PHYS 3446 – Lecture #22

Monday, Nov. 21, 2016

Dr. Jaehoon Yu

- Symmetries
  - Why do we care about the symmetry?
  - Symmetry in Lagrangian formalism
  - Symmetries in quantum mechanical system
  - Types of Symmetry

Announcements

- Quiz #4
  - Monday, Nov. 28
  - Covers up to what we learn coming Monday, Nov. 21
  - BYOF
- Term Exam #2
  - Monday, Dec. 5
  - Comprehensive: CH1.1 through what we cover Nov. 28
  - BYOF
- Quiz 3 results
  - Class Average: 41.2/90
    - Equivalent to 45.8/100
    - Previous quizzes: 47.1/100 & 43.5/100
  - Class top score: 69/90
- No class this Wednesday!
  - Happy and safe Thanksgiving!

Reminder: Research Project Report

1. Must contain the following at the minimum
   - Motivation, including the justification for it
   - Current status and some background behind them
   - Improvements and bases for such improvements
   - Conclusions
   - The reference to the original paper must be included!
   - Bibliography referring to web site must be minimized (<20%)
2. Each member of the group writes a 10 (max) page report, including figures
   - 20% of the total grade
   - Can share the theme and facts but you must write your own!
   - Text of the report must be your original!
   - ***Due Wed., Dec. 7, 2016***

Write Up Requirements and Evaluation

- Due date: Prior to class on Wednesday, Dec. 7
- Requirements
  - Need to put the name(s) of the person(s) who wrote the given sections
  - Professionally prepared in MS words
    - No spelling or grammar mistakes
    - The style of the write up should be unified so that it looks like written by one person
  - All contents on the template and more should be contained in the write up
  - Pictures, diagrams and photos should be added with appropriate figure captions and numbered in order of appearance. The captions should go at the bottom of the figure.
  - References must be indicated throughout the text in order of appearance. They must be properly matched in the list of bibliography at the end of the document.
  - Tables must be added and numbered in order of appearance. The caption should go on top of the table.
- Key evaluation points
  - Quality of the document – 30%
  - Content and organization of the document – 20 %
  - Satisfaction of the above requirements – 25%
  - Thoughtfulness, usefulness and relevance of contents of the document – 25%
    - E.g. The contact information of vendors must be usable for the construction

Project Report Template

PHYS3446-Your-Name-Here

Title Goes Here Like This With The First Letter of Each Word Capital

PHYS-3446, Fall 2016
Nov. dd, 2016

Author Name
Department of XYZ
The University of Texas at Arlington

Abstract

Describe briefly and to the point the content of the note in about a paragraph or so, including the brief conclusion. The font of the main body must be Times New Roman 12pt. Tables and figures must be numbered in sequence as they appear as Table 1 or Figure 1. Each has its own numbering system. They must be placed as close to the text in which they are referred. They must have associated captions attached to them. These explain what the contents of the figure or table are. Captions must be Times New Roman 11pt. References must be placed to where the reference is relevant in a square bracket with a number counted in sequence as they appear but only in the main body not in the abstract.

1. Introduction

Describe what this paper is all about and how this note is organized [1] and motivate the readers.

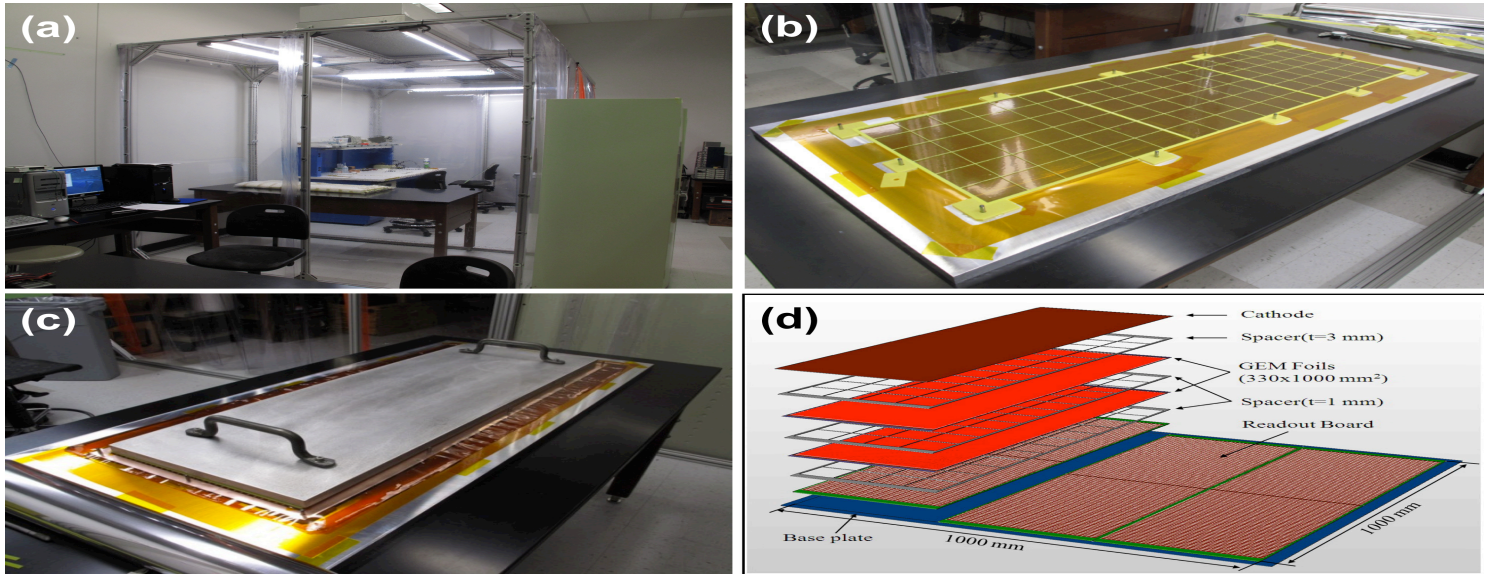

Figure. 1 (a) 12'x8' clean room for LGEM construction (b) An LGEM layer on an assembly jig held by alignment pins throughout the sides (c) glue curing process with heavy flattening pressing plane (d) Layout of a full 100cm ×100cm GEM DHCAL active layer.

Research Presentations

- Each of the 5 research groups makes a 10+3min presentation
  - 10min presentation + 3min Q&A
  - All presentations must be in power point
  - I must receive all final presentation files by 8pm, Monday, Dec. 5, 2016
    - No changes are allowed afterward
  - The representative of the group makes the presentation followed by all group members' participation in the Q&A session
- Date and time:
  - In class Wednesday, Dec. 7, 2016
- Important metrics
  - Contents of the presentation: 60%
    - Inclusion of all important points as mentioned in the report
    - The quality of the research and making the right points
  - Quality of the presentation itself: 15%
  - Presentation manner: 10%
  - Q&A handling: 10%
  - Staying in the allotted presentation time: 5%
  - Judging participation and sincerity: 5%

Presentation Sequence

1. Group 4
2. Group 3
3. Group 1
4. Group 2
5. Group 5

Evaluation Policy

- Two Term Exams: 15 % each ➔ 30%
- Lab Score: 15%
- Final Semester project paper: 20%
- 10+2 minute Project oral presentation: 10%
- Homework: 15%
- Quizzes: 10%
- Extra Credit: 10%
  - Consists of random attendances, colloquium (4pm Wednesdays) attendances, special projects, planetarium show and other opportunities

Quantum Numbers

- We've learned about various newly introduced quantum numbers as a patch work to explain experimental observations
  - Lepton numbers
  - Baryon numbers
  - Isospin
  - Strangeness
- Some of these numbers are conserved in certain situation but not in others
  - Very frustrating indeed….
- These are due to lack of quantitative description by an elegant theory

Why symmetry?

- Some quantum numbers are conserved in strong interactions but not in electromagnetic and weak interactions
  - Inherent reflection of underlying forces
- Understanding conservation or violation of quantum numbers in certain situations is important for formulating quantitative theoretical framework

Why symmetry?

- When is a quantum number conserved?
  - When there is an underlying symmetry in the system
  - When a quantum number is not affected (or is conserved) by (under) the changes in a physical system
- **Noether's theorem**: If there is a conserved quantity associated with a physical system, there exists an underlying invariance or symmetry principle responsible for this conservation.
- Symmetries provide critical restrictions in formulating theories

Symmetries in Lagrangian Formalism

- Symmetry of a system is defined by any set of transformations that keep the equation of motion unchanged or invariant
- Equations of motion can be obtained through
  - Lagrangian formalism: $\mathcal{L}$=T-V where the Equation of motion is what minimizes the Lagrangian $\mathcal{L}$ under changes of coordinates
  - Hamiltonian formalism: $\mathcal{H}$=T+V with the equation of motion that minimizes the Hamiltonian under changes of coordinates
- Both these formalisms can be used to discuss symmetries in non-relativistic (or classical cases) or relativistic cases and quantum mechanical systems

Symmetries in Lagrangian Formalism?

- Consider an isolated non-relativistic physical system of two particles interacting through a potential that only depends on the relative distance between them
  - EM and gravitational force
- The total kinetic and potential energies of the system are: $T = \frac{1}{2} m_1 \dot{\vec{r}}_1^{\,2} + \frac{1}{2} m_2 \dot{\vec{r}}_2^{\,2}$ and $V = V\left(\vec{r}_1 - \vec{r}_2\right)$
- The equations of motion are then

$$m_1 \ddot{\vec{r}}_1 = -\vec{\nabla}_1 V\left(\vec{r}_1 - \vec{r}_2\right) = -\frac{\partial}{\partial \vec{r}_1} V\left(\vec{r}_1 - \vec{r}_2\right)$$

$$m_2 \ddot{\vec{r}}_2 = -\vec{\nabla}_2 V\left(\vec{r}_1 - \vec{r}_2\right) = -\frac{\partial}{\partial \vec{r}_2} V\left(\vec{r}_1 - \vec{r}_2\right)$$

$$where \ \frac{\partial}{\partial \vec{r}_i} V\left(\vec{r}_1 - \vec{r}_2\right) = \hat{x}\frac{\partial}{\partial x_i} V + \hat{y}\frac{\partial}{\partial y_i} V + \hat{z}\frac{\partial}{\partial z_i} V$$

Symmetries in Lagrangian Formalism

- If we perform a linear translation of the origin of coordinate system by a constant vector $-\vec{a}$
  - The displacement vectors of the two particles become

$$\vec{r}_1 \rightarrow \vec{r}_1 - \vec{a} \qquad \vec{r}_2 \rightarrow \vec{r}_2 - \vec{a}$$

  - But the equations of motion do not change since $-\vec{a}$ is a constant vector
  - This is due to the invariance of the potential V under the translation

$$V' = V\left(\vec{r}_1' - \vec{r}_2'\right) = V\left(\vec{r}_1 - \vec{a} - \vec{r}_2 + \vec{a}\right) = V\left(\vec{r}_1 - \vec{r}_2\right)$$

Symmetries in Lagrangian Formalism?

- This means that the translation of the coordinate system for an isolated two particle system defines a symmetry of the system (remember Noether's theorem?)
- This particular physical system is invariant under spatial translation
- What is the consequence of this invariance?
  - From the form of the potential, the total force is

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 = -\vec{\nabla}_1 V\left(\vec{r}_1 - \vec{r}_2\right) - \vec{\nabla}_2 V\left(\vec{r}_1 - \vec{r}_2\right) = 0$$

  - Since $\dfrac{\partial V}{\partial \vec{r}_1} = -\dfrac{\partial V}{\partial \vec{r}_2}$

Symmetries in Lagrangian Formalism?

- What does this mean?
  - Total momentum of the system is invariant under spatial translation

$$\vec{F}_{tot} = \frac{d\vec{P}_{tot}}{dt} = 0$$

- In other words, **the translational symmetry results in linear momentum conservation**
- This holds for a multi-particle system as well

Symmetries in Lagrangian Formalism

- For multi-particle system, using Lagrangian $\mathcal{L}$=T-V, the equations of motion can be generalized

$$\frac{d}{dt}\frac{\partial L_i}{\partial \dot{\vec{r}}} - \frac{\partial L_i}{\partial \vec{r}} = 0$$

- By construction,

$$\frac{\partial L_i}{\partial \dot{\vec{r}}} = \frac{\partial T_i}{\partial \dot{\vec{r}}} = \frac{\partial}{\partial \dot{\vec{r}}}\left(\frac{1}{2} m_i \dot{\vec{r}}_i^{\,2}\right) = m_i \dot{\vec{r}}_i = \vec{p}_i$$

- As previously discussed, for the system with a potential that depends on the relative distance between particles, The Lagrangian is independent of particulars of the individual coordinate $\frac{\partial L_i}{\partial r_m} = 0$ and thus $\frac{d\vec{p}_i}{dt} = \frac{\partial L_i}{\partial r_i} = 0$

Symmetries in Lagrangian Formalism

- Momentum p_i can be expanded to other kind of momenta for the given spatial translation
  - Spatial translation: Linear momentum
  - Rotational translation: Angular momentum
  - Time translation: Energy
  - Rotation in isospin space: Isospin
- The equation $\frac{d\vec{p}_i}{dt} = \frac{\partial L_i}{\partial r_i} = 0$ says that if the Lagrangian of a physical system does not depend on specifics of a given coordinate, the conjugate momentum is conserved
- One can turn this around and state that if a Lagrangian does not depend on some particular coordinate, it must be invariant under the translations of this coordinate.

Translational Symmetries & Conserved Quantities

- The translational symmetries of a physical system give invariance in the corresponding physical quantities
  - Symmetry under spatial translation
    - Linear momentum conservation
  - Symmetry under spatial rotation
    - Angular momentum conservation
  - Symmetry under time translation
    - Energy conservation
  - Symmetry under isospin space rotation
    - Isospin conservation

Symmetries in Quantum Mechanics

- In quantum mechanics, an observable physical quantity corresponds to the expectation value of the Hermitian operator in a given quantum state
  - The expectation value is given as a product of wave function vectors about the physical quantity (operator)

$$\langle Q \rangle = \langle \psi | Q | \psi \rangle$$

  - Wave function ($|\psi\rangle$) is the probability distribution function of a quantum state at any given space-time coordinates
  - The observable is invariant or conserved if the operator Q commutes with Hamiltonian

Types of Symmetry

- All symmetry transformations of the theory can be categorized in
  - Continuous symmetry: Symmetry under continuous transformation
    - Spatial translation
    - Time translation
    - Rotation
  - Discrete symmetry: Symmetry under discrete transformation
    - Transformation in discrete quantum mechanical system

Isospin

- If there is isospin symmetry, proton (isospin up, I_3= ½) and neutron (isospin down, I_3= -½) are indistinguishable
- Let's define new neutron and proton states as some linear combination of the proton, $|p\rangle$, and neutron, $|n\rangle$, wave functions
- Then the finite rotation of the vectors in isospin space by an arbitrary angle $\theta/2$ about an isospin axis leads to a new set of transformed vectors

$$|p'\rangle = \cos\frac{\theta}{2}|p\rangle - \sin\frac{\theta}{2}|n\rangle \qquad |n'\rangle = \sin\frac{\theta}{2}|p\rangle + \cos\frac{\theta}{2}|n\rangle$$

Isospin

- What does the isospin invariance mean to nucleon-nucleon interaction?
- Two nucleon quantum states can be written in the following four combinations of quantum states
  - Proton on proton (I_3=+1) $|\psi_1\rangle = |pp\rangle$
  - Neutron on neutron (I_3=-1) $|\psi_2\rangle = |nn\rangle$
  - Proton on neutron or neutron on proton for both symmetric or anti-symmetiric (I_3=0)

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}\left(|pn\rangle + |np\rangle\right) \quad |\psi_4\rangle = \frac{1}{\sqrt{2}}\left(|pn\rangle - |np\rangle\right)$$

Impact of Isospin Transformation

- For I_3=+1 wave function w/ isospin transformation:

$$|\psi_1'\rangle = \left|\left(\cos\frac{\theta}{2}p - \sin\frac{\theta}{2}n\right)\left(\cos\frac{\theta}{2}p - \sin\frac{\theta}{2}n\right)\right\rangle =$$

$$= \cos^2\frac{\theta}{2}|pp\rangle - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\left(|pn\rangle + |np\rangle\right) + \sin^2\frac{\theta}{2}|nn\rangle$$

$$= \cos^2\frac{\theta}{2}|\psi_1\rangle - \sqrt{2}\cos\frac{\theta}{2}\sin\frac{\theta}{2}|\psi_3\rangle + \sin^2\frac{\theta}{2}|\psi_2\rangle$$

Can you do the same for the other two wave functions of I=1&0?

Isospin Tranformation

- For I_3=0 anti-symmetric wave function

$$|\psi_4'\rangle = \frac{1}{\sqrt{2}}\left\{\left|\left(\cos\frac{\theta}{2}p - \sin\frac{\theta}{2}n\right)\left(\sin\frac{\theta}{2}p + \cos\frac{\theta}{2}n\right)\right\rangle\right.$$

$$\left. -\left|\left(\cos\frac{\theta}{2}p - \sin\frac{\theta}{2}n\right)\left(\sin\frac{\theta}{2}p + \cos\frac{\theta}{2}n\right)\right\rangle\right\}$$

$$= \frac{1}{\sqrt{2}}\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)\left(|pn\rangle - |np\rangle\right) = |\psi_4\rangle$$

  - This state is totally insensitive to isospin rotation➔ singlet combination of isospins (total isospin 0 state)

Isospin Tranformation

- The other three states corresponds to three possible projection state of the total isospin =1 state (triplet state)
  - If there is an isospin symmetry in strong interactions all these three sub-states are equivalent and indistinguishable
- Based on this, we learn that any two nucleon system can be in an independent singlet or triplet state
  - Singlet state is anti-symmetric under n-p exchange
  - Triplet state is symmetric under n-p exchange