PHYS 3446 – Lecture #6

Wednesday, Sept 21, 2016
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1. Relativistic Treatment
2. Feynman Diagram
3. Nuclear Phenomenology
4. Properties of Nuclei
   - Labeling
   - Masses
   - Sizes
   - Nuclear Spin and Dipole Moment
   - Stability and Instability of Nuclei

Announcement

- Faculty expo #3
  - 4pm today in SH100
- Reading assignment
  - Read and follow through Appendix A, special relativity

Reminder: Homework Assignment #3

1. Derive Eq. 1.55 starting from 1.48 and 1.49 (5 points)
2. Derive the formulae for the available CMS energy ($\sqrt{s}$) for
   - Fixed target experiment with masses m_1 and m_2 with incoming energy E_1. (5points)
   - Collider experiment with masses m_1 and m_2 with incoming energies E_1 and E_2. (5points)
3. End of chapter problem 1.7 (5points)

- These assignments are due next Monday, Sept. 26
- Reading assignment: Section 1.7

Some Quantities in Special Relativity

- Fractional velocity: $\vec{\beta} = \vec{v}/c$
- Lorentz γ factor $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$
- Relative momentum and the total energy of the particle moving at a velocity $\vec{v} = \vec{\beta}c$ is

$$\vec{P} = M\gamma\vec{v} = M\gamma\vec{\beta}c$$

$$E = \sqrt{T^2 + E_{Rest}^2} = \sqrt{P^2c^2 + M^2c^4} = M\gamma c^2$$

- Square of the four momentum P=(E,**pc**), rest mass energy $P^2 = Mc^2 = E^2 - p^2c^2$

Relativistic Variables

- Velocity of CM in the scattering of two particles with rest mass m_1 and m_2 is:

$$\vec{\beta}_{CM} = \frac{\vec{v}_{CM}}{c} = \frac{\left(\vec{P}_1 + \vec{P}_2\right)c}{\left(E_1 + E_2\right)}$$

- If m_1 is the mass of the projectile and m_2 is that of the target, for a fixed target we obtain

$$\vec{\beta}_{CM} = \frac{\vec{P}_1 c}{\left(E_1 + E_2\right)} = \frac{\vec{P}_1 c}{\sqrt{P_1^2 c^2 + m_1^2 c^4} + m_2 c^2}$$

Relativistic Variables

- At very low energies where $m_1c^2 >> P_1c$, the velocity reduces to:

$$\vec{\beta}_{CM} = \frac{m_1\vec{v}_1c}{m_1c^2 + m_2c^2} = \frac{m_1\vec{v}_1}{\left(m_1 + m_2\right)c}$$

- At very high energies where $m_1c^2 << P_1c$ and $m_2c^2 << P_1c$, the velocity can be written as:

$$\beta_{CM} = \left|\vec{\beta}_{CM}\right| = \frac{1}{\sqrt{1 + \left(\frac{m_1c^2}{P_1c}\right)^2 + \frac{m_2c^2}{P_1c}}} \approx 1 - \frac{m_2c}{P_1} - \frac{1}{2}\left(\frac{m_1c}{P_1}\right)^2$$

Expansion

Relativistic Variables

- For high energies, if $m_1 \sim m_2$,
- $$\beta_{CM} \approx \left(1 - \frac{m_2 c}{P_1}\right)$$
- γ_{CM} becomes:

$$\gamma_{CM} = \left(1 - \beta_{CM}^2\right)^{-1/2} \approx$$

$$\left[\left(1 - \beta_{CM}\right)\left(1 + \beta_{CM}\right)\right]^{-1/2} \approx$$

$$\left[2\left(\frac{m_2 c}{P_1}\right)\right]^{-1/2} = \sqrt{\frac{P_1}{2 m_2 c}}$$

Relativistic Variables

- In general, we can rewrite

$$1-\beta_{CM}^2 = \frac{m_1^2c^4 + 2E_1m_2c^2 + m_2^2c^4}{\left(E_1 + m_2c^2\right)^2}$$

- Thus the generalized notation of γ_{CM} becomes

$$\gamma_{CM} = \left(1-\beta_{CM}^2\right)^{-1/2} = \frac{E_1 + m_2c^2}{\sqrt{m_1^2c^4 + 2E_1m_2c^2 + m_2^2c^4}}$$

Invariant Scalar: s

Relativistic Variables

- The invariant scalar, s, is defined as:

$$s=\left(P_1+P_2\right)^2=\left(E_1+E_2\right)^2-\left(\vec{P}_1+\vec{P}_2\right)^2c^2$$
$$=m_1^2c^4+m_2^2c^4+2E_1m_2c^2$$

- So what is this the CMS frame?

$$s=m_1^2c^4+m_2^2c^4+2E_1m_2c^2$$
$$=\left(E_{1CM}+E_{2CM}\right)^2-\left(\cancelto{0}{\vec{P}_{1CM}+\vec{P}_{2CM}}\right)^2c^2$$
$$=\left(E_{1CM}+E_{2CM}\right)^2=\left(E_{ToT}^{CM}\right)^2$$

- Thus, $\sqrt{s}$ represents the total available energy in the CMS

Useful Invariant Scalar Variables

- Another invariant scalar, t, the four momentum transfer (differences in four momenta), is useful for scattering:

$$t = \left(P_1^f - P_1^i\right) = \left(E_1^f - E_1^i\right)^2 - \left(\vec{P}_1^f - \vec{P}_1^i\right)^2 c^2$$

- Since momentum and total energy are conserved in all collisions, t can be expressed in terms of target variables

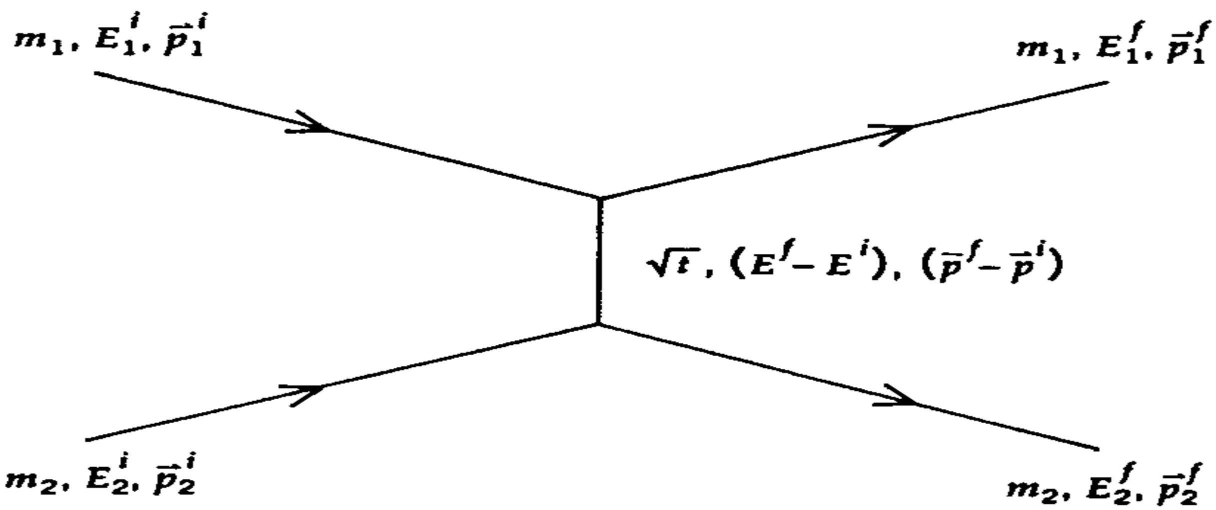
$$t = \left(P_2^f - P_2^i\right) = \left(E_2^f - E_2^i\right)^2 - \left(\vec{P}_2^f - \vec{P}_2^i\right)^2 c^2$$

- In CMS frame for an elastic scattering, where $P^i_{CM}=P^f_{CM}=P_{CM}$ and $E^i_{CM}=E^f_{CM}$:

$$t = -\left(P_{CM}^{f2} + P_{CM}^{i2} - 2\vec{P}_{CM}^f \cdot \vec{P}_{CM}^i\right)c^2 = -2P_{CM}^2 c^2\left(1 - \cos\theta_{CM}\right)$$

Feynman Diagram

- The variable t is always negative for an elastic scattering
- The variable t could be viewed as the square of the invariant mass of a particle with $E_2^f - E_2^i$ and $\vec{P}_2^f - \vec{P}_2^i$ exchanged in the scattering



t-channel diagram

Momentum of the carrier is the momentum difference between the two particles.

- While the virtual particle cannot be detected in the scattering, the consequence of its exchange can be calculated and observed!!!
  - A virtual particle is a particle whose mass is different than the rest mass

Useful Invariant Scalar Variables

- For convenience we define a variable q^2, $q^2c^2 = -t$
- In the lab frame, $\vec{P}^{i}_{2Lab} = 0$, thus we obtain:

$$q^2c^2 = -\left[\left(E^f_{2Lab} - m_2c^2\right)^2 - \left(P^f_{2Lab}c\right)^2\right]$$

$$= 2m_2c^2\left(E^f_{2Lab} - m_2c^2\right) = 2m_2c^2T^f_{2Lab}$$

- In the non-relativistic limit: $T^f_{2Lab} \approx \frac{1}{2}m_2v_2^2$
- q^2 represents "hardness of the collision". Small θ_{CM} corresponds to small q^2.

Relativistic Scattering Angles in Lab and CMS

- For a relativistic scattering, the relationship between the scattering angles in Lab and CMS is:

$$\tan\theta_{Lab} = \frac{\bar{\beta}\sin\theta_{CM}}{\gamma_{CM}\left(\bar{\beta}\sin\theta_{CM} + \beta_{CM}\right)}$$

- For Rutherford scattering ($m=m_1<<m_2$, $v\sim v_0<<c$):

$$dq^2 = -2P^2 d\left(\cos\theta\right) = \frac{P^2 d\Omega}{\pi}$$

Resulting in a cross section

$$\frac{d\sigma}{dq^2} = \frac{4\pi\left(kZZ'e^2\right)^2}{v^2}\frac{1}{q^4}$$

- Divergence at $q^2\sim 0$, a characteristics of a Coulomb field

Nuclear Phenomenology

- Rutherford scattering experiment clearly demonstrated the existence of a positively charged central core in an atom
- The formula deviated for high energy α particles (E>25MeV), especially for low Z nuclei.
- 1920's James Chadwick noticed serious discrepancies between Coulomb scattering expectation and the observed elastic scattering of α particle on He.
- None of the known effects, including quantum effect, described the discrepancy.
- Clear indication of something more than Coulomb force involved in the interactions.
- Before Chadwick's discovery of neutron in 1932, people thought nucleus contain protons and electrons. ➔ We now know that there are protons and neutrons (nucleons) in nuclei.

Properties of Nuclei: Labeling

$$^{A}X^{Z}$$

- The nucleus of an atom X can be labeled uniquely by its:
  - Electrical Charge or atomic number Z (number of protons).
  - Total number of nucleons A ($=N_p+N_n$)
- **Isotopes**: Nuclei with the same Z but different A
  - Same number of protons but different number of neutrons
  - Have similar chemical properties
- **Isobars**: Nuclei with same A but different Z
  - Same number of nucleons but different number of protons
- **Isomers** or resonances of the ground state: Excited nucleus to a higher energy level
- **Mirror nuclei**: Nuclei with the same A but with switched N_p and N_n

Nuclear Properties: Masses of Nuclei

- A nucleus of ${}^{A}X^{Z}$ has N_p=Z and N_n=A-Z
- Naively one would expect

$$M(A,Z) = Zm_p + (A-Z)m_n$$

- Where m_p~938.27MeV/c^2 and m_n=939.56MeV/c^2
- However measured mass turns out to be

$$M(A,Z) < Zm_p + (A-Z)m_n$$

- This is one of the explanations for nucleus not falling apart into its nucleon constituents

Nuclear Properties: Binding Energy

- The mass deficit

$$\Delta M(A,Z) = M(A,Z) - Zm_p - (A-Z)m_n$$

- Is always negative and is proportional to the nuclear binding energy
- How are the BE and mass deficit related?

$$B.E = \Delta M(A,Z)c^2$$

- What is the physical meaning of BE?
  - A minimum energy required to release all nucleons from a nucleus
  - So B= -BE is the energy required to keep a nucleus

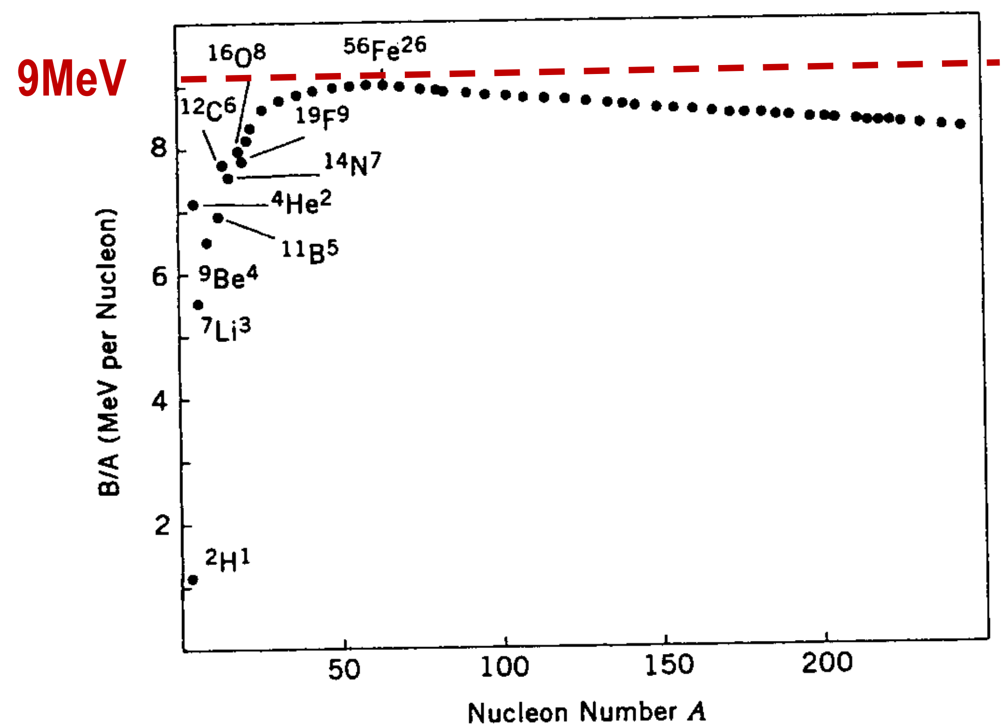
Nuclear Properties: Binding Energy

- BE per nucleon is

$$\frac{B}{A}=\frac{-BE}{A}$$

$$=\frac{-\Delta M\left(A,Z\right)c^2}{A}$$

$$=\frac{\left(Zm_p+\left(A-Z\right)m_n-M\left(A,Z\right)\right)c^2}{A}$$



- Rapidly increase with A till A~60 at which point B/A~9MeV.
- A>60, the B/A gradually decrease ➔ For most the large A nucleus, B/A~8MeV.

Nuclear Properties: Binding Energy

- de Broglie's wavelength: $$\bar{\lambda} = \frac{\hbar}{p}$$
  - Where $\hbar$ is the Planck's constant
  - And $\bar{\lambda}$ is the reduced wave length
- Assuming 8MeV was given to a nucleon (m~940MeV), the wavelength is

$$\bar{\lambda} = \frac{\hbar}{p} = \frac{\hbar}{\sqrt{2mT}} = \frac{\hbar c}{\sqrt{2mc^2T}} \sim \frac{197\, MeV - fm}{\sqrt{2 \cdot 940 \cdot 8}} \sim 1.6\, fm$$

- Makes sense for nucleons to be inside a nucleus since the size is small.
- If it were electron with 8MeV, the wavelength is ~10fm, a whole lot larger than a nucleus.

Nuclear Properties: Sizes

- At relativistic energies, the magnetic moment of electron also contributes to the scattering
  - Neville Mott formulated Rutherford scattering in QM and included the spin effects
  - R. Hofstadter, *et al.*, discovered the effect of spin, nature of nuclear (& proton) form factor in late 1950s
- Mott scattering x-sec (scattering of a point particle) is related to Rutherford x-sec: $\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 4\cos^2\frac{\theta}{2}\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}$
- Deviation from the distribution expected for point-scattering provides a measure of size (structure)

Nuclear Properties: Sizes

- Another way is to use the strong nuclear force of sufficiently energetic strongly interacting particles (π mesons, protons, etc)
  - What is the advantage of using these particles?
    - If the energy is high, Coulomb interaction can be neglected
    - These particles readily interact with nuclei, getting “absorbed” into the nucleus
    - Thus, probe strong interactions directly
  - These interactions can be treated the same way as the light absorptions resulting in diffraction, similar to that of light passing through gratings or slits

Nuclear Properties: Sizes

- The size of a nucleus can be inferred from the diffraction pattern
- All these phenomenological investigation provided the simple formula for the radius of the nucleus to its number of nucleons or atomic number, A:

$$R = r_0 A^{1/3} \approx 1.2 \times 10^{-13} A^{1/3} cm = 1.2 A^{1/3} \mathrm{fm}$$

How would you interpret this formula?

Nuclear Properties: Spins

- Both protons and neutrons are fermions with spins $1/2$
- Nucleons inside a nucleus can have orbital angular momentum
- In Quantum Mechanics orbital angular momenta are integers
- Thus the total angular momentum of a nucleus is
  - Integers: if even number of nucleons in the nucleus
  - Half integers: if odd number of nucleons in the nucleus
- Interesting facts are
  - All nucleus with even number of p and n are spin 0.
  - Large nuclei have very small spins in their ground state
- Hypothesis: Nucleon spins in the nucleus are very strongly paired to minimize their overall effect