PHYS 3446 – Lecture #9

Monday, Oct. 3, 2016

Dr. Jae Yu

- Nuclear Models
  - Fermi Gas Model
  - Shell Model
  - Collective Model
  - Super-deformed Nuclei
- Nuclear Radiation
  - Alpha decay
  - Beta decay

Announcement

- First term exam
  - Date and time: 2:30 – 3:50pm, Monday, Oct. 10
  - Location: SH125
  - Covers: Ch 1 – Ch 3 or what we finish Wednesday, Oct. 5, + Appendix A
  - Can bring your calculator but no phone or computer can be used as a replacement
- First quiz results
  - Class average: 56.5/120
    - Equivalent to 47.1/100
  - Class top score: 89/120

Nuclear Models: Fermi Gas Model

- An early attempt to incorporate quantum effects
- Assumes nucleus as a gas of free protons and neutrons confined to the nuclear volume
  - The nucleons occupy quantized (discrete) energy levels
  - Nucleons are moving inside a spherically symmetric well with the range determined by the radius of the nucleus
  - Depth of the well is adjusted to obtain correct binding energy
- Protons carry electric charge ➔ Senses slightly different potential than neutrons

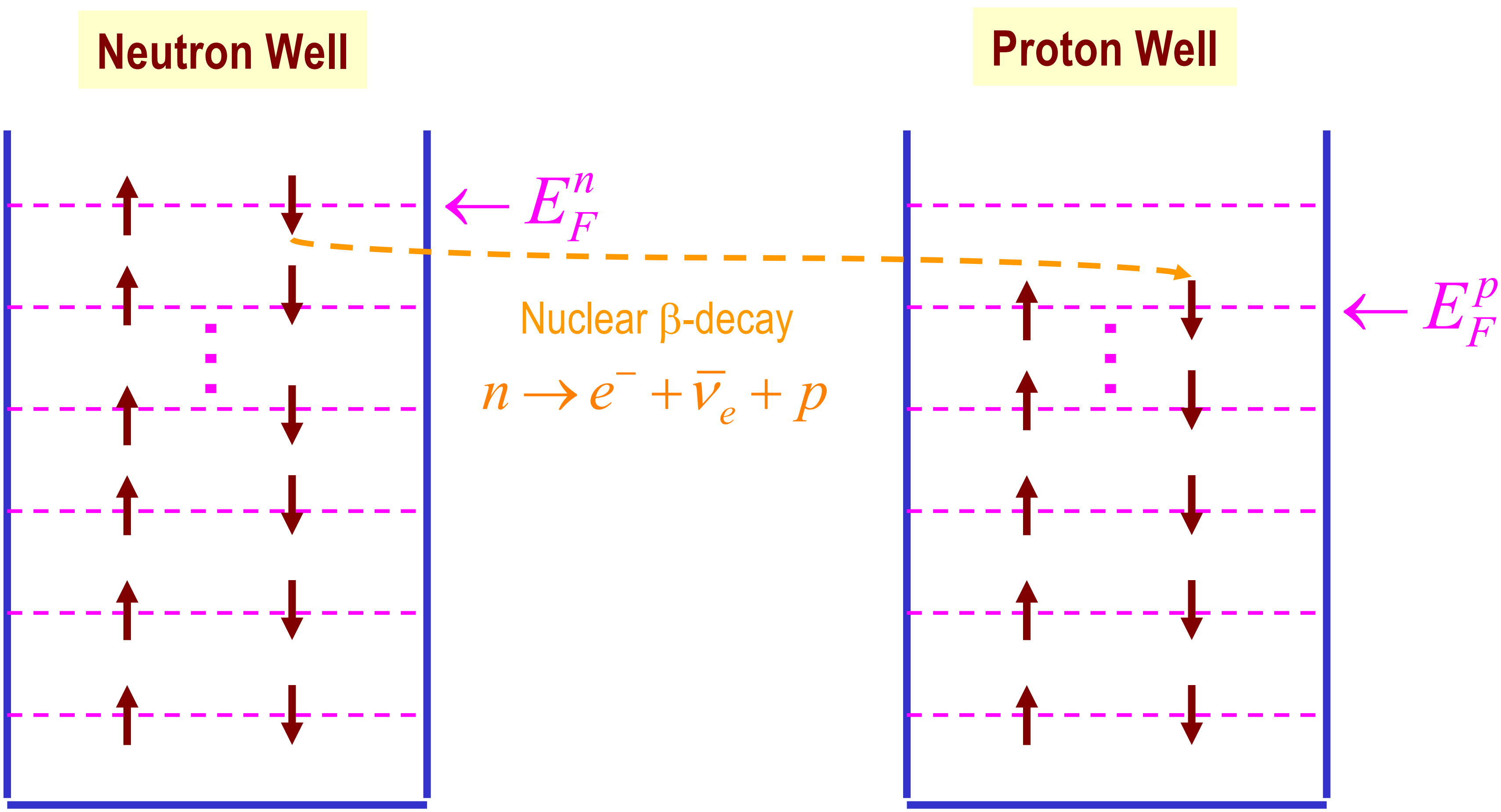
Nuclear Models: Fermi Gas Model

- Nucleons are Fermions (spin ½ particles) so
  - Obey Pauli exclusion principle
  - Any given energy level can be occupied by at most two identical nucleons – opposite spin projections
- For a greater stability, the energy levels fill up from the bottom up to the Fermi level
  - Fermi level: Highest, fully occupied energy level (E_F)
- Binding energies are given as follows:
  - BE of the last nucleon= E_F since no Fermions above E_F
  - In other words, the level occupied by Fermion reflects the BE of the last nucleon

Nuclear Models: Fermi Gas Model

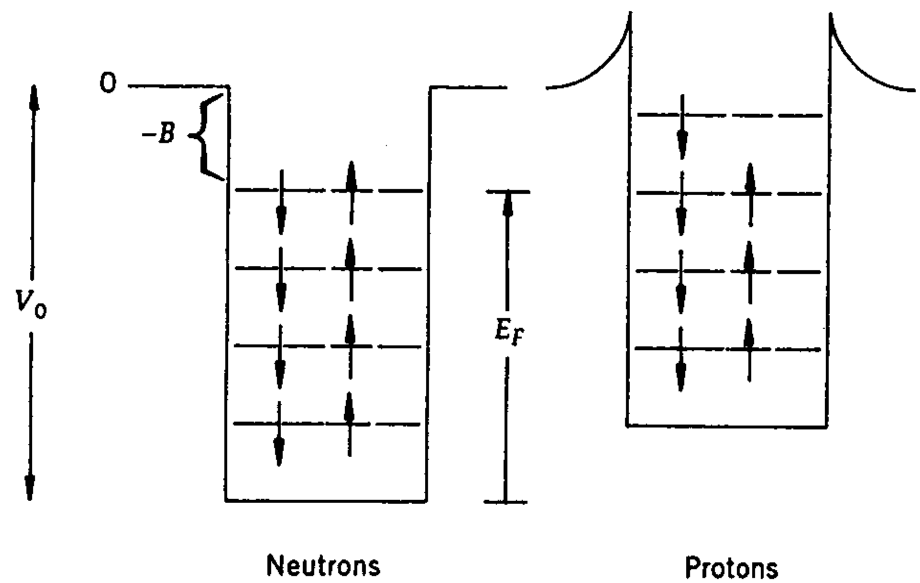
- Experimental observations show BE is charge independent
- If the well depth is the same for p and n, BE for the last nucleon would be charge dependent for heavy nuclei (Why?)
  - Since there are more neutrons than protons, neutrons sit higher E_F

Same Depth Potential Wells



Nuclear Models: Fermi Gas Model

- Experimental observations show BE is charge independent
- If the well depth is the same for p and n, BE for the last nucleon would be charge dependent for heavy nuclei (Why?)
  - Since there are more neutrons than protons, neutrons sit higher E_F
  - But experiments observed otherwise
- E_F must be the same for protons and neutrons. How do we make this happen?
  - Make protons move to a shallower potential well
- What happens if this weren't the case?
  - Nucleus is unstable.
  - All neutrons at higher energy levels would undergo a β-decay and transition to lower proton levels



Fermi Gas Model: E_F vs n_F

- Fermi momentum: $E_F = p_F^2/2m \Rightarrow p_F = \sqrt{2mE_F}$
- Volume for momentum space up to Fermi level $V_{p_F} = \frac{4\pi}{3} p_F^3$
- Total volume for the states (kinematic phase space)
  - Proportional to the total number of quantum states in the system

$$V_{TOT} = V \cdot V_{p_F} = \frac{4\pi}{3} r_0^3 A \cdot \frac{4\pi}{3} p_F^3 = \left(\frac{4\pi}{3}\right)^2 A \left(r_0 p_F\right)^3$$

- Using Heisenberg's uncertainty principle: $\Delta x \Delta p \geq \frac{1}{2}\hbar$
- The minimum volume associated with a physical system becomes $V_{state} = (2\pi\hbar)^3$
- The n_F that can fill up to E_F is

$$n_F = 2\frac{V_{TOT}}{(2\pi\hbar)^3} = \frac{2}{(2\pi\hbar)^3}\left(\frac{4\pi}{3}\right)^2 A\left(r_0 p_F\right)^3 = \frac{4}{9\pi} A \left(\frac{r_0 p_F}{\hbar}\right)^3$$

Why?

Fermi Gas Model: E_F vs n_F

- Let's consider a nucleus with N=Z=A/2 and assume that all states up to Fermi level are filled

$$N = Z = \frac{A}{2} = \frac{4}{9\pi} A \left(\frac{r_0 p_F}{\hbar} \right)^3 \quad \text{or} \quad p_F = \frac{\hbar}{r_0} \left(\frac{9\pi}{8} \right)^{1/3}$$

- What do you see about p_F above?
  - Fermi momentum is constant, independent of the number of nucleons

$$E_F = \frac{p_F^2}{2m} = \frac{1}{2m} \left(\frac{\hbar}{r_0} \right)^2 \left(\frac{9\pi}{8} \right)^{2/3} \approx \frac{2.32}{2mc^2} \left(\frac{\hbar c}{r_0} \right)^2 \approx \frac{2.32}{2 \cdot 940} \left(\frac{197 MeV - fm}{1.2 fm} \right) \approx 33 MeV$$

- Using the average BE of -8MeV, the depth of potential well (V_0) is ~40MeV
  - Consistent with other findings
- This model is a natural way of accounting for a_4 term in Bethe-Weizsacker mass formula

Nuclear Models: Shell Model

- Exploit the success of atomic model
  - Uses orbital structure of nucleons
  - Electron energy levels are quantized
  - Limited number of electrons in each level based on the available spin and angular momentum configurations
    - For n^{th} energy level, ℓ angular momentum (ℓ<n), one expects a total of $2(2\ell+1)$ possible degenerate states for electrons

Atomic Shell Model Reminder

- Orbits and energy levels an electron can occupy are labeled by
  - Principle quantum number: n
    - n can only be integer
  - For given n, energy degenerate orbital angular momentum: ℓ
    - The values are given from 0 to $n-1$ for each n
  - For any given orbital angular momentum, there are $(2\ell+1)$ sub-states: m_ℓ
    - $m_\ell = -\ell, -\ell+1, \ldots, 0, 1, \ldots, \ell-1, \ell$
    - Due to rotational symmetry of the Coulomb potential, all these sub-states are degenerate in energy
  - Since electrons are fermions w/ intrinsic spin angular momentum $\frac{1}{2}\hbar$,
    - Each of the sub-states can be occupied by two electrons
  - So the total number of state is $2(2\ell+1)$

Nuclear Models: Shell Model

- Exploit the success of atomic model
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  - Limited number of electrons in each level based on available spin and angular momentum configurations
    - For n^{th} energy level, ℓ angular momentum ($\ell<n$), one expects a total of $2(2\ell+1)$ possible degenerate states for electrons
- Quantum numbers of individual nucleons are taken into account to affect the fine structure of spectra

Nuclear Models: Shell Model

- Nuclei have magic numbers just like inert atoms
  - Atoms: Z=2, 10, 18, 36, 54
  - Nuclei: N=2, 8, 20, 28, 50, 82, and 126;   Z=2, 8, 20, 28, 50, and 82
  - Magic Nuclei: Nuclei with either N or Z a magic number ➔ Stable
  - Doubly magic nuclei: Nuclei with both N and Z magic numbers ➔ Particularly stable (some examples?)
- Explains well the stability of nucleus

Shell Model: Various Potential Shapes

- To solve the equation of motion in quantum mechanics, Schrödinger equation, one must know the shape of the potential
  - $$\left(\vec{\nabla}^2 + \frac{2m}{\hbar^2}\left(E - V\left(r\right)\right)\right)\varphi\left(\vec{r}\right) = 0$$
  - Details of nuclear potential not well known
- A few shapes of potential energies tried out
  - Infinite square well: Each shell can contain up to $2(2\ell+1)$ nucleons

Nuclear Models: Shell Model – Square well potential case

N_M	n	$\ell=n-1$	$N_s=2(2\ell+1)$	N_T
2	1	0	2	**2**
8	2	0,1	2+6	**8**
20	3	0,1,2	2+6+10	18
28	4	0,1,2,3	2+6+10+14	32
50	5	0,1,2,3,4	2+6+10+14+18	**50**
82	6	0,1,2,3,4,5	2+6+10+14+18+22	72

Shell Model: Various Potential Shapes

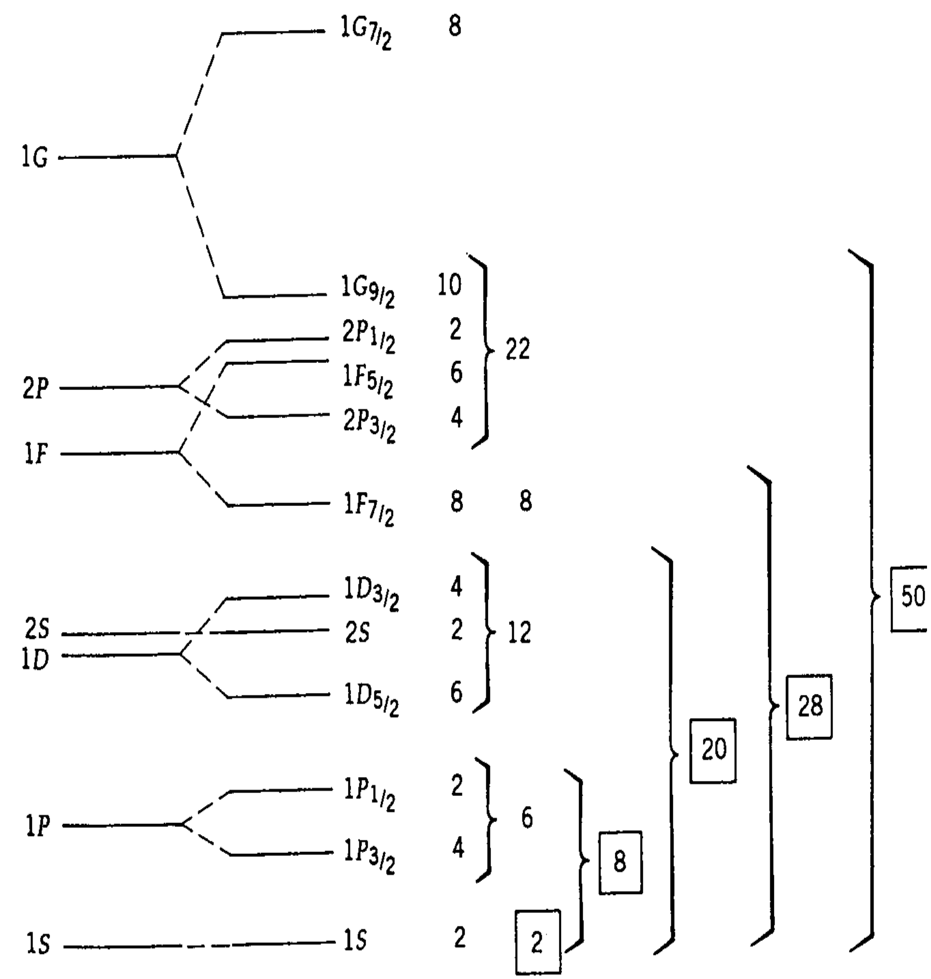
- To solve equation of motion in quantum mechanics, Schrödinger equation, one must know the shape of the potential
  - $\left(\vec{\nabla}^2 + \frac{2m}{\hbar^2}\left(E - V(r)\right)\right)\varphi(\vec{r}) = 0$
  - Details of nuclear potential not well known
- A few models of potential tried out
  - Infinite square well: Each shell can contain up to $2(2\ell+1)$ nucleons
    - Can predict 2, 8 and 50 but no other magic numbers
  - Three dimensional harmonic oscillator: $V(r) = \frac{1}{2}m\varpi^2 r^2$
    - Predicts 2, 8, 20, 40 and 70 ➔ Some magic numbers predicted

Shell Model: Spin-Orbit Potential

- Central potential could not reproduce all magic numbers
- In 1940, Mayer and Jesen proposed a central potential + strong spin-orbit interaction w/

$$V_{TOT} = V(r) - f(r)\vec{L}\cdot\vec{S}$$

  – $f(r)$ is an arbitrary empirical function of radial coordinates and chosen to fit the data
- For the spin-orbit interaction with the properly chosen $f(r)$, a finite square well can split
- Reproduces all the desired magic numbers



Spectroscopic notation: n L j

Orbit number

Orbital angular momentum

Projection of total momentum

Predictions of the Shell Model

- Spin-Parity of large number of odd-A nuclei predicted well
  - Nucleons are Fermions and obey Pauli exclusion principle
  - ➔ Fill up ground state energy levels in pairs
  - Ground state of all even-even nuclei have zero total angular momentum
- The shell model cannot predict stable odd-odd nuclei spins
  - No prescription for how to combine the unpaired proton and neutron spins

Predictions of the Shell Model

- Magnetic Moment of neutron and proton are

$$\mu_p \approx 2.79\mu_N \qquad \mu_n \approx -1.91\mu_N$$

- Intrinsic magnetic moment of unpaired nucleons contribute to total magnetic moment of nuclei
  - What does a deuteron consist of?

$$\mu_D = \mu_p + \mu_n = 2.79\,\mu_N - 1.91\mu_N = 0.88\,\mu_N$$

    - Measured value is $\mu_D = 0.86\,\mu_N$
  - For Boron ($^{10}B^{5}$) , the 5 neutrons and 5 protons have the same level structure: $(1S_{1/2})^2(1P_{3/2})^3$, leaving one of each unpaired proton and neutron in angular momentum ℓ=1 state ➔ $\mu = \frac{e\hbar}{2m_N c}\cdot l = \frac{e\hbar}{2m_N c}\cdot 1 = \mu_N$

$$\mu_B = \mu_p + \mu_n + \mu_{orbit} = 2.79\,\mu_N - 1.91\mu_N + \mu_N = 1.88\,\mu_N$$

    - Measured value is $\mu_B = 1.80\,\mu_N$
- Does not work well with heavy nuclei

Collective Model

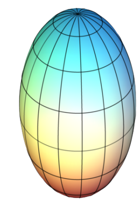
- For heavy nuclei, shell model predictions do not agree with experimental measurements
  - Especially in magnetic dipole moments
- Measured values of quadrupole moments for closed shells differ significantly with experiments
  - Some nuclei's large quadrupole moments suggests significant nonspherical shapes
  - The assumption of rotational symmetry in shell model does not seem quite right
- These deficiencies are somewhat covered through the reconciliation of liquid drop model and the Shell model
  - Bohr, Mottelson and Rainwater's collective model, 1953

Collective Model

- Assumption
  - Nucleus consists of hard core of nucleons in filled shells
  - Outer valence nucleons behave like the surface molecules in a liquid drop
  - Non-sphericity of the central core is caused by the surface motion of the valence nucleons
- Thus, in the collective model, the potential is a shell model with a spherically asymmetric potential
  - Aspherical nuclei can produce additional energy levels upon rotation while spherical ones cannot
- Important predictions of the collective model:
  - Existence of rotational and vibrational energy levels in nuclei
  - Accommodate decrease of spacing between first excite state and the ground level for even-even nuclei as A increases, since moment of inertia increases with A
  - Spacing is largest for closed shell nuclei, since they tend to be spherical

Super-deformed Nuclei

- Nuclei tend to have relatively small intrinsic spins
- Particularly stable nuclei predicted for A between 150 and 190 with spheroidal character
  - Semi-major axis about a factor of 2 larger than semi-minor



- Heavy ion collisions in late 1980s produced super-deformed nuclei with angular momentum of $60\hbar$
- The energy level spacing of these observed through photon radiation seem to be fixed
- Different nuclei seem to have identical emissions as they spin down
- Problem with collective model and understanding of strong pairing of nucleon binding energy
- Understanding nuclear structure still in progress

Nuclear Radiation: Alpha Decay

- Represents the disintegration of a parent nucleus to a daughter through an emission of a He nucleus
- Reaction equation is

$$^{A}X^{Z} \rightarrow {}^{A-4}Y^{Z-2} + {}^{4}He^{2}$$

- α-decay is a spontaneous fission of the parent nucleus into two daughters of highly asymmetric masses
- Assuming parent at rest, from the energy conservation

$$M_P c^2 = M_D c^2 + T_D + M_\alpha c^2 + T_\alpha$$

- Re-organizing the terms, we obtain

$$T_D + T_\alpha = \left(M_P - M_D - M_\alpha\right)c^2 = \Delta M c^2$$

Nuclear Radiation: Alpha Decay

- Since electron masses cancel, we could use atomic mass expression

$$T_D + T_\alpha = \big(M(A,Z) - M(A-4,Z-2) - M(4,2)\big)c^2 \equiv Q$$

- This is the definition of the **disintegration energy** or **Q-value**
  - Difference of rest masses of the initial and final states
  - Q value is equal to the sum of the final state kinetic energies
  - Energy lost during the disintegration process
- For non-relativistic particles, KE are

$$T_D = \frac{1}{2} M_D v_D^2 \qquad T_\alpha = \frac{1}{2} M_\alpha v_\alpha^2$$

Nuclear Radiation: Alpha Decay

- Since the parent is at rest, from the momentum conservation

$$M_D v_D = M_\alpha v_\alpha \Rightarrow v_D = \frac{M_\alpha}{M_D} v_\alpha$$

- If $M_D = M_\alpha,\ v_D = v_\alpha$, then $T_D = T_\alpha$
- We can write the relationship of KE and Q-value as

$$T_D + T_\alpha = \frac{1}{2} M_D v_D^2 + \frac{1}{2} M_\alpha v_\alpha^2 = \frac{1}{2} M_D \left(\frac{M_\alpha}{M_D} v_\alpha \right)^2 + \frac{1}{2} M_\alpha v_\alpha^2$$

$$T_D + T_\alpha = T_\alpha \frac{M_\alpha + M_D}{M_D} \qquad \boxed{T_\alpha = \frac{M_D}{M_\alpha + M_D} Q}$$

- This means that T_α is unique for the given nuclei
- Direct consequence of 2-body decay of a parent at rest

Nuclear Radiation: Alpha Decay

- KE of the emitted α must be positive
- Thus for an α-decay to occur, it must be an exorthermic process $\Delta M \geq 0, \quad Q \geq 0$
- For a massive nuclei, the daughter's KE is

$$T_D = Q - T_\alpha = \frac{M_\alpha}{M_\alpha + M_D} Q = \frac{M_\alpha}{M_D} T_\alpha \ll T_\alpha$$

- Since $M_\alpha / M_D \approx 4/(A-4)$, we obtain

$$T_\alpha \approx \frac{A-4}{A} Q \qquad T_D \approx \frac{4}{A} Q$$

Nuclear Radiation: Alpha Decay

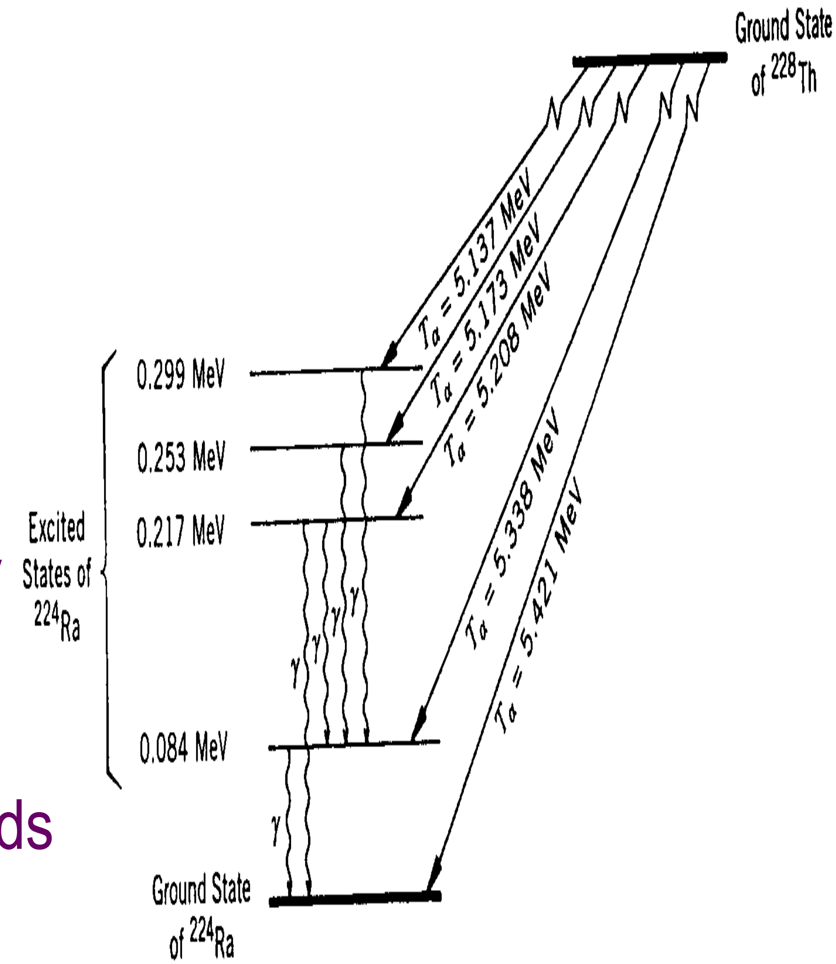
- Most energetic α-particles produced alone
  - Parent nucleus decays to the ground state of a daughter and produces an α-particle whose KE is the entire Q value
- Less energetic ones accompany photons – mostly delayed…
  - Indicates quantum energy levels
  - Parent decays to an excited state of the daughter after emitting an α

$$^{A}X^{Z} \rightarrow {}^{A-4}Y^{*Z-2} + {}^{4}He^{2}$$

  - Daughter then subsequently de-excite by emitting a photon

$$^{A-4}Y^{*Z-2} \rightarrow {}^{A-4}Y^{Z-2} + \gamma$$

  - Difference in the two Q values corresponds to the energy of the photon



Nuclear Radiation: α-Decay Example

- $^{240}Pu^{94}$ decay reaction is

$$^{240}Pu^{94} \rightarrow {}^{236}U^{92} + {}^{4}He^{2}$$

- α particles are observed with 5.17MeV and 5.12 MeV
- Since $Q = \frac{A}{A-4} T_{\alpha}$
- We obtain the two Q-values

$$Q_1 \approx \frac{240}{236} 5.17MeV = 5.26MeV \qquad Q_2 \approx \frac{240}{236} 5.12MeV = 5.21MeV$$

- Which yields photon energy of $E_{\gamma} = \Delta Q = Q_1 - Q_2 = 0.05MeV$
- Consistent with experimental measurement, 45KeV
- Indicates the energy level spacing of order 100KeV for nuclei
  – Compares to order 1eV spacing in atomic levels

Discovery of Alpha and Beta Radiations

- After Bacquerel's discovery of uranium effect on photo-films in 1896
- The Curies began study of radio activity in 1898
- Rutherford also studied using a more systematic experimental equipments in 1898
  - Measured the currents created by the radiations
- While Bacquerel concluded that rays are observed in different levels
- Rutherford made the observation using electrometer and determined that there are at least two detectable rays
  - Named α and β rays

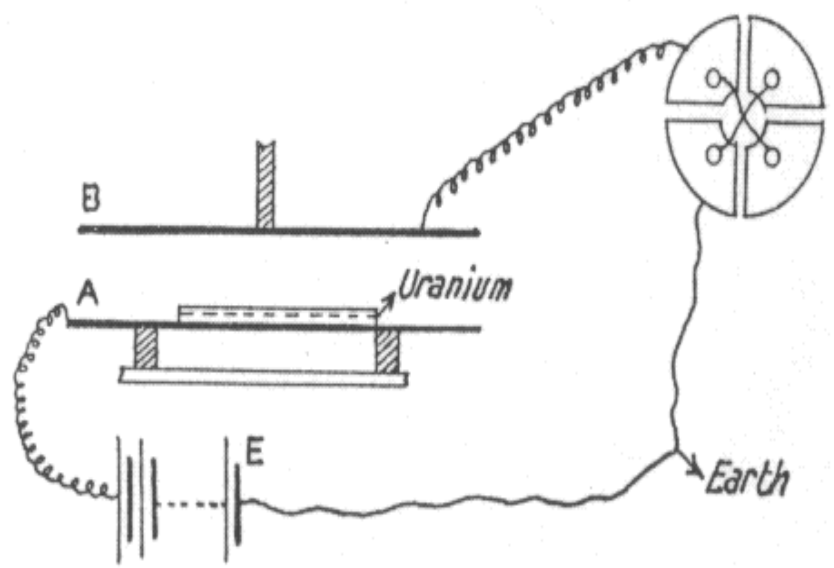


Fig. 1

Nuclear Radiation: β-Decays

- Three kinds of β-decays
  - Electron emission
    - Nucleus with large N_n
    - Atomic number increases by one
    - Nucleon number stays the same

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-}$$

  - Positron emission
    - Nucleus with many protons
    - Atomic number decreases by one
    - Nucleon number stays the same

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+}$$

  - You can treat nuclear reaction equations algebraically
    - The reaction is valid in the opposite direction as well
    - Any particle moved over the arrow becomes its anti particle

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} \quad \Longrightarrow \quad {}^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1}$$

Nuclear Radiation: β-Decays

- Electron capture
  - Nucleus with many protons
  - Absorbs a K-shell atomic electron $\quad {}^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1}$
  - Proton number decreases by one
  - Causes cascade X-ray emission from the transition of remaining atomic electrons

- For β-decay: ΔA=0 and |ΔZ|=1

Nuclear Radiation: β-Decays

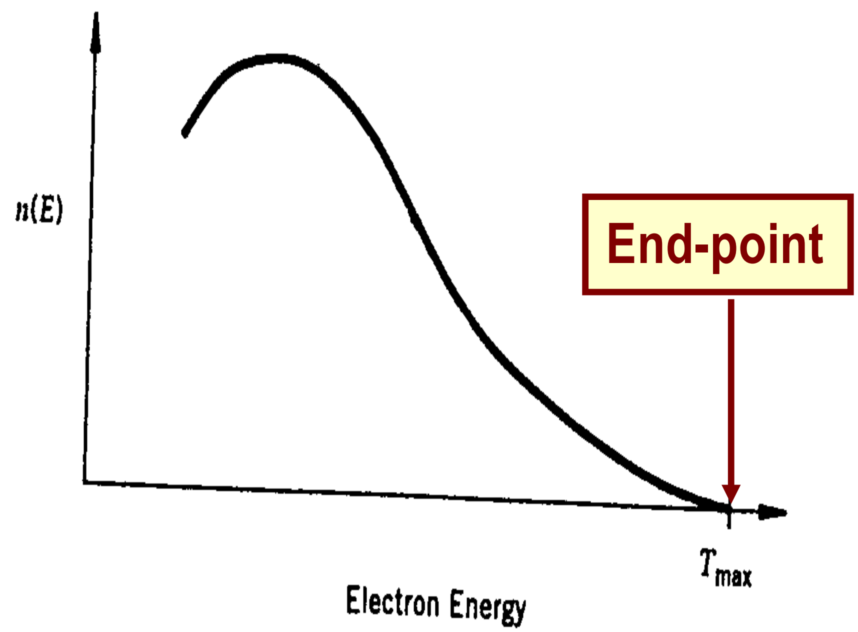
- Initially assumed to be 2-body decay $^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-}$
- From energy conservation

$$E_X = E_Y + E_{e^-} = E_Y + T_e + m_e c^2$$

- Since lighter electron carries most the KE

$$T_e = \left(E_X - E_Y - m_e c^2\right) = \left(m_X - m_Y - m_e\right)c^2 - T_Y = Q - T_Y \approx Q$$

- Results in a **unique Q value** as in α-decay.
- In reality, electrons emitted with continuous E spectrum with an end-point given by the formula above
- Energy conservation is violated!!!!



Nuclear Radiation: β-Decays

- Angular momentum is also in trouble
- In β-decays total number of nucleons is conserved
  - Recall $|\Delta A|=0$ and $|\Delta Z|=1$ in β-decays?
- Electrons are fermions with spin $\frac{1}{2}\hbar$
- Independent of any changes of an integer orbital angular momentum, the total angular momentum cannot be conserved
  - How much does it always differ by? $\frac{1}{2}\hbar$
- Angular momentum conservation is violated!!!

Nuclear Radiation: β-Decays

- In 1931 Pauli postulated an additional particle emitted in β-decays
  - No one observed this particle in experiments
    - Difficult to detect
    - First observation of ν_e in 1956, ν_μ in 1962 and ν_τ in 1977 (direct 2000)
  - Charge is conserved in β-decays
    - Electrically neutral
  - Maximum energy of electrons is the same as the Q value
    - Massless
  - Must conserve the angular momentum
    - Must be a fermion with spin $\frac{1}{2}\hbar$
- This particle is called the neutrino (by Feynman) and expressed as ν

Nuclear Radiation: Neutrinos

- Have anti-neutrinos $\bar{\nu}$, just like other particles
- Neutrinos and anti-neutrinos are distinguished through the spin projection on momentum
  - Helicity is used to distinguish them $H \propto \vec{p} \cdot \vec{s}$
    - Left-handed (spin and momentum opposite direction) anti-electron-neutrinos are produced in β-decays
    - Right-handed electron-neutrinos are produced in positron emission
  - e^- is a particle and e^+ is the anti-particle to e^-
  - ν_e is a particle and $\bar{\nu}_e$ is the anti-particle to ν_e

β–Decay Reaction Equations with Neutrinos

- Electron emission

$$ {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-} + \bar{\nu}_{e} $$

- Positron emission

$$ {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} + \nu_{e} $$

- Electron capture

$$ {}^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1} + \nu_{e} $$

β-Decays with neutrinos

- If the parent nucleus decays from rest, from the conservation of energy

$$M_p c^2 = T_D + M_D c^2 + T_{e^-} + m_e c^2 + T_{\bar{\nu}_e} + m_{\bar{\nu}_e} c^2$$

- Thus the Q-value of a β-decay can be written

$$T_D + T_{e^-} + T_{\bar{\nu}_e} = \left(M_p - M_D - m_e - m_{\bar{\nu}_e}\right)c^2 = \Delta M c^2 = Q$$

- Electron emission can only occur if Q>0
- Neglecting all small atomic BE, e emission can occur if

$$Q = \left(M\left(A,Z\right) - M\left(A,Z+1\right) - m_e - m_{\bar{\nu}_e}\right)c^2$$
$$\approx \left(M\left(A,Z\right) - M\left(A,Z+1\right)\right)c^2 \geq 0$$

β-Decays with neutrinos

- Since the daughter nucleus is much heavier than e or ν, the small recoil energy of daughter can be ignored
  - Thus we can obtain $T_{e^-} + T_{\bar{\nu}_e} \approx Q$
- This means that the energy of the electron is not unique and can be any value in the range $0 \le T_{e^-} \le Q$
  - The maximum electron kinetic energy can be Q
  - This is the reason why the electron energy spectrum is continuous and has an end point (=Q)
- The same can apply to the other two β-decays

Particle Numbers

- Baryon numbers: A quantum number assigned to baryons (particles consist of quarks)
  - Mostly conserved in many interactions
  - Baryons: +1
  - Anti-baryons: -1
  - Protons and neutrons are baryons with baryon number +1 each
- Lepton numbers: A quantum number assigned to leptons (electrons, muons, taus and their corresponding neutrinos)
  - Leptons: +1
  - Anti-leptons: -1
  - Must be conserved at all times under SM in each species

Lepton Numbers

- Three charged leptons exist in nature with their own associated neutrinos

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

- These three types of neutrinos are distinct from each other
  - muon neutrinos never produce other leptons than muons or anti-muons

$$\nu_\mu + {}^A X^Z \rightarrow {}^A Y^{Z+1} + \mu^-$$

$$\nu_\mu + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + e^-$$

$$\nu_\mu + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + \tau^-$$

Lepton Numbers

For electron neutrinos

$$\nu_e + {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^-$$

$$\nu_e + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + \mu^-$$

$$\nu_e + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + \tau^-$$

For tau neutrinos

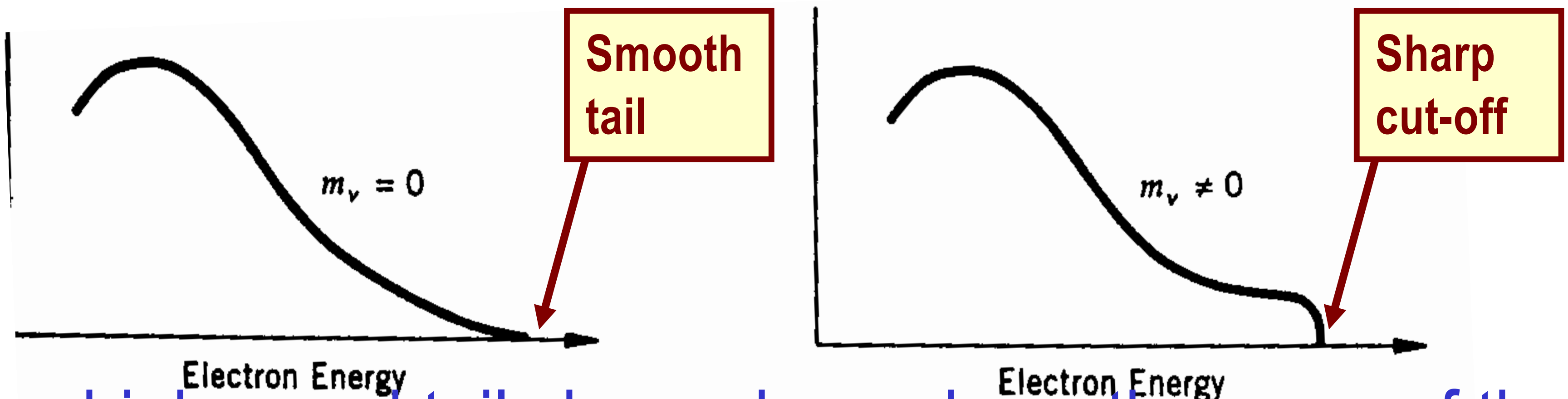
$$\nu_\tau + {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + \tau^-$$

$$\nu_\tau + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + e^-$$

$$\nu_\tau + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + \mu^-$$

Neutrino Mass

- What does neutrino mass do to the β-spectrum?



- The higher end tail shape depends on the mass of the neutrino
  - β-spectrum could be used to measure the mass of neutrinos
    - Very sensitive to the resolution on the device
  - Most stringent direct limit is $m_\nu < 2eV/c^2$
- Non-zero mass of the neutrino means
  - Neutrino Oscillation: Mixing of neutrino species

Weak Interactions

- β-decay can be written at the nucleon level as:

$$n \rightarrow p + e^- + \bar{\nu}_e \qquad p \rightarrow n + e^+ + \nu_e \qquad p + e^- \rightarrow n + \nu_e$$

- Since neutrons are heavier than protons, they can decay to a proton in a free space
  - On the other hand, protons are lighter than neutrons therefore they can only undergo a β-decay within a nucleus
  - Life time of a neutron is about 900sec
  - This life time is a lot longer than nuclear reaction time scale 10^{-23} s or EM scale 10^{-16} s.
- This means that a β-decay is a nuclear phenomenon that does not involve strong nuclear or EM forces
- Fermi postulated a new weak force responsible for β-decay

Weak Interactions

- Weak forces are short ranged
  - How do we know this?
    - Occurs in the nuclear domain
    - Weakness of the strength is responsible for long life time seen in β-decays
- Nucleus does not contain electrons
  - Electrons in β-decays must come from somewhere else
  - Electrons are emitted without time delay
    - The electron must come at the time of decay just like the alphas from a nuclear disintegration
  - β-decay can be considered to be induced by the weak force

Weak Interactions

- The transition probability per unit time, the width, can be calculated from perturbation theory using Fermi's Golden rule

$$P = \frac{2\pi}{\hbar}\left|H_{fi}\right|^2 \rho\left(E_f\right)$$

- Where the weak interaction Hamiltonian is

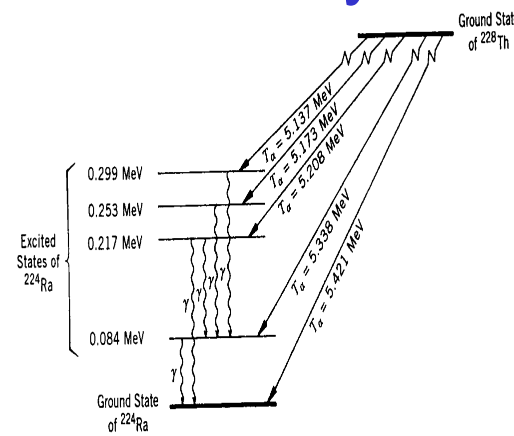
$$H_{fi} = \left\langle f\left|H_{wk}\right|i\right\rangle = \int d^3x\psi_f^*\left(x\right)H_{wk}\psi_i\left(x\right)$$

Weak Interactions

- Based on β-decay reaction equations, the H_{wk} must be a four fermionic states
  - H_{wk} proposed by Fermi in 1933 is a **four-fermion interaction** or current-current interaction
  - Relativistic
  - Agreed rather well with experiments for low energy β-decays
- Parity violation
  - There are only left-handed neutrinos and right-handed anti-neutrinos
  - A system is parity invariant if it does not change under inversion of spatial coordinates
  - The spin $\vec{r} \rightarrow -\vec{r}, \ \vec{p} \rightarrow -\vec{p} \ \Rightarrow \vec{L} = \vec{r} \times \vec{p} = (-\vec{r}) \times (-\vec{p}) = \vec{L}$
  - The handedness, helicity, changes upon the spatial inversion since the direction of the motion changes while the spin direction does not
  - Since there is no right handed neutrinos, parity must be violated in weak interactions

Gamma Decays

- When a heavy nuclei undergo alpha and beta decays, the daughters get into an excited state
  - Must either break apart
  - Or emit another particle
  - To bring the daughter into its ground state



- Typical energies of photons in γ-decays are a few MeV's
  - These decays are EM interactions thus the life time is on the order of 10^{-16}sec.
- Photons carry one unit of angular momentum
  - Parity is conserved in this decay

Assignments

1. Reading assignment: CH 4.3
2. End of the chapter problems: 3.2
3. Derive the following equations:
   - Eq. 4.8 starting from conservation of energy
   - Eq. 4.11 both the formula

- Due for these homework problems is next Monday, Oct. 16.