PHYS 3446 – Lecture #13

Monday, Oct. 17, 2016

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- Energy Deposition in Media
  - Bremstraarhlung Process
  - Photon Energy Loss
  - Neutron Interactions
  - Hadron Interactions

Announcement

- First term exam results
  - Class average: 40.1/97
    - Equivalent to 41.3/100
  - Top score: 68/97
- Evaluation Policy Reminder:
  - Two Term Exams: 15 % each ➔ 30%
  - Lab Score: 15%
  - Final Semester project paper: 20%
  - 5+2 minute Project oral presentation: 10%
  - Homework: 15%
  - Quizzes: 10%
  - Extra Credit: 10%

Homework #6

- Perform the detailed calculations in examples 1 – 7 in CH6
- Due for these homework problems is next Monday, Oct. 24.

Straggling, Multiple Scattering and Statistical process

- Phenomenological calculations can describe average behavior but large fluctuations are observed in an event-by-event bases
  - This is due to the statistical nature of the scattering process
    - Finite dispersion of energy deposit or scattering angular distributions is measured
- Statistical effect of angular deviation experienced in Rutherford scattering off atomic electrons in the medium
  - Consecutive collisions add up in a random fashion and provide net deflection of any incident particles from its original path
  - Called "Multiple Coulomb Scattering" ➔ Increases as a function of path length

$$\theta_{rms} \approx \frac{20MeV}{\beta pc} z \sqrt{\frac{L}{X_0}}$$

    - z: charge of the incident particle, L: material thickness, X_0: the radiation length of the medium

Ex: Multiple Scattering Angles

- Compare the multiple scattering angles of 5MeV proton to 5MeV electron through 1cm of Ar gas whose radiation length is 105m at atmospheric pressure and 0°C.
- For proton which is non-relativistic

$$p_p = \sqrt{2M_pT_p} = \sqrt{2 \cdot 1000\,MeV/c^2 \cdot 5\,MeV} \approx 100\,MeV/c$$

$$v_p = \sqrt{2T_p/M_p} = \sqrt{2 \cdot 5\,MeV/1000\,MeV/c^2} \approx 0.1c$$

$$\theta_{rms} \approx \frac{20MeV}{\beta pc} z\sqrt{\frac{L}{X_0}} = \frac{20}{0.1 \cdot 100} \cdot 1 \cdot \sqrt{\frac{0.01}{105}} \approx 0.02rad \approx 20mrad$$

- For the electron which is relativistic

$$p_e = \frac{E}{c} = \frac{T + m_e c^2}{c} \approx 5.5\,MeV/c \qquad \beta \sim 1$$

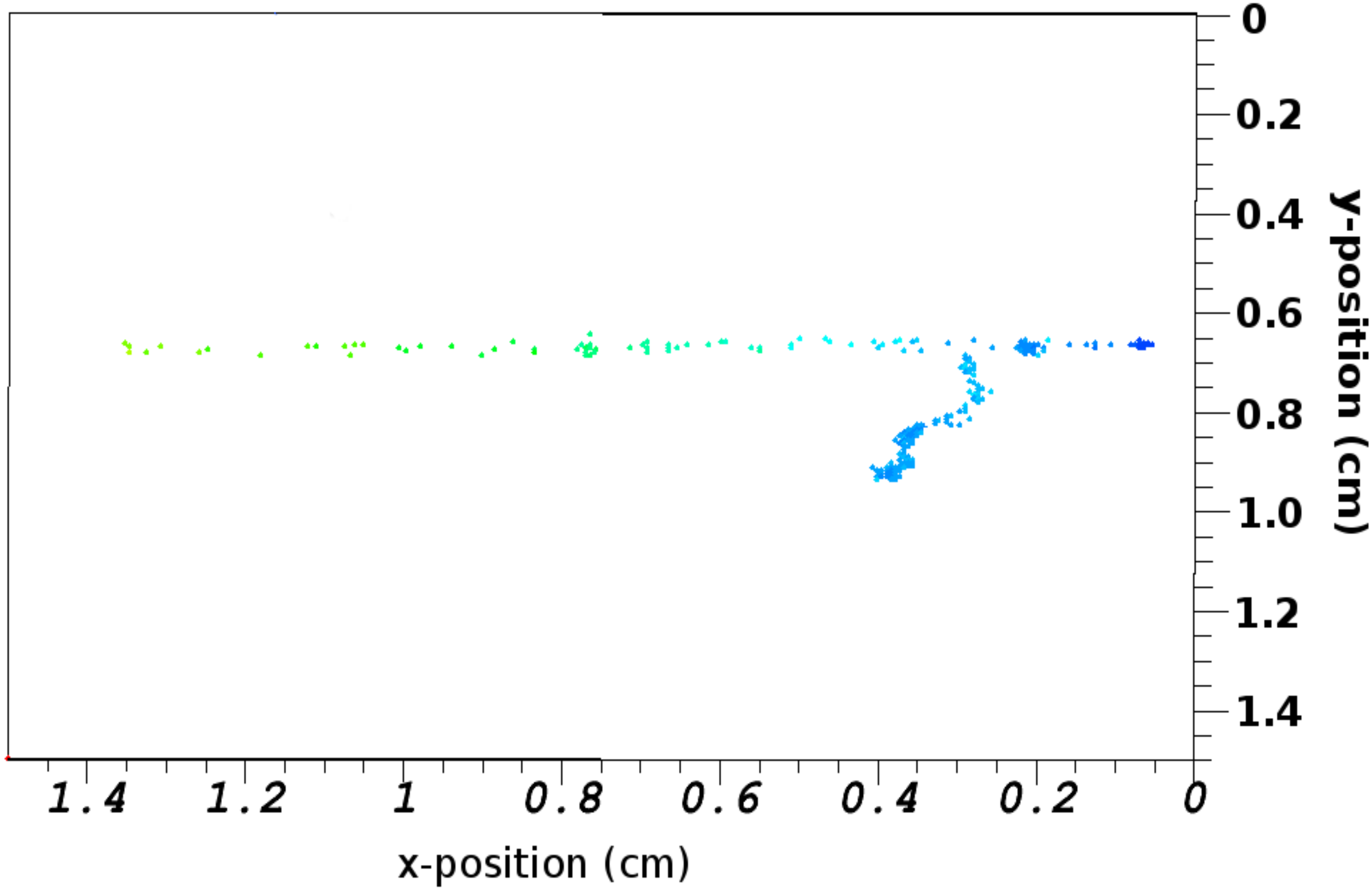
$$\theta_{rms} \approx \frac{20MeV}{\beta pc} z\sqrt{\frac{L}{X_0}} = \frac{20}{1 \cdot 5.5} \cdot 1 \cdot \sqrt{\frac{0.01}{105}} \approx 0.04rad \approx 40mrad$$

Energy Loss Through Bremsstrahlung

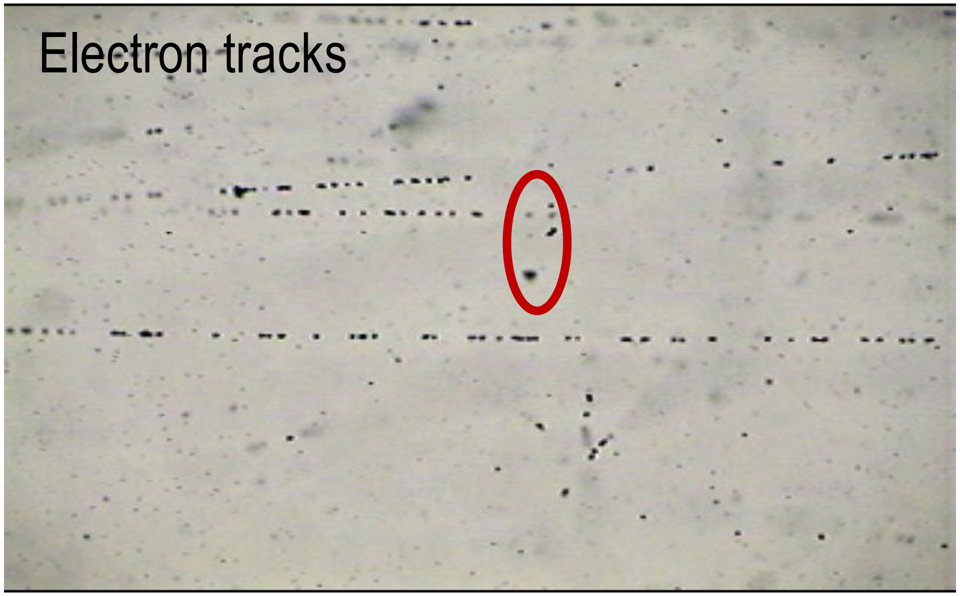
- Energy loss of incident electrons
  - Bethe-Bloch formula works well (up to above 1MeV for electrons)
  - But due to the small mass, electron's energy loss gets complicated
    - Relativistic corrections take large effect even down to a few keV level
    - Electron projectiles can transfer large fractions of energies to the atomic electrons they collide
      - Produce δ-rays or knock-on electrons ➔ Which have the same properties as the incident electrons
  - Electrons suffer a large acceleration as a result of the interaction with electric field by nucleus. What do these do?
  - Causes electrons to radiate or emit photons
    - Bremsstrahlung ➔ An important mechanism of relativistic electron energy loss

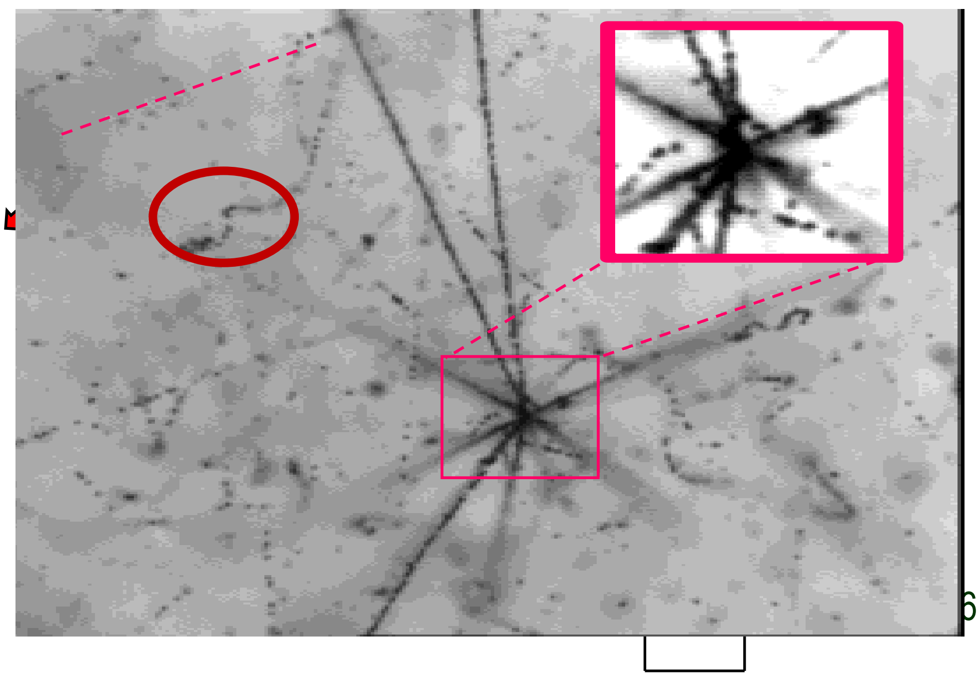
*** Bremsstrahlung: Braking Radiation or Decelerating Radiation**

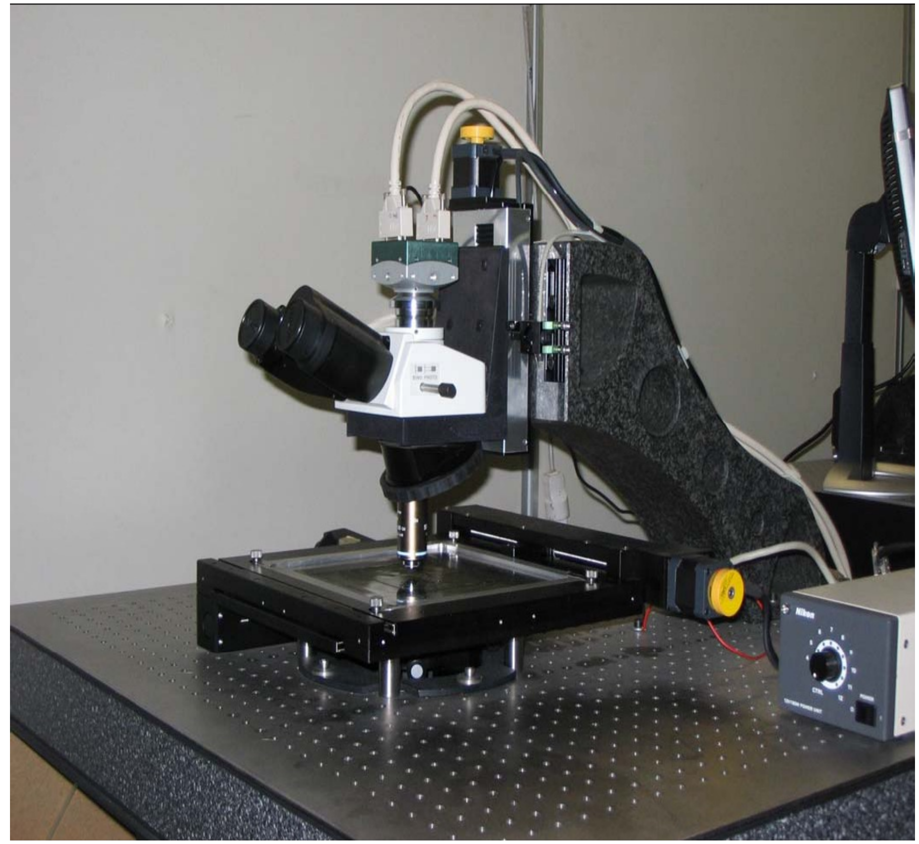
A 3D depiction of a delta ray



Nuclear Emulsion Photos







Total Electron Energy Loss

- The total electron energy loss in matter can be written as

$$\left(-\frac{dT}{dx}\right)_{tot} = \left(-\frac{dT}{dx}\right)_{ion} + \left(-\frac{dT}{dx}\right)_{brem}$$

- Relative magnitude between Bremsstrahlung and ionization is

$$\left(-\frac{dT}{dx}\right)_{brem} \Big/ \left(-\frac{dT}{dx}\right)_{ion} \approx \frac{TZ}{1200 m_e c^2}$$

  - Z: Atomic number of the medium, m_e: rest mass of the electron, T: Kinetic energy of the electron in MeV

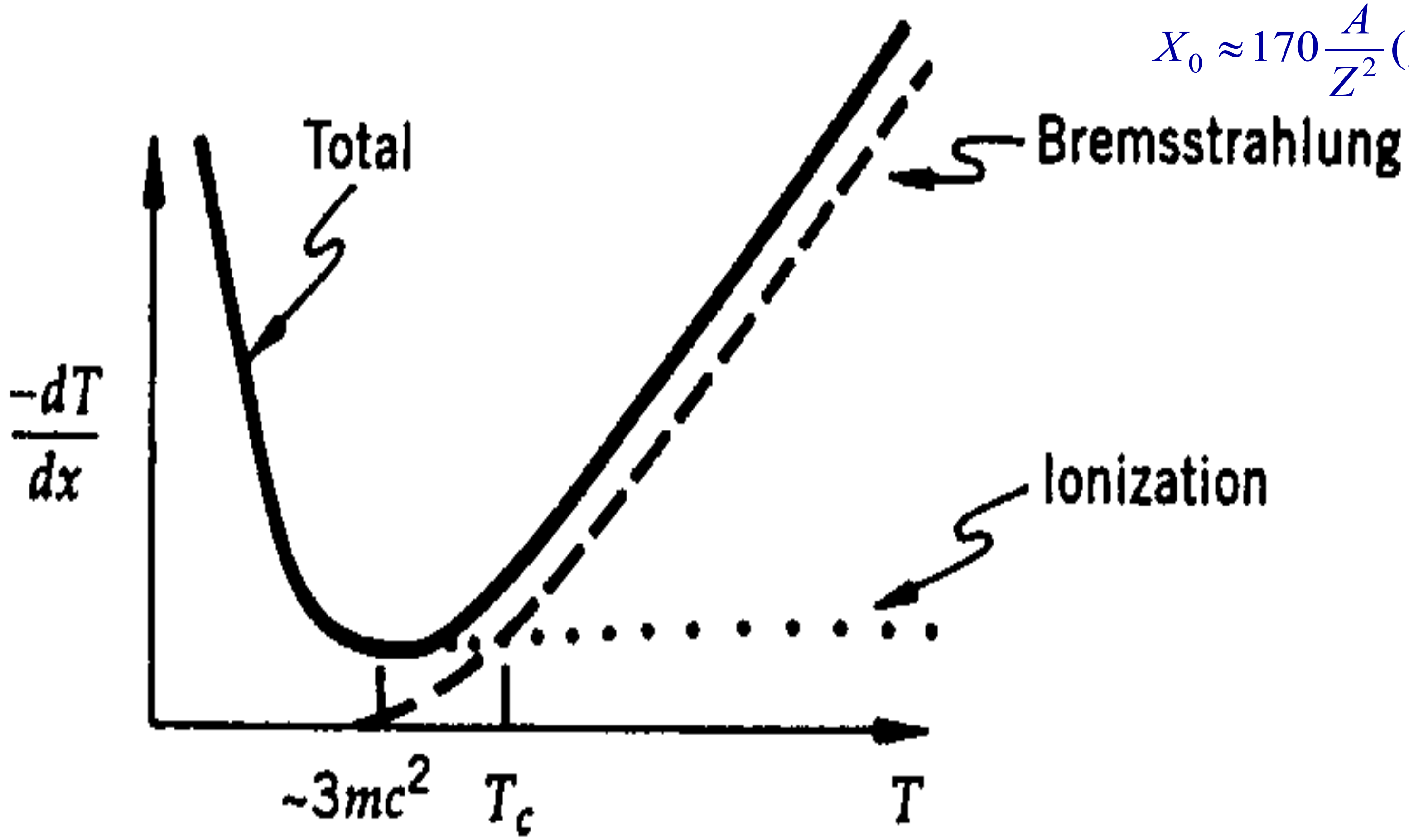
- At high energies, the ionization loss is constant
  - Radiation is the dominant energy loss mechanism
  - The energy loss is directly proportional to the incident kinetic energy

Total Electron Energy Loss

- Above the critical energy (T_c) the brem process dominates

$$\left(\frac{dT}{dx}\right)_{brem} = \left(\frac{dT}{dx}\right)_{ion} = -\frac{T_c}{X_0}$$

$$X_0 \approx 170\frac{A}{Z^2}(g/cm^2)$$



Photon Energy Loss

- Photons are electrically neutral
  - They do not feel the Coulomb force
  - They cannot directly ionize atoms
- Photons are EM force carriers
  - Can interact with matter and result in ionization
  - What are the possible processes?
    - Photo-electric effect
    - Compton scattering
    - Pair production

Light Attenuation

- Reduction of intensity in a medium
- Can be described by the effective absorption coefficient μ
  - μ reflects the total cross section for the interaction
  - μ depends on energy or frequency of the incident light
- The intensity of light at any given point through the medium, x, is given as $I(x) = I_0 e^{-\mu x}$
- Half thickness, the thickness of material for photon's intensity to be half the initial intensity: $x_{1/2} = \frac{\ln 2}{\mu} = \frac{0.693}{\mu}$
- μ^{-1} is the mean free path for absorption

Photo-electric Effect

- Low energy photon is absorbed by a bound electron in an atom
  - The electron is subsequently emitted with T_e
  - The energy of electron T_e is $T_e = h\nu - I_B$
- I_B: Energy needed to free the given atomic electron
- ν: Frequency of the incident photon

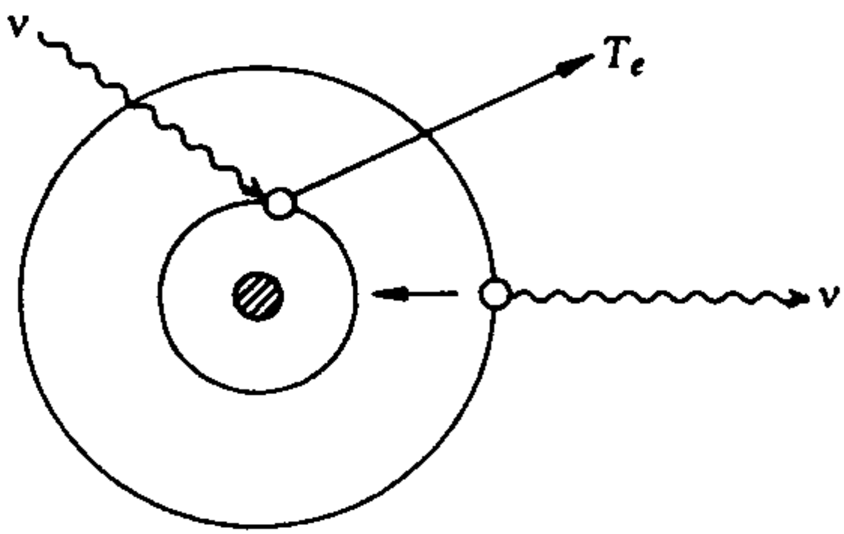


Photo-electric Effect

- The energy $I_{\mathcal{B}}$ sets the threshold photon energies for this process to take place
- Photo-electric effect cross section is large in the range of X-ray energies (keV)

Photo-electric Effect X-sec

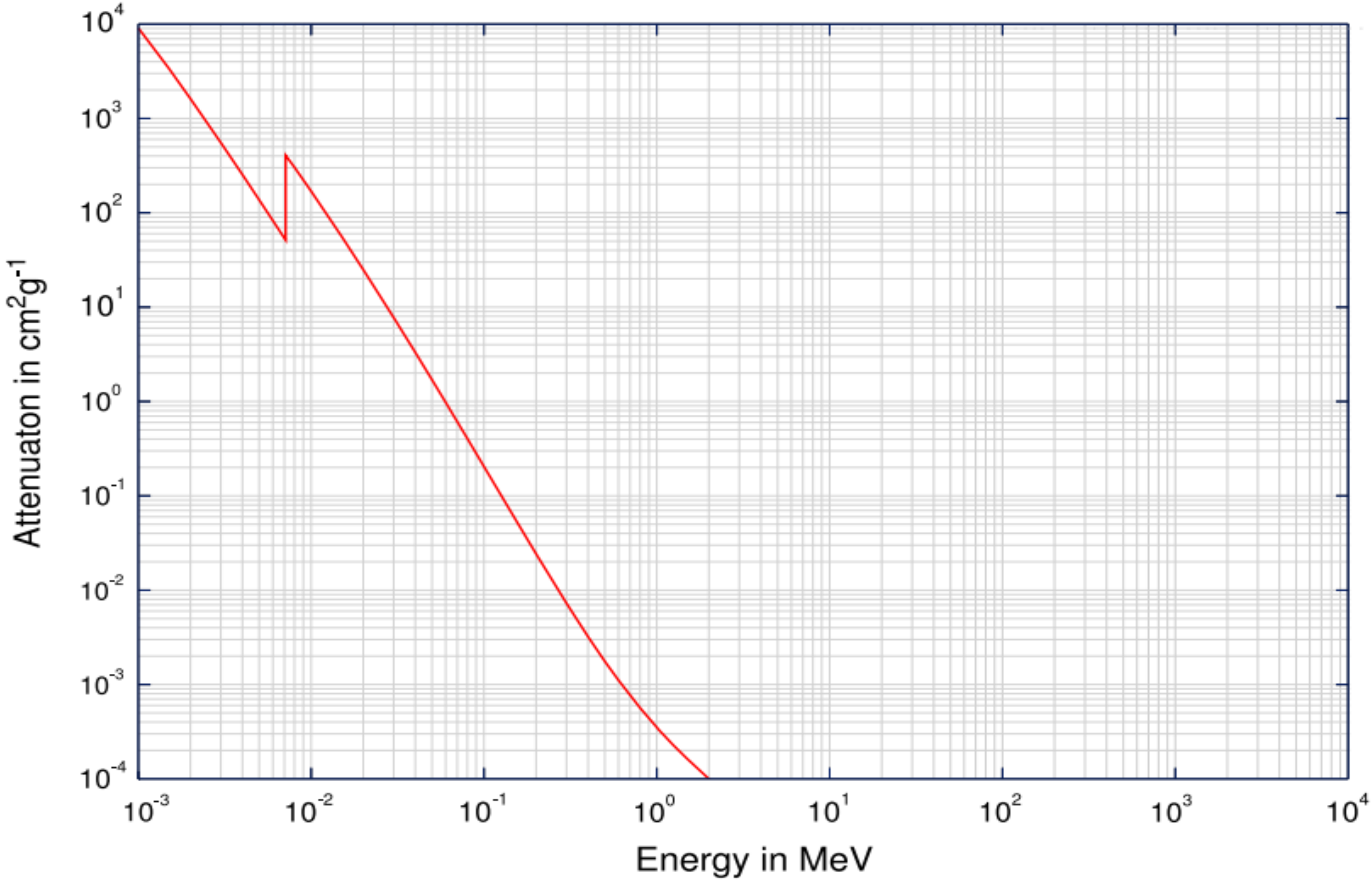


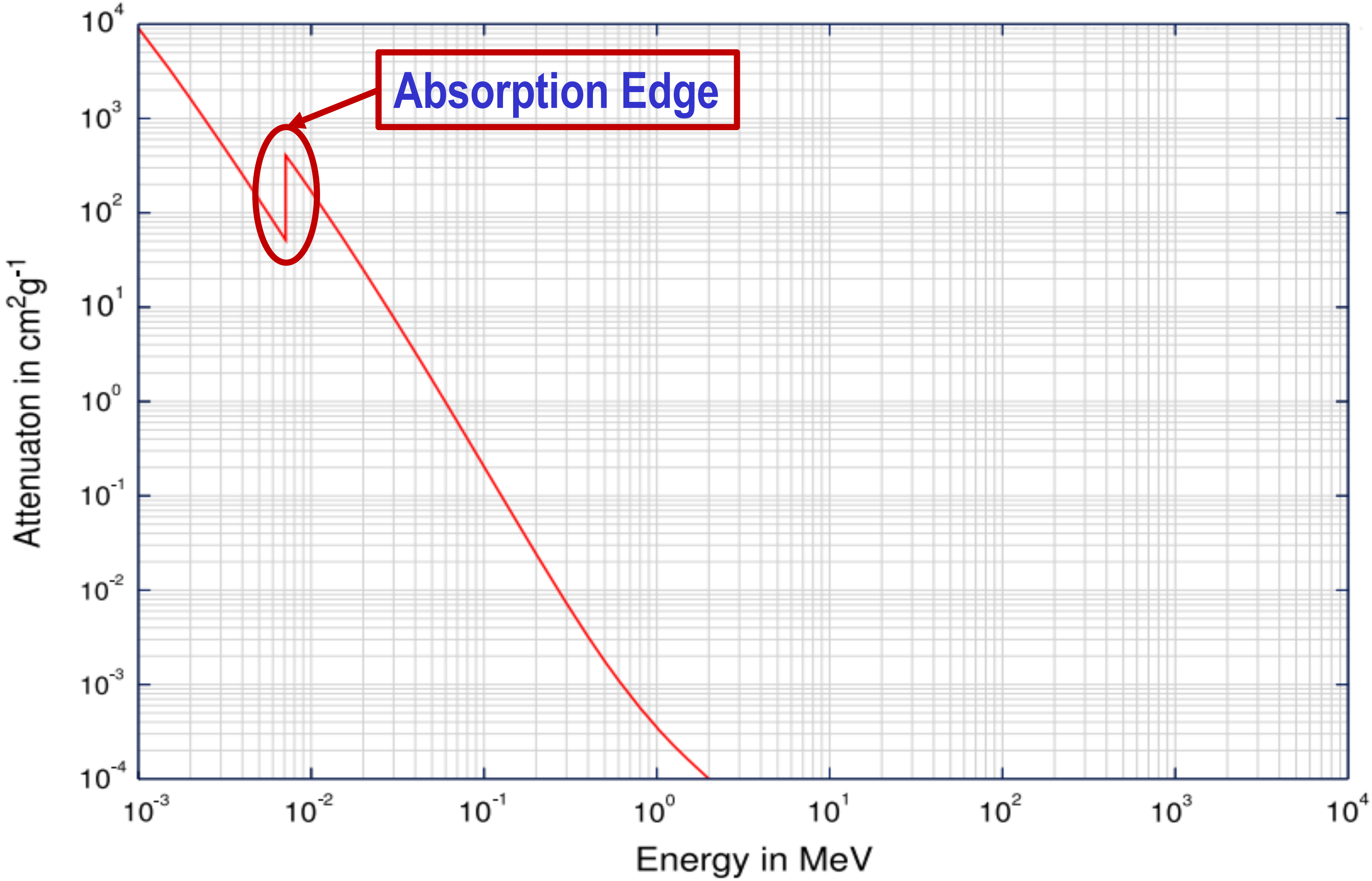
Photo-electric Effect

- The energy $I_{\mathcal{B}}$ sets the threshold photon energies for this process to take place
- Photo-electric effect cross section is large in the range of X-ray energies (keV)
- The scale of cross section is

$$\sigma \approx \frac{Z^5}{(h\nu)^{7/2}} \quad \text{for} \quad E_\gamma < m_e c^2 \quad \text{and} \quad \sigma \approx \frac{Z^5}{h\nu} \quad \text{for} \quad E_\gamma > m_e c^2$$

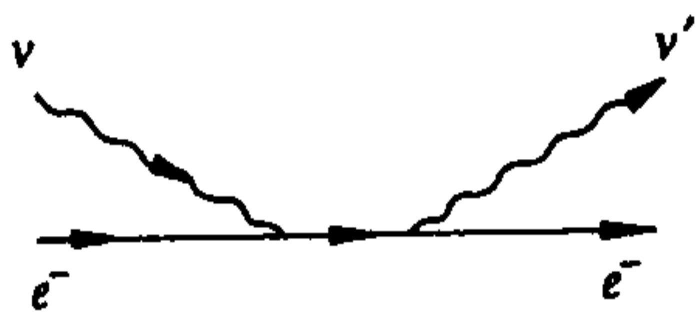
- What do you conclude from these?
  - This process is particularly important for high Z medium
  - Not very significant above 1MeV photon energies
- When an inner layer electron is emitted, photons from transition accompany the electron

Photo-electric Effect X-sec



Compton Scattering

- Equivalent to photo-electric effect on a free electron
  - Like a collision between a photon with energy $E=h\nu$ and momentum $p=E/c$ on a stationary electron
  - Electron absorbs a photon
    - Forms an electron like system with excited state and with an unphysical mass (virtual system)
    - Emits a photon with different frequency as it de-excites into a physical electron



Compton Scattering

- The kinematics of the scattering assumes the free electron
- Thus the results will not work for low energy (<100keV) incident photons where the effect of atomic binding can be important
- The emitted photon frequency of scattering angle θ is

$$\nu' = \frac{\nu}{1 + \frac{h\nu}{m_e c^2}\left(1 - \cos\theta\right)}$$

  - For finite scattering angle (θ), the energy of the scattered photon is smaller than that of the incident one
  - Some incident photon energy is transferred to the electron, having recoil energy dependent on the scattering angle
  - This was a piece of evidence for particle property of light

Pair Production

- When a photon has sufficient energy, it can be absorbed in matter and produces a pair of oppositely charged particles
  - Should not violate any conservation laws, including quantum number conservations
  - Most common one is the conversion to an electron and positron pair
- Massless photons cannot produce a pair of massive particles without violating energy-momentum conservation
  - In photon's rest frame, the initial state energy is 0.
  - While final state energy is non-zero.
- Thus the pair production can only occur in a medium
  - Why?
    - A recoiling nucleus can absorb any momentum required to assure energy-momentum conservation

Pair Production

- What is the minimum energy needed to produce an electron-positron pair?
  - Twice the rest mass energy of the electron

$$h\nu \approx 2m_e c^2 = 2 \times 0.511 MeV = 1.022 MeV$$

- The pair production cross section is proportional to Z^2
  - Z: atomic number of the medium
  - Rises rapidly and dominates all energy-loss mechanisms for photon energies above <u>**10MeV**</u> or so.
  - It saturates and can be characterized by a constant mean free path for the conversion
    - A constant absorption coefficient ➔ Electron radiation length of medium

$$X_{pair} = \left(\mu_{pair}\right)^{-1} \approx \frac{9}{7} X_0$$

Pair Production

- What happens to the positron created in the conversion?
  - Positron is the anti-particle of the electron
  - Behaves exactly like electrons as they traverse through the matter
    - Deposits energy through ionization or Bremsstrahlung
  - When it loses most of its kinetic energy, it captures an electron to form a hydrogen like atom, a positronium.
  - Positronium is unstable and decays (annihilate) in a life time of 10^{-10} s
    - $e^{+} + e^{-} \rightarrow \gamma + \gamma$
  - Why two photons?
    - To conserve the angular momentum
  - To conserve energy-momentum, the photon energies are exactly 0.511 MeV each
    - Good way to detect positronium