PHYS 3446 – Lecture #10

Wednesday, Oct. 5, 2016

*Dr. **Jae** Yu*

- Nuclear Radiation
- Beta Decay & Weak Interactions
- Gamma Decay
- Energy Deposition in Media
- Charged Particle Detection
- Ionization Process
- Photon Energy Loss

Announcement

- First term exam
  - Date and time: 2:30 – 3:50pm, Monday, Oct. 10
  - Location: SH125
  - Covers: Ch 1 – Ch 4 + Appendix A
  - Can bring your calculator but no phone or computer can be used as a replacement

Assignments

1. Reading assignment: CH 4.3; CH5
2. End of the chapter problems: 3.2
3. Derive the following equations:
   - Eq. 4.8 starting from conservation of energy
   - Eq. 4.11 both the formula

- Due for these homework problems is next Monday, Oct. 16.

Nuclear Radiation: α-Decay Example

- $^{240}Pu^{94}$ decay reaction is

$$^{240}Pu^{94} \rightarrow {}^{236}U^{92} + {}^{4}He^{2}$$

- α particles are observed with T_α=5.17 and 5.12 MeV
- Since $Q = \dfrac{A}{A-4} T_\alpha$
- We obtain the two Q-values

$$Q_1 \approx \frac{240}{236} 5.17 MeV = 5.26 MeV \qquad Q_2 \approx \frac{240}{236} 5.12 MeV = 5.21 MeV$$

- Which yields photon energy of $E_\gamma = \Delta Q = Q_1 - Q_2 = 0.05 MeV$
- Consistent with the experimental measurement, 45KeV
- Indicates the energy level spacing of order 100KeV for nuclei
  - Compares to order 1eV spacing in atomic levels

Discovery of Alpha and Beta Radiations

- After Bacquerel's discovery of uranium effect on photo-films in 1896
- The Curies began to study radio activity in 1898
- Rutherford also studied using a more systematic experimental equipment in 1898
  - Measured the currents created by the radiations
- While Bacquerel concluded that the rays are observed in different levels
- Rutherford made the observation using electrometer and determined that there are at least two detectable rays
  - Named α and β rays

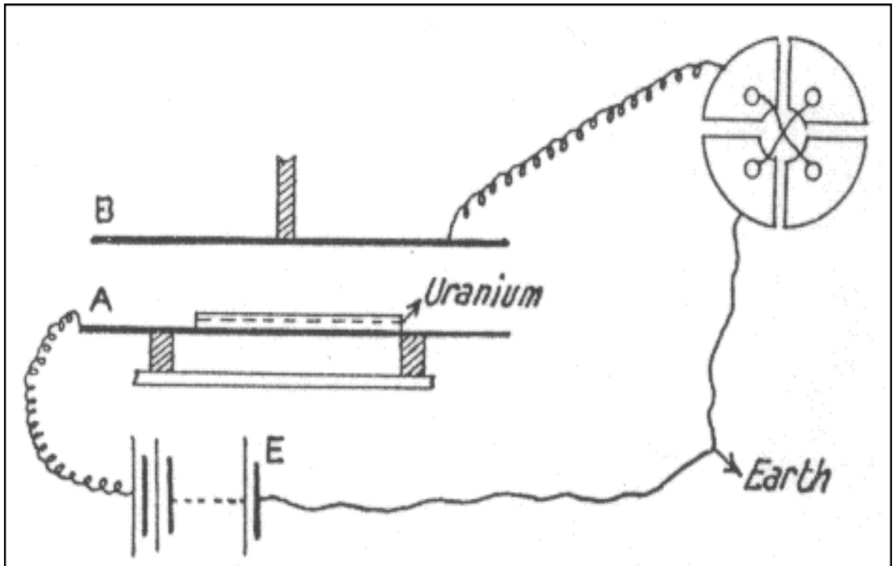


Fig. 1

Nuclear Radiation: β-Decays

- Three kinds of β-decays
  - Electron (β) emission
    - Nucleus with large N_n
    - Atomic number increases by one
    - Nucleon number stays the same

$$ {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-} $$

  - Positron (β^{+}) emission
    - Nucleus with many protons
    - Atomic number decreases by one
    - Nucleon number stays the same

$$ {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} $$

  - You can treat nuclear reaction equations algebraically
    - The reaction is valid in the opposite direction as well
    - Any particle moved over the arrow becomes its anti particle

$$ {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} \quad \Longrightarrow \quad {}^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1} $$

Nuclear Radiation: β-Decays

- Electron capture
  - Nucleus with many protons
  - Absorbs a K-shell atomic electron
  - Proton number decreases by one
  - Causes cascade X-ray emission from the transition of remaining atomic electrons

$$ {}^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1} $$

- For β-decay: ΔA=0 and |ΔZ|=1

Nuclear Radiation: β-Decays

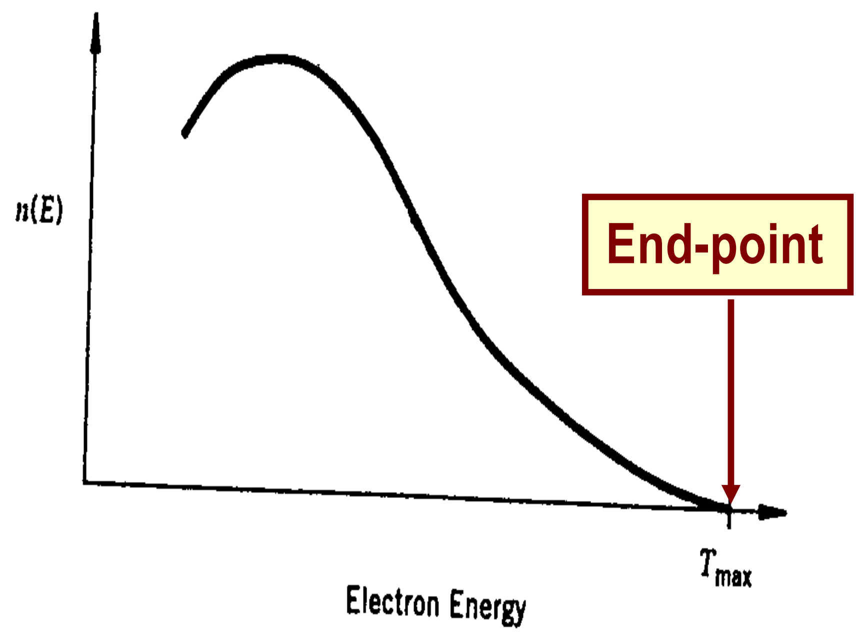
- Initially assumed to be a 2-body decay $^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-}$
- From the energy conservation

$$E_X = E_Y + E_{e^-} = E_Y + T_e + m_e c^2 = m_Y c^2 + m_e c^2 + T_e + T_Y$$

- Since lighter electron carries most the KE

$$T_e = \left(E_X - E_Y - m_e c^2\right) = \left(m_X - m_Y - m_e\right)c^2 - T_Y = Q - T_Y \approx Q$$

- Results in a **unique Q value** as in α-decay.
- In reality, electrons emitted with continuous E spectrum with an end-point given by the formula above
- Energy conservation is violated!!!!



Nuclear Radiation: β-Decays

- Angular momentum is also in trouble
- In β-decays total number of nucleons is conserved
  - Recall |ΔA|=0 and |ΔZ|=1 in β-decays?
- Electrons are fermions with spin $\frac{1}{2}\hbar$
- No matter how you change the integer orbital angular momentum, the total angular momentum cannot be conserved
  - How much does it always differ by? $\frac{1}{2}\hbar$
- Angular momentum conservation is violated!!!

Nuclear Radiation: β-Decays

- In 1931 Pauli postulated an additional particle emitted in β-decays
  - No one observed this particle in experiments
    - Difficult to detect
    - First observation of ν_e in 1956, ν_μ in 1962 and ν_τ in 1977 (direct 2000)
  - Charge is conserved in β-decays
    - Electrically neutral
  - Maximum energy of electrons is the same as the Q value
    - Massless
  - Must conserve the angular momentum
    - Must be a fermion with spin $\frac{1}{2}\hbar$
- This particle is called the neutrino (by Feynman) and expressed as ν

Nuclear Radiation: Neutrinos

- Have anti-neutrinos $\bar{\nu}$, just like other particles
- Neutrinos and anti-neutrinos are distinguished through the spin projection on momentum axis
  - Helicity is used to distinguish them $H \propto \vec{p} \cdot \vec{s}$
    - This is intrinsic for neutrinos
    - Right-handed (spin and momentum in opposite direction) anti-electron-neutrinos are produced in β-decays
    - Left-handed electron-neutrinos are produced in positron emission
  - e^- is a particle and e^+ is the anti-particle to e^-
  - ν_e is a particle and $\bar{\nu}_e$ is the anti-particle to ν_e

β–Decay Reaction Equations with Neutrinos

- Electron emission

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-} + \bar{\nu}_{e}$$

- Positron emission

$$^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^{+} + \nu_{e}$$

- Electron capture

$$^{A}X^{Z} + e^{-} \rightarrow {}^{A}Y^{Z-1} + \nu_{e}$$

β-Decays with neutrinos

- If the parent nucleus decays from rest, from the conservation of energy

$$M_p c^2 = T_D + M_D c^2 + T_{e^-} + m_e c^2 + T_{\bar{\nu}_e} + m_{\bar{\nu}_e} c^2$$

- Thus the Q-value of a β-decay can be written

$$T_D + T_{e^-} + T_{\bar{\nu}_e} = \left(M_p - M_D - m_e - m_{\bar{\nu}_e}\right)c^2 = \Delta M c^2 = Q$$

- Electron emission can only occur if Q>0
- Neglecting all small atomic BE, e emission can occur if

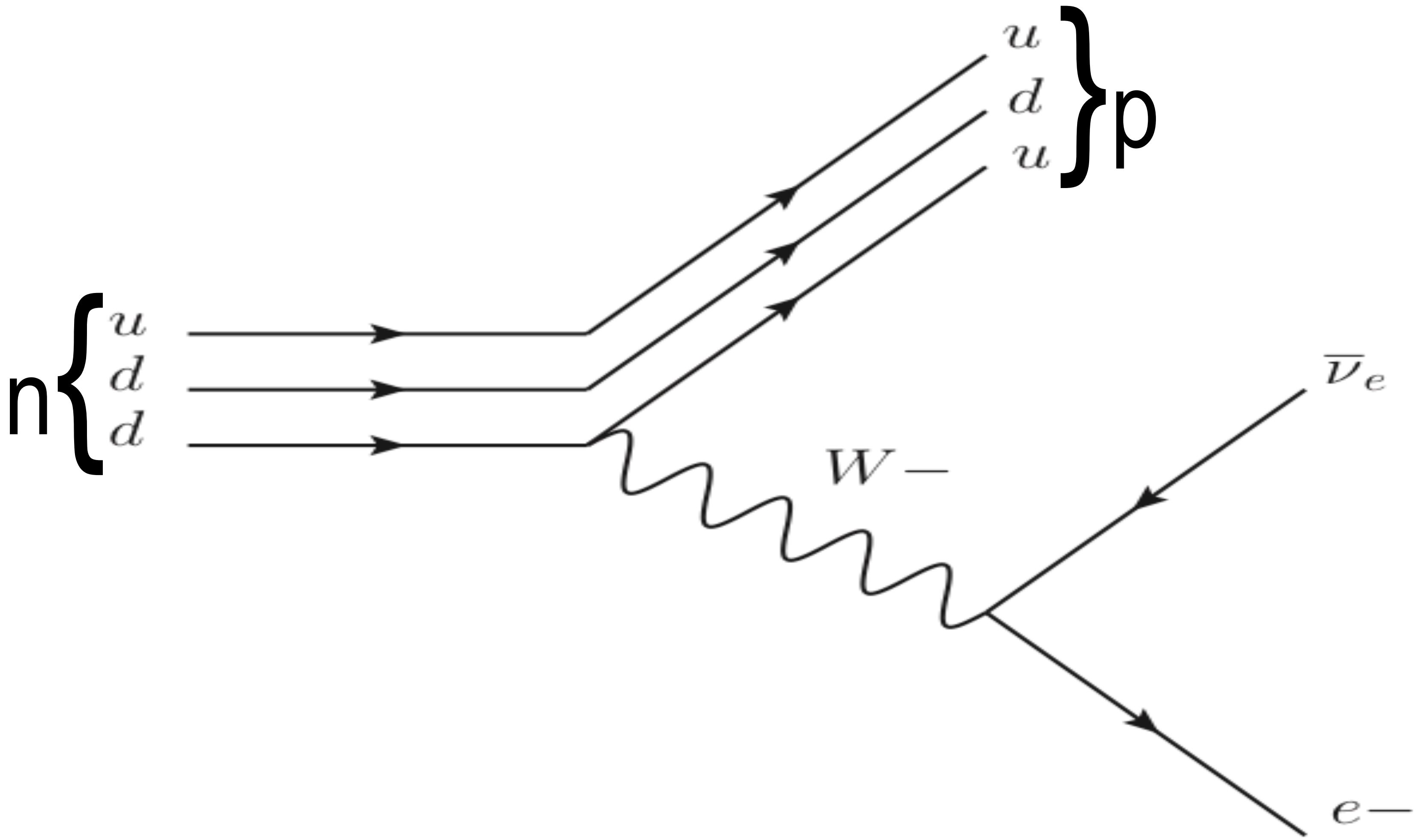
$$Q = \left(M\left(A,Z\right) - M\left(A,Z+1\right) - m_e - m_{\bar{\nu}_e}\right)c^2$$

$$\approx \left(M\left(A,Z\right) - M\left(A,Z+1\right)\right)c^2 \geq 0$$

β-Decays with neutrinos

- Since the daughter nucleus is much heavier than e or ν, the small recoil kinetic energy of the daughter can be ignored
  - Thus we can obtain $T_{e^-} + T_{\bar{\nu}_e} \approx Q$
- This means that the energy of the electron is not unique and can be any value in the range $0 \le T_{e^-} \le Q$
  - The maximum electron kinetic energy can be Q
  - This is the reason why the electron energy spectrum is continuous and has an end point (=Q)
- The same can apply to the other two β-decays

Neutron Beta Decay Feynman Diagram



Particle Numbers

- Baryon numbers: The quantum number assigned to baryons (particles consist of quarks)
  - Mostly conserved in many interactions
  - Baryons: +1
  - Anti-baryons: -1
  - Protons and neutrons are baryons with baryon number +1 each
- Lepton numbers: The quantum number assigned to leptons (electrons, muons, taus and their corresponding neutrinos)
  - Leptons: +1
  - Anti-leptons: -1
  - Must be conserved at all times under SM in each species

Lepton Numbers

- Three charged leptons exist in nature with their own associated neutrinos

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

- These three types of neutrinos are distinct from each other
  - muon neutrinos never produce other leptons than muons or anti-muons

$$\nu_\mu + {}^A X^Z \rightarrow {}^A Y^{Z+1} + \mu^-$$

$$\nu_\mu + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + e^-$$

$$\nu_\mu + {}^A X^Z \not\rightarrow {}^A Y^{Z+1} + \tau^-$$

Lepton Numbers

For electron neutrinos

$$\nu_e + {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^{-}$$

$$\nu_e + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + \mu^{-}$$

$$\nu_e + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + \tau^{-}$$

For tau neutrinos

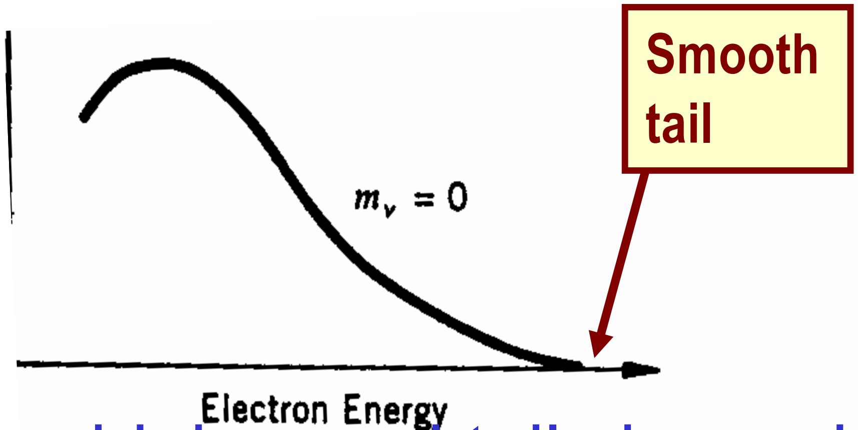
$$\nu_\tau + {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + \tau^{-}$$

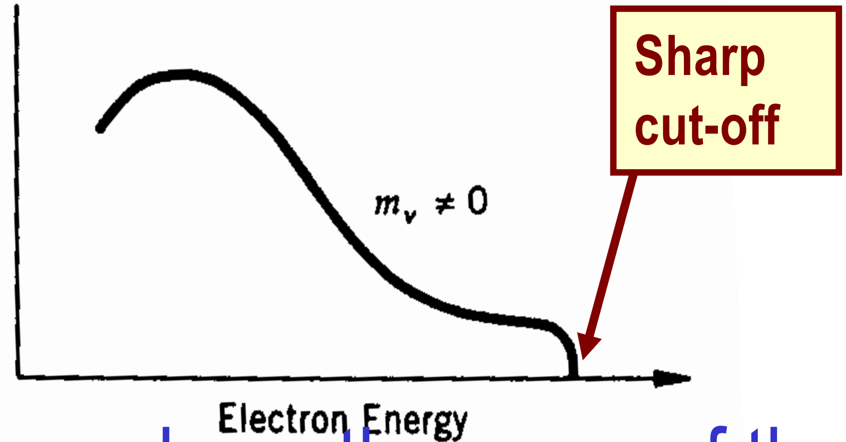
$$\nu_\tau + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + e^{-}$$

$$\nu_\tau + {}^{A}X^{Z} \not\rightarrow {}^{A}Y^{Z+1} + \mu^{-}$$

Neutrino Mass

- What does neutrino mass do to the β-spectrum?





- The higher end tail shape depends on the mass of the neutrino
  - β-spectrum could be used to measure the mass of neutrinos
    - Very sensitive to the resolution on the device
  - Most stringent direct limit is $m_\nu < 2 eV/c^2$
- Non-zero mass of the neutrino means
  - Neutrino Oscillation: Mixing of neutrino species

Weak Interactions

- The β-decay can be written at the nucleon level as:

$$n \rightarrow p + e^- + \bar{\nu}_e \qquad p \rightarrow n + e^+ + \nu_e \qquad p + e^- \rightarrow n + \nu_e$$

- Since neutrons are heavier than protons, they can decay to a proton in a free space
  - On the other hand, protons are lighter than neutrons therefore they can only undergo a β-decay within a nucleus
  - Lifetime of a neutron is about 900sec
  - This lifetime is a lot longer than nuclear reaction time scale 10^{-23} s or EM scale 10^{-16} s.
- This means that a β-decay is a nuclear phenomenon that does not involve the strong nuclear or EM forces
- Fermi postulated a new weak force (1933) responsible for β-decay (discovered the force mediators in 1983!)

Weak Interactions

- Weak forces are short ranged
  - How do we know this?
    - Occurs in the nuclear domain
    - Weakness of the strength is responsible for long lifetime seen in β-decays
- Nucleus does not contain electrons
  - Electrons in β-decays must come from somewhere else
  - Electrons are emitted without a time delay
    - The electron must come at the time of decay just like the alphas from a nuclear disintegration
  - β-decay can be considered to be induced by the weak force

Weak Interactions

- The transition probability per unit time, the width, can be calculated from perturbation theory using Fermi's Golden rule

$$P = \frac{2\pi}{\hbar}\left|H_{fi}\right|^2 \rho\left(E_f\right)$$

- Where the weak interaction Hamiltonian is

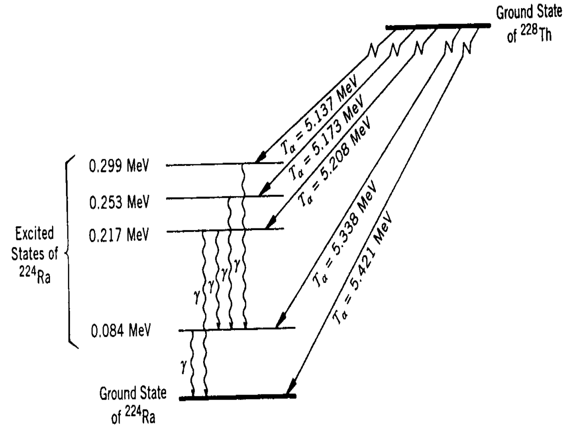
$$H_{fi} = \left\langle f \middle| H_{wk} \middle| i \right\rangle = \int d^3x \psi_f^*(x) H_{wk} \psi_i(x)$$

Weak Interactions

- Based on β-decay reaction equations, the H_{wk} must be a four fermionic states
  - H_{wk} proposed by Fermi in 1933 is a **four-fermion interaction** or current-current interaction
  - Relativistic
  - Agreed rather well with experiments for low energy β-decays
- Parity violation
  - There are only left-handed neutrinos and right-handed anti-neutrinos
  - A system is parity invariant if it does not change under inversion of spatial coordinates
  - The spin $\vec{r} \rightarrow -\vec{r} \quad \vec{p} \rightarrow -\vec{p} \Rightarrow \vec{L} = \vec{r} \times \vec{p} = (-\vec{r}) \times (-\vec{p}) = \vec{L}$
  - The handedness, helicity, changes upon the spatial inversion since the direction of the motion changes while the spin direction does not
  - Since there is no right handed neutrinos, parity must be violated in weak interactions

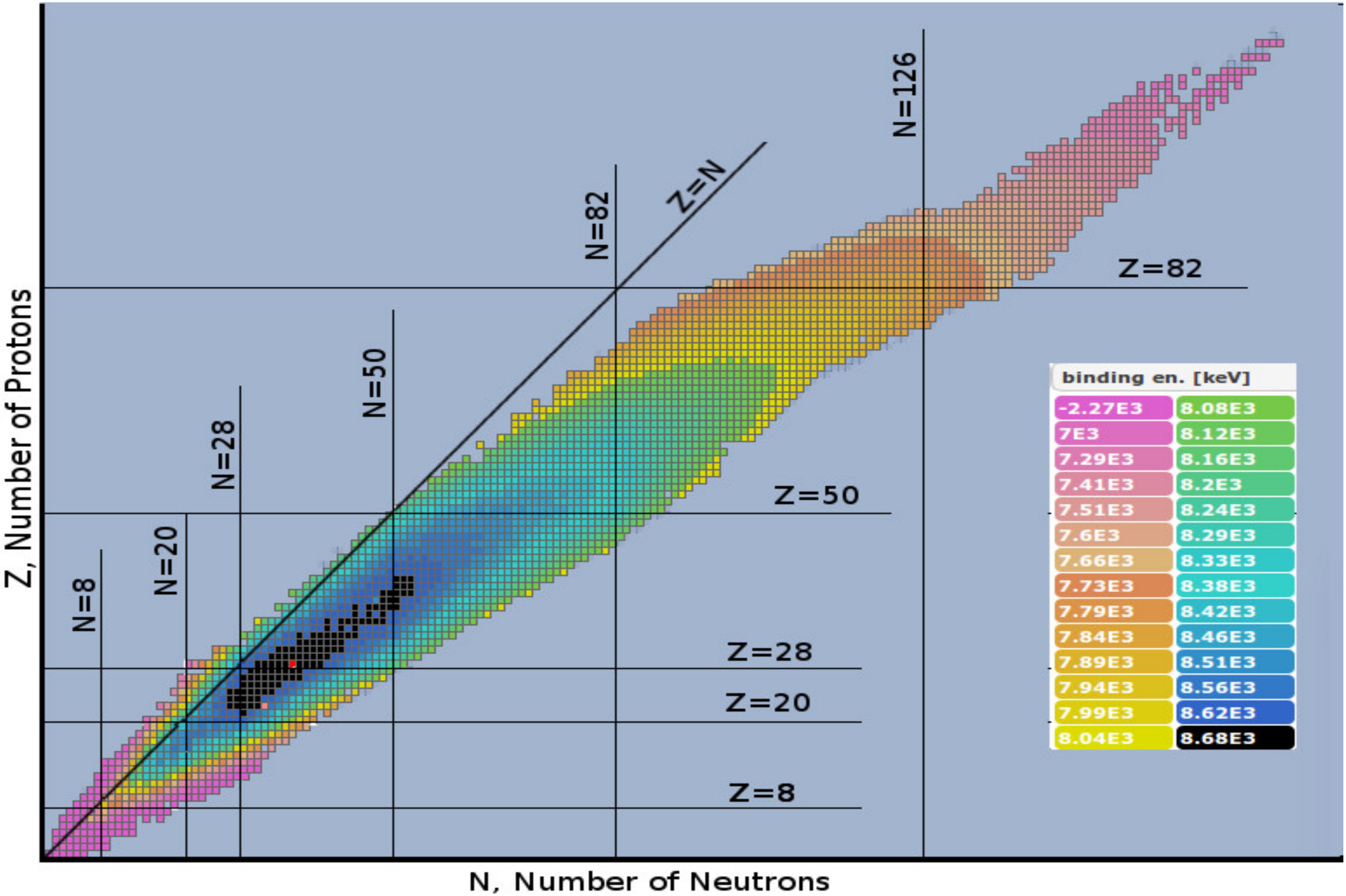
Gamma Decays

- When a heavy nuclei undergo alpha and beta decays, the daughters get into an excited state
  - Must either break apart
  - Or emit another particle
  - To bring the daughter into its ground state

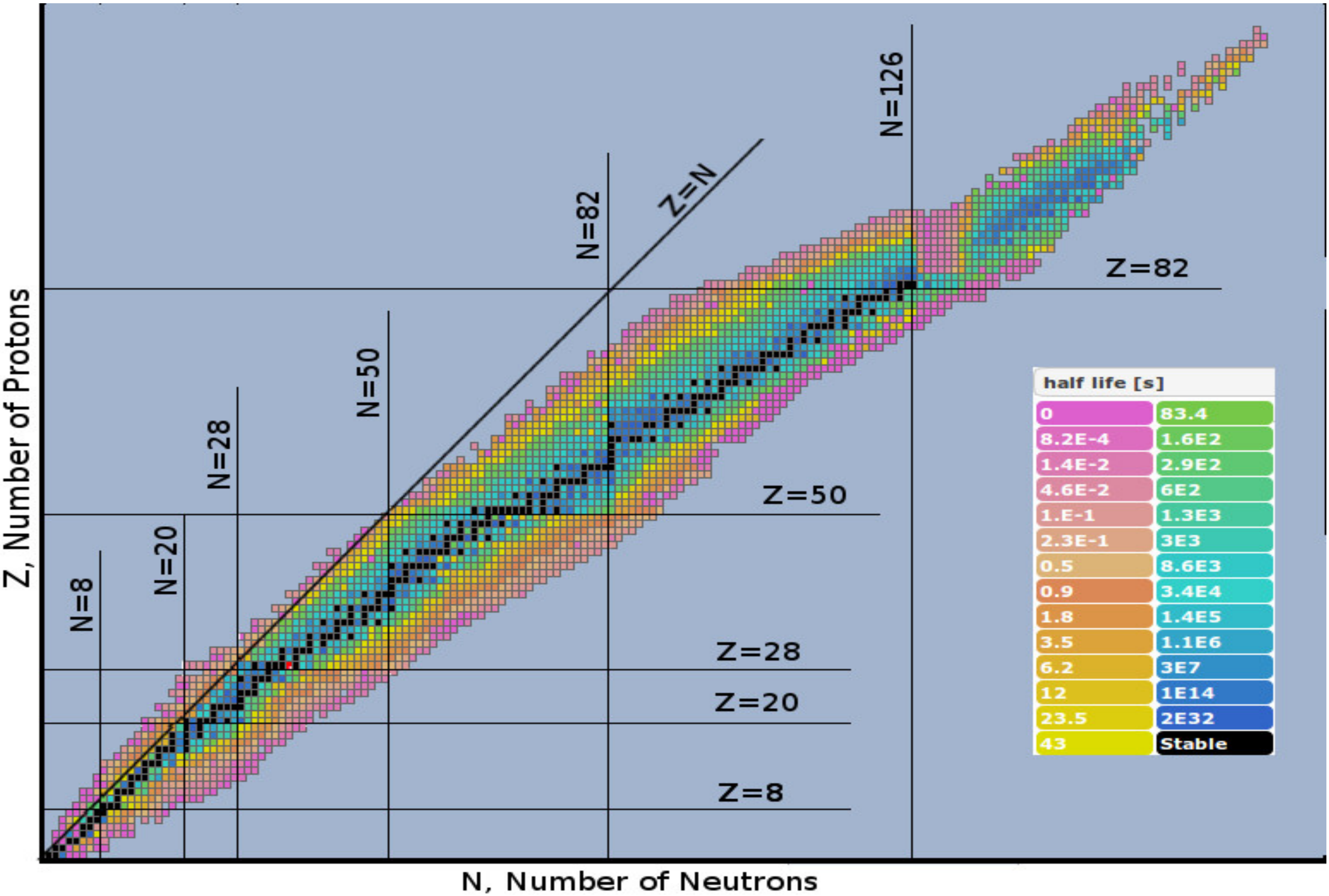


- Typical energies of photons in γ-decays are a few MeV's
  - These decays are EM interactions thus the lifetime is on the order of 10^{-16}sec.
- Photons carry one unit of angular momentum
  - Parity is conserved in this decay

Nuclear Binding Energy – Valley of Stability



Half Life



Z, Number of Protons
N, Number of Neutrons
N=8
N=20
N=28
N=50
N=82
N=126
Z=N
Z=8
Z=20
Z=28
Z=50
Z=82
half life [s]
0
8.2E-4
1.4E-2
4.6E-2
1.E-1
2.3E-1
0.5
0.9
1.8
3.5
6.2
12
23.5
43
83.4
1.6E2
2.9E2
6E2
1.3E3
3E3
8.6E3
3.4E4
1.4E5
1.1E6
3E7
1E14
2E32
Stable

Nuclear Decay Type