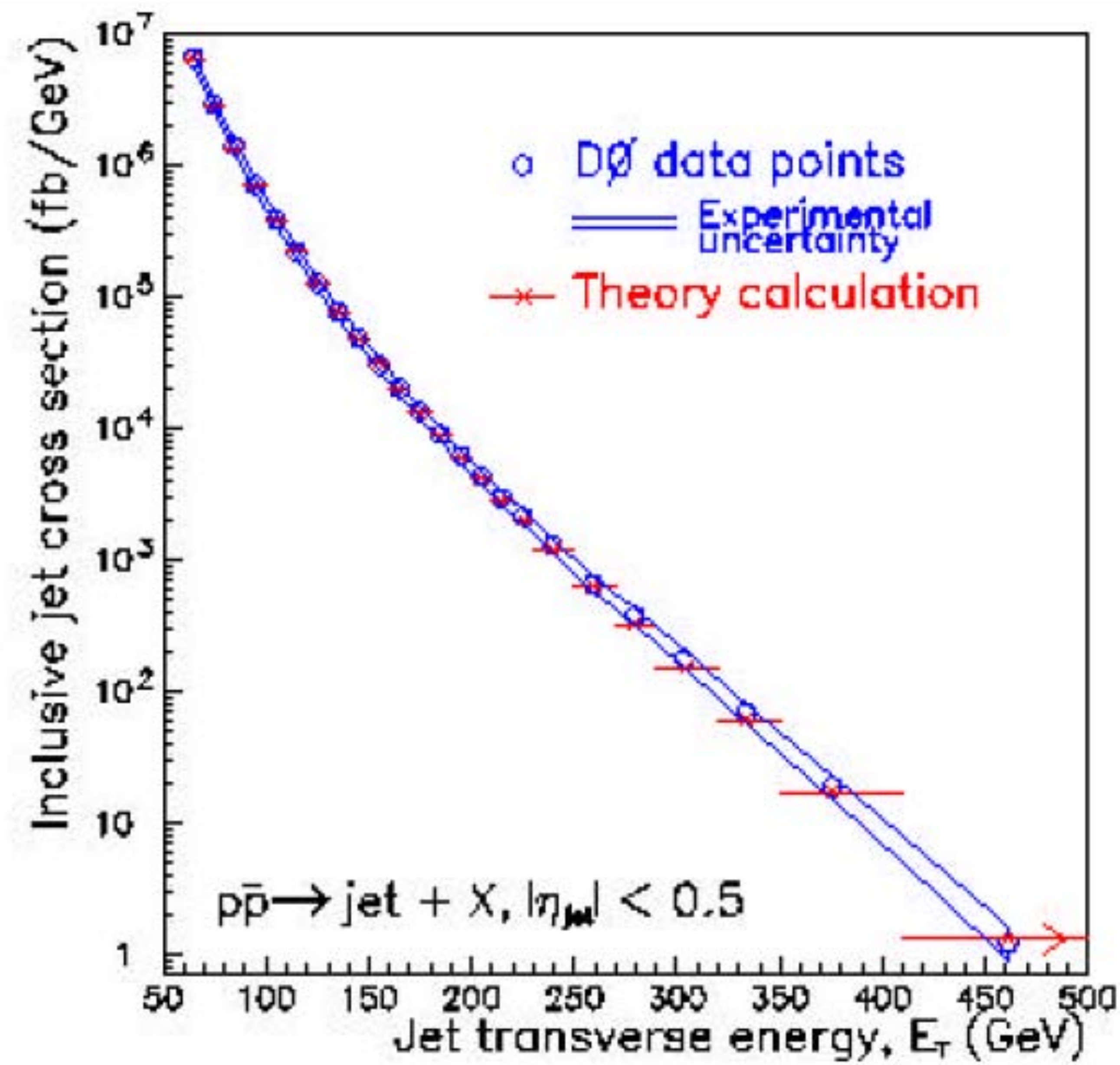
- Transverse mass distribution of electrons in W+X events

Example Cross Section: W(→ee) +X



- Invariant mass distribution of electrons in Z+X events
- Mass of the Z boson is 91GeV

Example Cross Section: Jet +X



- Inclusive jet production cross section as a function of transverse energy