PHYS 3446 – Lecture #5

Monday, Sept 19, 2016
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1. Lab Frame and Center of Mass Frame
2. Relativistic Treatment
3. Feynman Diagram
4. Invariant kinematic variable
5. Nuclear properties
   - Mott scattering
   - Spin and Magnetic Moments
   - Stability and Instability of Nuclei

Homework Assignment #3

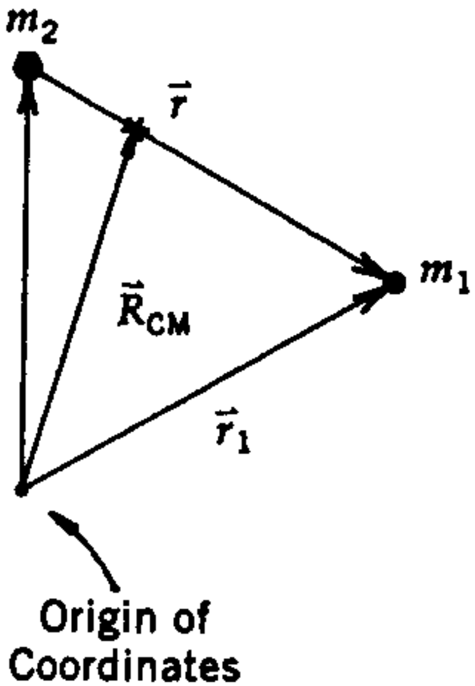
1. Derive Eq. 1.55 starting from 1.48 and 1.49 (5 points)
2. Derive the formulae for the available CMS energy ( ) for
   - Fixed target experiment with masses m_1 and m_2 with incoming energy E_1. (5points)
   - Collider experiment with masses m_1 and m_2 with incoming energies E_1 and E_2. (5points)
3. End of chapter problem 1.7 (5points)

- These assignments are due next Monday, Sept. 26
- Reading assignment: Section 1.7

Lab Frame and Center of Mass Frame

- We assumed that the target nuclei do not move throughout the collision in Rutherford Scattering
- In reality, they recoil as a result of scattering
- Sometimes we use two beams of particles for scattering experiments (target is moving)
- This situation could be complicated but,
- If the motion can be described in the Center of Mass frame under a central potential, it can be simplified

Lab Frame and CM Frame



$$\vec{r}_1 = r_1\,\hat{r}_1$$
$$\vec{r}_2 = r_2\,\hat{r}_2$$
$$\vec{r} = r\,\hat{r}$$
$$\vec{R}_{CM} = R_{CM}\,\hat{R}_{CM}$$

- The equations of motion can be written

$$m_1\ddot{\vec{r}}_1 = -\vec{\nabla}_1 V\left(\left|\vec{r}_1 - \vec{r}_2\right|\right)$$
$$m_2\ddot{\vec{r}}_2 = -\vec{\nabla}_2 V\left(\left|\vec{r}_1 - \vec{r}_2\right|\right)$$

where $\vec{\nabla}_i = \hat{r}_i \frac{\partial}{\partial r_i} + \frac{\hat{\theta}_i}{r_i}\frac{\partial}{\partial \theta_i} + \frac{\hat{\phi}_i}{r_i \sin\theta_i}\frac{\partial}{\partial \phi_i} \quad i = 1,2$

Since the potential depends only on relative separation of the particles, we redefine new variables, relative coordinates & coordinate of CM

$$\vec{r} = \vec{r}_1 - \vec{r}_2 \quad \text{and} \quad \vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

Lab Frame and CM Frame

- From the equations in previous slides

$$\boxed{\frac{m_1 m_2}{m_1+m_2}\ddot{\vec{r}} \equiv \mu\ddot{\vec{r}} = -\vec{\nabla}V\left(\left|\vec{r}\right|\right) = -\frac{\partial V\left(\left|\vec{r}\right|\right)}{\partial r}\hat{r}}$$

Reduced Mass

and $\boxed{\left(m_1+m_2\right)\ddot{\vec{R}}_{CM} = M\ddot{\vec{R}}_{CM} = 0}$

Thus $\dot{\vec{R}}_{CM} = \text{constant}\times\hat{R}$

- What do we learn from this exercise?
- For a central potential, the motion of the two particles can be decoupled from that of the reference frame when re-written in terms of
  - a relative coordinate
  - The coordinate of center of mass

Now with some simple arithmatics

- From the equations of motion, we obtain

$$m_1\ddot{\vec{r}}_1 - m_2\ddot{\vec{r}}_2 = m_1\ddot{\vec{r}}_1 - m_2\left(\ddot{\vec{r}}_1 - \ddot{\vec{r}}\right) = \left(m_1 - m_2\right)\ddot{\vec{r}}_1 + m_2\ddot{\vec{r}}$$

$$= -\left(\hat{r}_1\frac{\partial}{\partial r_1} - \hat{r}_2\frac{\partial}{\partial r_2}\right)V\left(\left|\vec{r}\right|\right) = -\hat{r}\frac{\partial}{\partial r}V\left(\left|\vec{r}\right|\right)$$

- Since the momentum of the system is conserved:

$$\frac{m_1\ddot{\vec{r}}_1 + m_2\ddot{\vec{r}}_2}{m_1 + m_2} = \ddot{\vec{R}}_{CM} = 0 \Rightarrow \quad m_1\ddot{\vec{r}}_1 = -m_2\ddot{\vec{r}}_2 = -m_2\left(\ddot{\vec{r}}_1 - \ddot{\vec{r}}\right)$$

- Rearranging the terms, we obtain

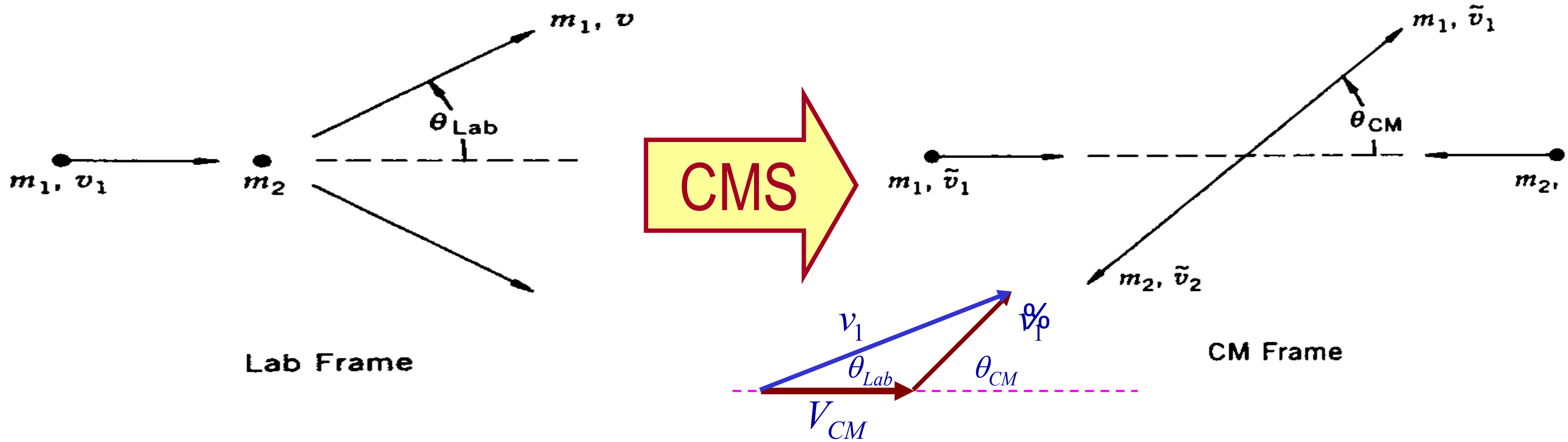
$$\left(m_1 + m_2\right)\ddot{\vec{r}}_1 = m_2\ddot{\vec{r}} \Rightarrow \quad \ddot{\vec{r}}_1 = \frac{m_2}{\left(m_1 + m_2\right)}\ddot{\vec{r}}$$

$$\left(m_1 - m_2\right)\frac{m_2}{\left(m_1 + m_2\right)}\ddot{\vec{r}} + m_2\ddot{\vec{r}} = \boxed{\frac{m_1 m_2}{\left(m_1 + m_2\right)}\ddot{\vec{r}} = -\hat{r}\frac{\partial}{\partial r}V\left(\left|\vec{r}\right|\right)}$$

Lab Frame and CM Frame

- The CM is moving at a constant velocity in the lab frame independent of the form of the central potential
- The overall motion is as if that of a fictitious particle with mass μ (the reduced mass) and coordinate **r**.
- In the frame where CM is stationary, the dynamics becomes equivalent to that of a single particle of mass μ scattering off of a fixed scattering center.
- Frequently we define the Center of Mass frame as the frame where the sum of the momenta of all the interacting particles is 0.

Relationship of variables in Lab and CMS



- The speed of CM is

$$v_{CM} = \dot{R}_{CM} = \frac{m_1 v_1}{(m_1 + m_2)}$$

- Speeds of the particles in CMS are

$$\tilde{v} = v_1 - v_{CM} = \frac{m_2 v_1}{(m_1 + m_2)} \quad \text{and} \quad \tilde{v}_2 = v_{CM} = \frac{m_1 v_1}{(m_1 + m_2)}$$

- The momenta of the two particles are equal and opposite!!

Scattering angles in Lab and CMS

- θ_{CM} represents the changes in the direction of the relative position vector **r** as a result of the collision in CMS
- Thus, it must be identical to the scattering angle for the particle with the reduced mass, μ.
- Z components of the velocities of particle wit mass m_1 after the scattering in lab and CMS are:

$$v\cos\theta_{Lab} - v_{CM} = \tilde{v}\cos\theta_{CM}$$

- The perpendicular components of the velocities are:

$$v\sin\theta_{Lab} = \tilde{v}\sin\theta_{CM}$$

- Thus, the angles are related, for elastic scattering only, as:

$$\tan\theta_{Lab} = \frac{\sin\theta_{CM}}{\cos\theta_{CM} + v_{CM}/\tilde{v}} = \frac{\sin\theta_{CM}}{\cos\theta_{CM} + m_1/m_2} = \frac{\sin\theta_{CM}}{\cos\theta_{CM} + \zeta}$$

Differential cross sections in Lab and CMS

- The particles that scatter in lab at an angle θ_{Lab} into solid angle $d\Omega_{Lab}$ scatter at θ_{CM} into the solid angle $d\Omega_{CM}$ in CM.
- Since ϕ is invariant, $d\phi_{Lab} = d\phi_{CM}$.
  - Why?
  - ϕ is perpendicular to the direction of boost, thus is invariant.
- Thus, the differential cross section becomes:

$$\frac{d\sigma}{d\Omega_{Lab}}(\theta_{Lab})\sin\theta_{Lab}d\theta_{Lab} = \frac{d\sigma}{d\Omega_{CM}}(\theta_{CM})\sin\theta_{CM}d\theta_{CM}$$

reorganize

$$\frac{d\sigma}{d\Omega_{Lab}}(\theta_{Lab}) = \frac{d\sigma}{d\Omega_{CM}}(\theta_{CM})\frac{d(\cos\theta_{CM})}{d(\cos\theta_{Lab})}$$

Using Eq. 1.53

$$\boxed{\frac{d\sigma}{d\Omega_{Lab}}(\theta_{Lab}) = \frac{d\sigma}{d\Omega_{CM}}(\theta_{CM})\frac{\left(1+2\zeta\cos\theta_{CM}+\zeta^2\right)^{3/2}}{\left|1+\zeta\cos\theta_{CM}\right|}}$$