PHYS 3446 – Lecture #7

Monday, Sept 26, 2016

Dr. Jae Yu

- Properties of Nuclei
  - Nuclear Spin and Dipole Moment
  - Stability and Instability of Nuclei
- Nature of the Nuclear Force
  - Short Range Nature of the Nuclear Force
  - Shape of the Nuclear Potential
  - Yukawa Potential
  - Range of Yukawa Potential
- Nuclear Models

Announcement

- First quiz
  - Beginning of class Wednesday, Sept. 28
- First term exam
  - Date and time: 2:30 – 4:50pm, Monday, Oct. 10
  - Location: SH125
  - Covers: Ch 1 – Ch 3 or what we finish Wednesday, Oct. 5, + Appendix A
- Colloquium
  - Dr. Amir Farbin on Deep Learning

Physics Department
The University of Texas at Arlington
Colloquium

Deep Learning in High Energy Physics

Dr. Amir Farbin

University of Texas at Arlington

Wednesday September 28, 2016
4:00 p.m. Room 100 Science Hall

The recent Deep Learning (DL) renaissance has yielded impressive feats in industry and science, replacing laborious feature engineering with automatic feature learning, providing better algorithms, and enabling analysis of unlabeled data. DL is applicable to a large number of High Energy Physics (HEP) problems such as tracking, calorimetry, particle identification, simulation, monitoring, anomaly detection, noise reduction, data compression, workflow optimization, and data analysis. I will discuss how DL can be applied to many of these areas and overview the first attempts of DL in HEP. I will also present several public datasets that we have been compiling to enable collaborations with the Machine Learning community and contributions from the public.

Refreshments will be served at 3:30 p.m. in the Physics Library

Homework Assignment #4

1. Compute the mass density of a nucleus (10points)
   - Pick two nuclei for this. I would like you guys to do different ones.
2. Compute the de Broglie wavelengths for (15 points)
   - Protons in Fermilab's Tevatron Collider
   - Protons in CERN's Large Hadron Collider (LHC)
   - 500 GeV electrons in the International Linear Collider
3. Compute the actual value of the nuclear magneton (5 points)

- Due for the above is next Monday, Oct. 3

Nuclear Properties: Spins

- Both protons and neutrons are fermions with intrinsic spin $\frac{1}{2}\hbar$
- Nucleons inside a nucleus can have orbital angular momenta
- In Quantum Mechanics orbital angular momenta are integers
- Thus the total angular momentum (vector sum of orbital and intrinsic momenta of all constituents) of a nucleus is
  - Integers: if even number of nucleons in the nucleus
  - Half integers: if odd number of nucleons in the nucleus
- Interesting facts are
  - All nucleus with even number of p and even n are spin 0.
  - Large nuclei have very small spins in their ground state
- Hypothesis: Nucleon spins in a nucleus are very strongly paired to minimize their overall effect (entropy)

Nuclear Properties: Magnetic Dipole Moments

- Every charged particle has a magnetic dipole moment associated with its spin $\vec{\mu} = g\frac{e}{2mc}\vec{S}$
- e, m and **S** are the charge, mass and the intrinsic spin of the charged particle
- The constant g is called Landé factor with its value:
  - $g = 2$: for a point like particle, such as the electron
  - $g \neq 2$: Particle possesses an anomalous magnetic moment, an indication of having a substructure

Nuclear Properties: Magnetic Dipole Moments

- For electrons, $\mu_e \sim \mu_B$, where μ_B is **Bohr Magneton** of a point-like particle

$$\mu_B = \frac{e\hbar}{2m_e c} = 5.79 \times 10^{-11} \text{ MeV/T}$$

- For nucleons, magnetic dipole moment is measured in **nuclear magneton**, defined using proton mass

$$\mu_N = \frac{e\hbar}{2m_p c}$$

- Measured magnetic moments of proton and neutron:

$$\mu_p \approx 2.79 \mu_N \qquad \mu_n \approx -1.91 \mu_N$$

Nuclear Properties: Magnetic Dipole Moments

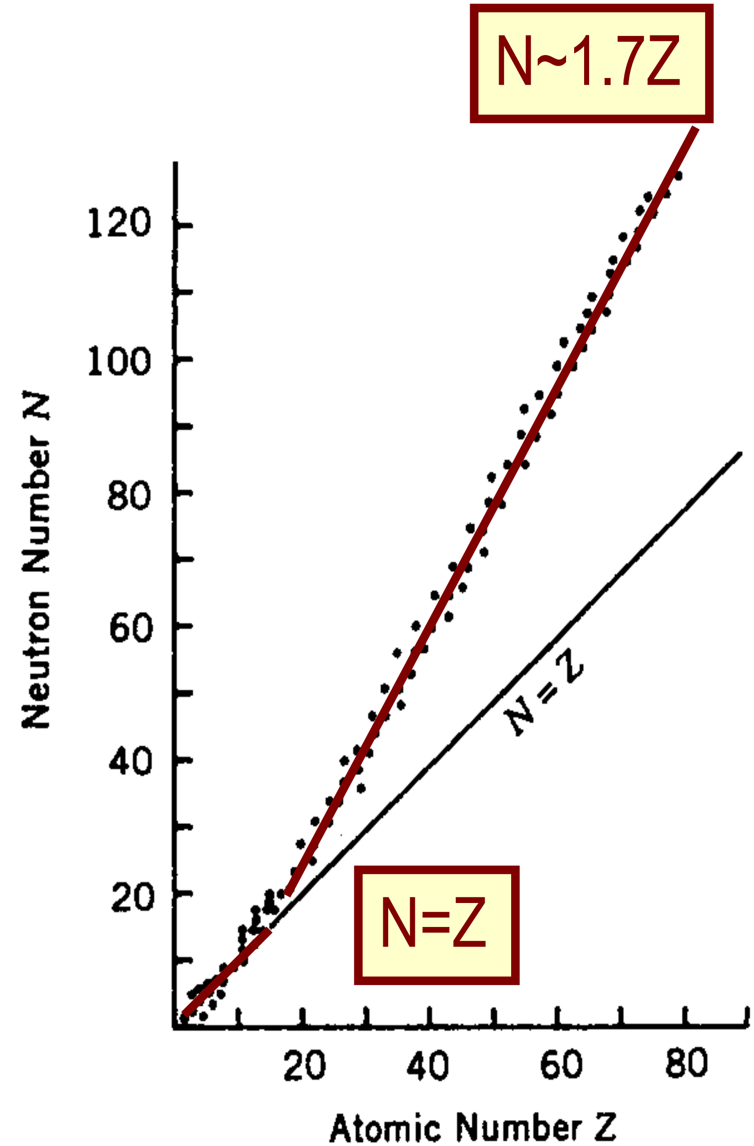
- What important information do you get from these?
  - The Landé factors of the nucleons deviate significantly from 2.
    - Strong indication of substructure
  - An electrically neutral neutron has a significant magnetic moment
    - Must have extended charge distributions
- Measurements show that magnetic moment of nuclei lie $-3\mu_N \sim 10\mu_N$
  - Indication of strong pairing
  - Electrons cannot reside in nucleus ($\mu_e \sim \mu_B \sim 2000\mu_N$)

Nuclear Properties: Stability

- The number of protons and neutrons inside the stable nuclei are
  - A<40: Equal (N=Z)
  - A>40: N~1.7Z
  - Neutrons outnumber protons
  - Most are even-p + even–n

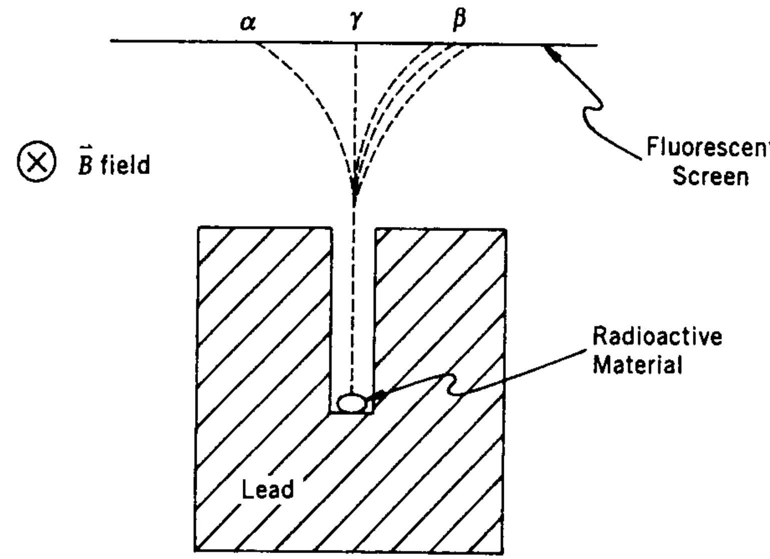
N	Z	N_{nucl}
Even	Even	**156**
Even	Odd	48
Odd	Even	50
Odd	Odd	5

    - See table 2.1
  - Supports strong pairing



Nuclear Properties: Instability

- In 1896 H. Becquerel accidently discovered natural radioactivity
  - Study of Uranium salts' fluorescent properties
- Nuclear radio activity involves emission of three radiations: α, β, and γ
- These can be characterized using the device on the right
  - α: Nucleus of He (v~0.1c), short range (a sheet of paper can stop it)
  - β: electrons (v<0.99c), has longer range and produces less ionization than α, can be stopped by 3mm lead
  - γ: photons, long range, few cm lead to stop



- What do you see from above?
  - α and β are charged particles while γ is neutral.
  - α is mono-energetic
  - β has broad spectrum
- What else could you see?

Nature of the Nuclear Force

- Scattering experiments help to
  - Determine the properties of nuclei
  - Learn more global information on the characteristics of the nuclear force
- From what we have learned, it is clear that there is no classical analog to nuclear force
  - Gravitational force is too weak to provide the binding
  - Can't have an electromagnetic origin
    - Deuteron nucleus has one neutron and one proton
    - Coulomb force would destabilize the nucleus

Short-range Nature of the Nuclear Force

- Atomic structure is well explained by the electromagnetic interaction
  - Thus the range of nucleus cannot be much greater than the radius of the nucleus
  - Nuclear force should range ~ $10^{-13} - 10^{-12}$cm
- Binding energy is constant per each nucleon, essentially independent of the size of the nucleus
  - If the nuclear force is long-ranged, like the Coulomb force
  - For A nucleons, there would be ½ A(A-1) pair-wise interactions
  - Thus, the BE which reflects all possible interactions among the nucleons would grow as a function of A

$$B \propto A(A-1)$$

For large A

$$\frac{B}{A} \propto A$$

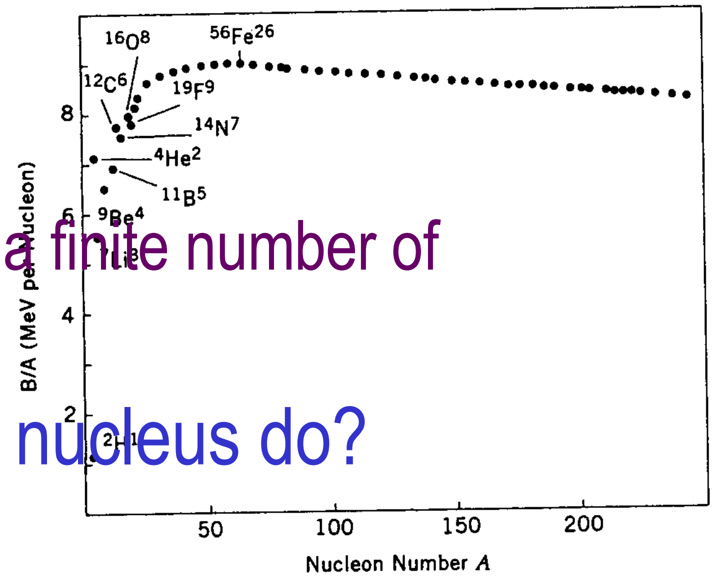
Short-range Nature of the Nuclear Force

- If the nuclear force is long-ranged and is independent of the presence of other nucleons, BE per nucleon will increase linearly with A
  - This is because long-range forces do not saturate
  - Since any single particle can interact with as many other particle as are available

  ⇒Binding becomes tighter as the number of interacting objects increases
  - The size of the interacting region stays fairly constant
  - Atoms with large number of electrons have the sizes compatible to those with smaller number of electrons

Short-range Nature of the Nuclear Force

- Long-rangeness of nuclear force is disputed by the experimental measurement that the BE/nucleon stays constant
  - Nuclear force must saturate
  - Any given nucleon can only interact with a finite number of nucleons in its vicinity



- What does adding more nucleons to a nucleus do?
  - Only increases the size of the nucleus
    - Recall that $R \sim A^{1/3}$
  - The size of a nucleus grows slowly with A and keeps the nuclear density constant

$\Rightarrow$Another supporting evidence of short-range nature of nuclear force

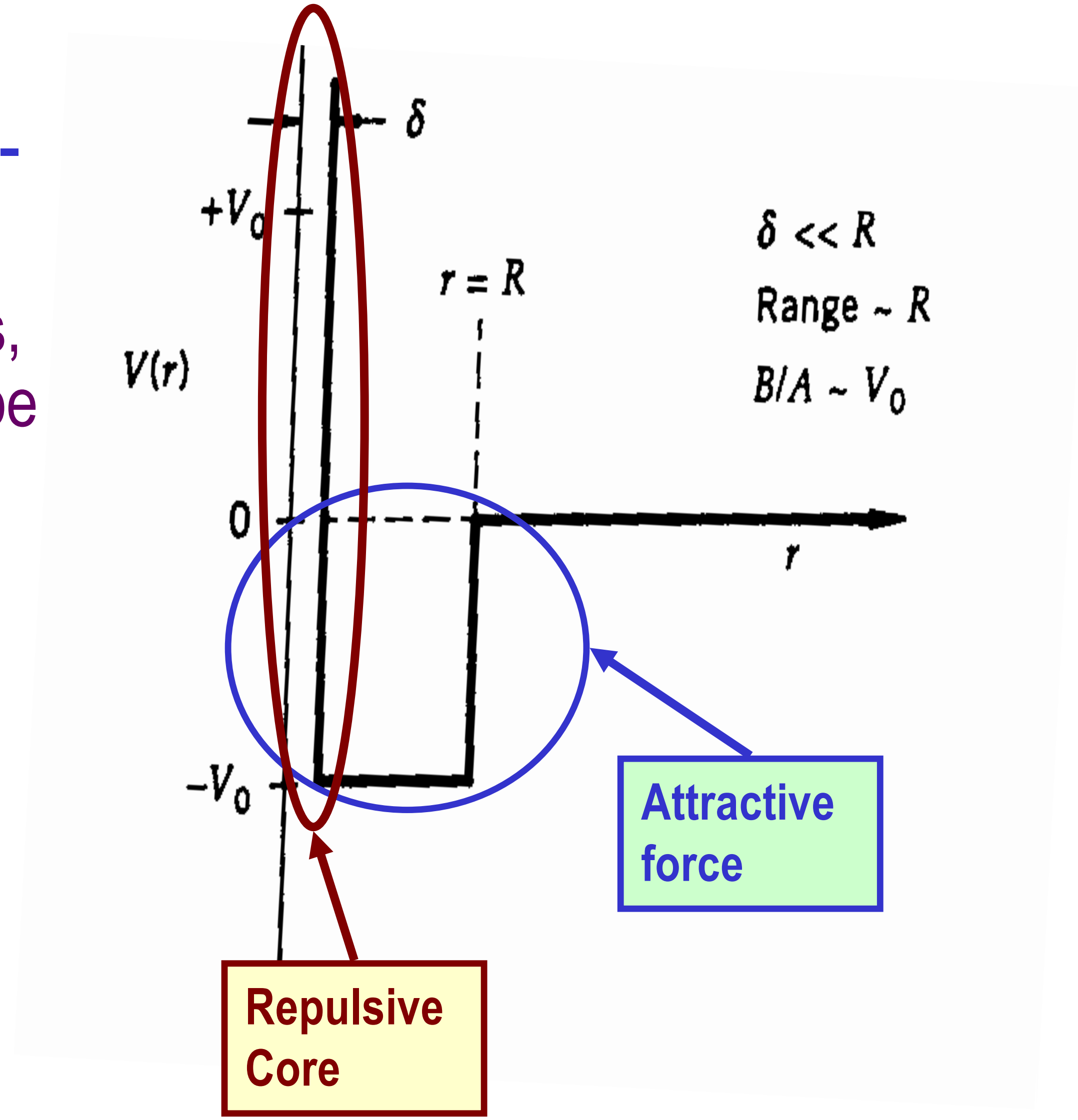
Shape of the Nuclear Potential

- Nuclear force keeps the nucleons within the nucleus.
  - What does this tell you about the nature of the nuclear force?

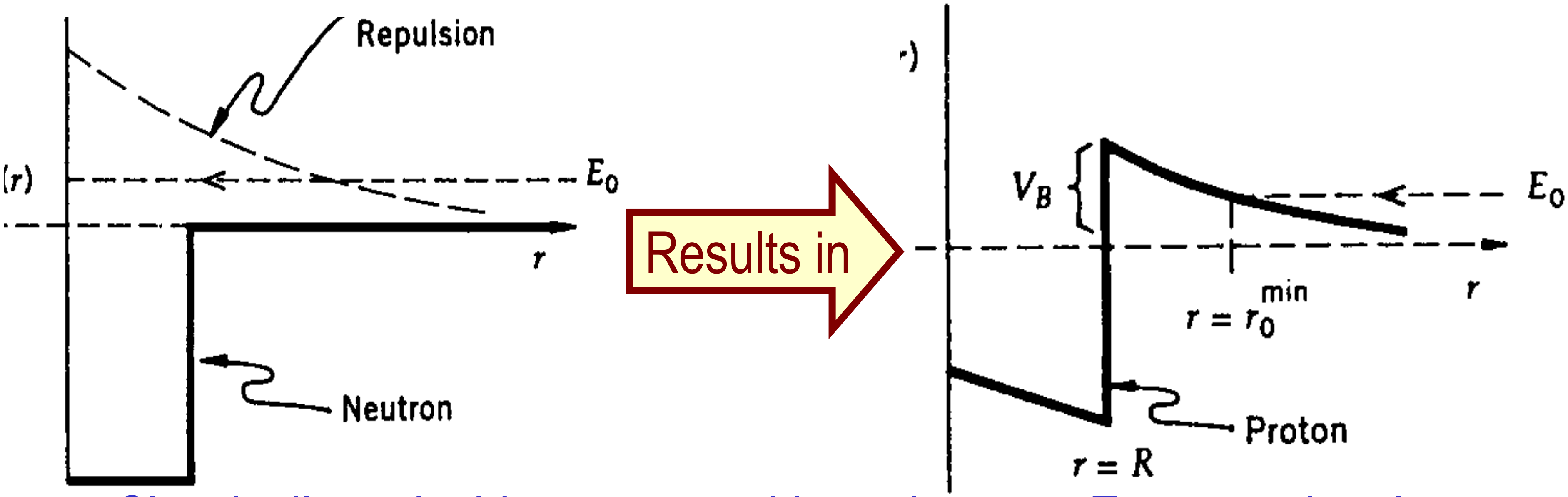
  $\Rightarrow$It must be attractive!!
- However, scattering experiments with high energy revealed a repulsive core!!
  - Below a certain length scale, the nuclear force changes from attractive to repulsive.
  - What does this tell you?
    - Nucleons have a substructure….
- This feature is good, why?
  - If the nuclear force were attractive at all distances, the nucleus would collapse in on itself.

Shape of the Nuclear Potential

- We can turn these behaviors into a square-well potential
  - For low energy particles, the repulsive core can be ignored, why?
    - Can't get there..
- This model is too simplistic, since there are too many abrupt changes in potential.
  - There would be additional effects by the Coulomb force



Nuclear Potential w/ Coulomb Corrections



- Classically an incident proton with total energy E_0 cannot be closer than $r=r_0$. Why?
  - For $R<r<r_0$, $V(r) > E_0$ and $KE<0$ ➔ Physically impossible
- What about a neutron?
  - Could penetrate into the nuclear center.
- Low energy scattering experiment did not provide the exact shape of the potential but the range and height of the potential
- The square-well shape provides a good phenomenological description of the nuclear force.

Nuclear Potential

- A square well nuclear potential → provides the basis of quantum theory with discrete energy levels and corresponding bound state just like in atoms
  - Presence of nuclear quantum states have been confirmed through
    - Scattering experiments
    - Studies of the energies emitted in nuclear radiation
- Studies of mirror nuclei and the scatterings of protons and neutrons demonstrate
  - Without the Coulomb effects, the forces between two neutrons, two protons or a proton and a neutron are the same
    - Nuclear force is independent of electrical charge
    - Protons and neutrons behave the same under the nuclear force
  - Inferred as **charge independence** of nuclear force.

Nuclear Potential – Iso-spin symmetry

- Strong nuclear force is independent of the electric charge carried by nucleons
  - Concept of strong isotopic-spin symmetry.
    - proton and neutron are the two different iso-spin state of the same particle called nucleon
  - In other words,
    - If the Coulomb effect were turned off, protons and neutrons would be indistinguishable in their nuclear interactions
    - Can you give another case just like this???
  - This is analogues to the indistinguishability of spin up and down states in the absence of a magnetic field!!
- This is called Iso-spin symmetry!!!

Range of the Nuclear Force

- EM force can be understood as a result of a photon exchange
  - Photon propagation is described by the Maxwell's equation
  - Photons propagate at the speed of light.
  - What does this tell you about the mass of the photon?
    - Massless
- Coulomb potential is $$V(r) \propto \frac{1}{r}$$

Massless particle exchange

- What does this tell you about the range of the Coulomb force?
  - Long range. Why?

Yukawa Potential

- For massive particle exchanges, the potential takes the form

$$V(r) \propto \frac{e^{-\frac{mc}{\hbar}r}}{r}$$

  - What is the mass, m, in this expression?
    - Mass of the particle exchanged in the interaction
      - The force mediator mass
- This form of potential is called Yukawa Potential
  - Formulated by Hideki Yukawa in 1934
- What does Yukawa potential turn to in the limit m →0?
  - Coulomb potential

Ranges in Yukawa Potential

- From the form of the Yukawa potential

$$V(r) \propto \frac{e^{-\frac{mc}{\hbar}r}}{r} = \frac{e^{-r/\lambda\!\!\!^{-}}}{r}$$

- The range of the interaction is given by some characteristic value of r. What is this?
  - Compton wavelength of the mediator with mass, m: $\lambda\!\!\!^{-} = \frac{\hbar}{mc}$
- What does this mean?
  - Once the mass of the mediator is known, range can be predicted
  - Once the range is known, the mass can be predicted

Ranges in Yukawa Potential

- Let's put Yukawa potential to work
- What is the range of the nuclear force?
  - About the same as the typical size of a nucleus
    - 1.2×10^{-13}cm
  - thus the mediator mass is

$$mc^2 = \frac{\hbar c}{\lambda} \approx \frac{197\,MeV-fm}{1.2\,fm} \approx 164\,MeV$$

- This is close to the mass of a well known π meson (pion)

$$m_{\pi^+} = m_{\pi^-} = 139.6\,MeV/c^2;\quad m_{\pi^0} = 135\,MeV/c^2$$

- Thus, it was thought that π are the mediators of the nuclear force