PHYS 1441 – Section 002
Lecture #6

Monday, Sept. 18, 2017

Dr. Jaehoon Yu

- Chapter 21
  - Motion of a Charged Particle in an Electric Field
  - Electric Dipoles
- Chapter 22
  - Electric Flux
  - Gauss' Law with many charges
  - What is Gauss' Law good for?

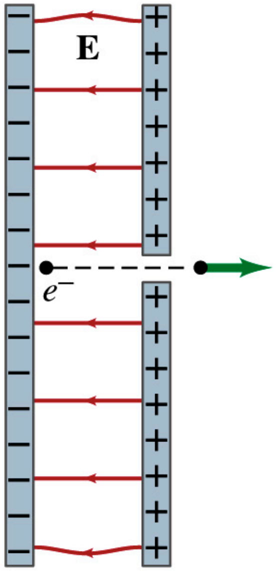
Today's homework is homework #4, due 11pm, Monday, Sept. 25!!

Announcements

- 1st Term exam
  - In class, this Wednesday, Sept. 20: DO NOT MISS THE EXAM!
  - CH21.1 to what we learn today+ Appendices A1 – A8
  - You can bring your calculator but it must not have any relevant formula pre-input
    - No phone or computers can be used as a calculator!
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations, word definitions, or solutions of ANY problems !
  - No additional formulae or values of constants will be provided!
- Reading assignments
  - CH21.11 and CH22.4
- Bring out the special project #2

Special Project #3

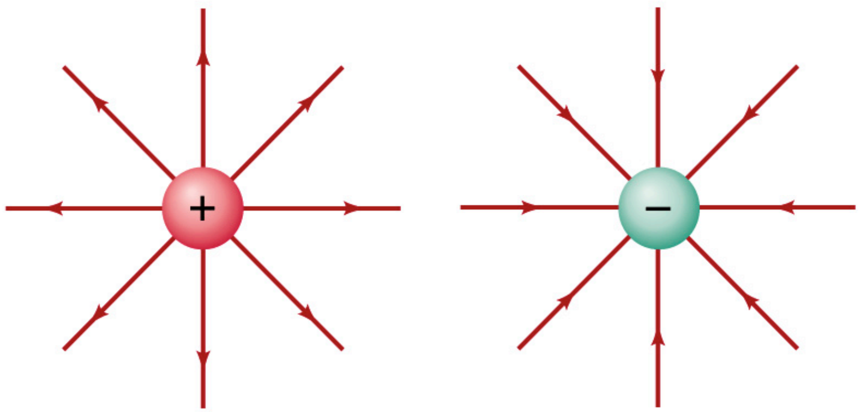
- **Particle Accelerator**. A charged particle of mass **M** with charge **-Q** is accelerated in the uniform field **E** between two parallel charged plates whose separation is **D** as shown in the figure on the right. The charged particle is accelerated from an initial speed $\mathbf{v_0}$ near the negative plate and passes through a tiny hole in the positive plate.
  - Derive the formula for the electric field E to accelerate the charged particle to a fraction *f* of the speed of light *c*. Express E in terms of **M, Q, D,** ***f*****, c** and $\mathbf{v_0}$.
  - (a) Using the Coulomb force and kinematic equations. (8 points)
  - (b) Using the work-kinetic energy theorem. (8 points)
  - (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)



- Due beginning of the class Monday, Oct. 2

Motion of a Charged Particle in an Electric Field

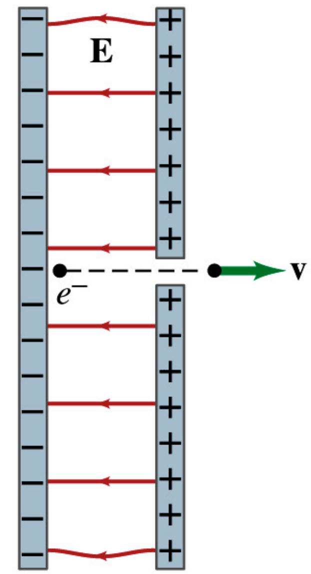
- If an object with an electric charge q is at a point in space where electric field is **E**, the force exerting on the object is $\vec{F} = q\vec{E}$.
- What do you think will happen to the charge?
  - Let's think about the cases like these on the right.
  - The object will move along the field line…Which way?
    - Depends on the sign of the charge
  - The charge gets accelerated under an electric field.



(a) (b)

Example 21 – 14

- **Electron accelerated by electric field.** An electron (mass m = 9.1×10^{-31}kg) is accelerated in a uniform field E (E=2.0×10^{4}N/C) between two parallel charged plates. The separation of the plates is 1.5cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate. (a) With what speed does it leave the hole? (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.



The magnitude of the force on the electron is F=qE and is directed to the right. The equation to solve this problem is

$$F = qE = ma$$

The magnitude of the electron's acceleration is $a = \frac{F}{m} = \frac{qE}{m}$

Between the plates the field **E** is uniform, thus the electron undergoes a uniform acceleration

$$a = \frac{eE}{m_e} = \frac{\left(1.6 \times 10^{-19} C\right)\left(2.0 \times 10^{4} N/C\right)}{\left(9.1 \times 10^{-31} kg\right)} = 3.5 \times 10^{15}\ m/s^2$$

Example 21 – 14

Since the travel distance is $1.5\text{x}10^{-2}$m, using one of the kinetic eq. of motions,

$$v^2 = v_0^2 + 2ax \;\therefore v = \sqrt{2ax} = \sqrt{2\cdot 3.5\times10^{15}\cdot 1.5\times10^{-2}} = 1.0\times10^{7}\, m/s$$

Since there is no electric field outside the conductor, the electron continues moving with this speed after passing through the hole.

- (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.

The magnitude of the electric force on the electron is

$$F_e = qE = eE = \left(1.6\times10^{-19}\,C\right)\left(2.0\times10^{4}\,N/C\right) = 3.2\times10^{-15}\,N$$

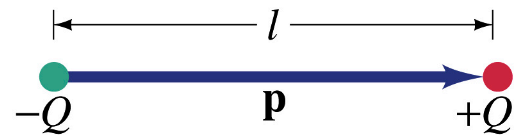
The magnitude of the gravitational force on the electron is

$$F_G = mg = 9.8\,m/s^2 \times\left(9.1\times10^{-31}\,kg\right) = 8.9\times10^{-30}\,N$$

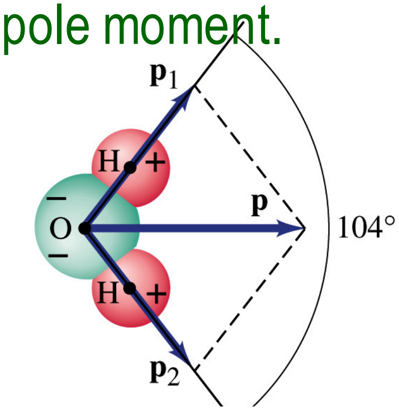
Thus the gravitational force on the electron is negligible compared to the electromagnetic force.

Electric Dipoles

- An electric dipole is the combination of two equal charges of opposite signs, +Q and –Q, separated by a distance l, which behaves as one entity.
- The quantity Ql is called the electric dipole moment and is represented by the symbol p.

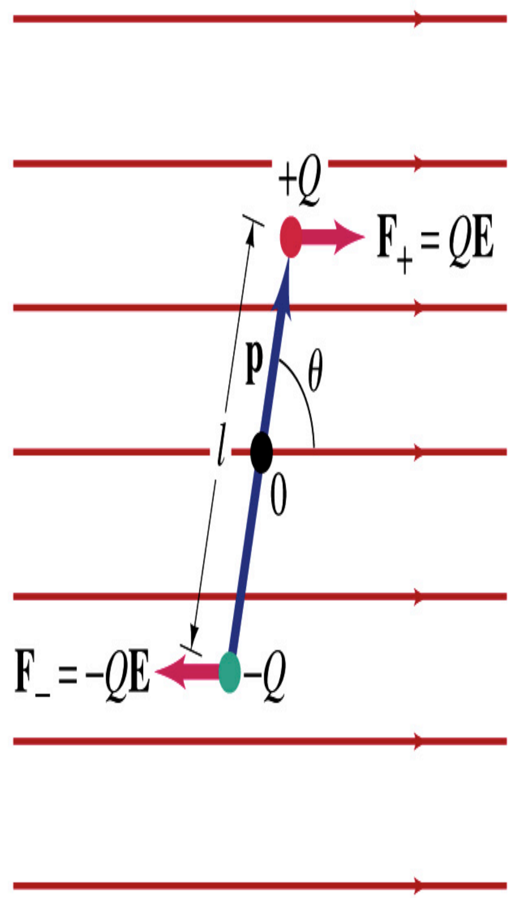


  - The dipole moment is a vector quantity, $\mathbf{p}$
  - The magnitude of the dipole moment is **Ql**. Unit? **C-m**
  - Its direction is from the negative to the positive charge.
  - Many of diatomic molecules like CO have a dipole moment. ➔ These are referred as polar molecules.
    - Even if the molecule is electrically neutral, their sharing of electron causes separation of charges
    - Symmetric diatomic molecules, such as O_2, do not have dipole moment.
  - The water molecule also has a dipole moment which is the vector sum of two dipole moments between Oxygen and each of Hydrogen atoms.

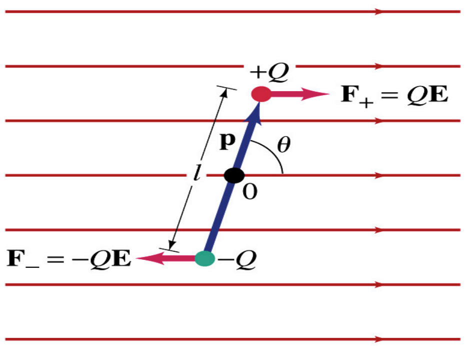


Dipoles in an External Field

- Let's consider a dipole placed in a uniform electric field **E**.
- What do you think will happen to the dipole in the figure?
  - Forces will be exerted on the charges.
    - The positive charge will get pushed toward right while the negative charge will get pulled toward left.
  - What is the net force acting on the dipole?
    - Zero
  - So will the dipole not move?
    - Yes, it will.
  - Why?
    - There is a torque applied on the dipole.



Dipoles in an External Field, cnt'd



- How much is the torque on the dipole?
  - Do you remember the formula for torque?
    - $\vec{\tau} = \vec{r} \times \vec{F}$
  - The magnitude of the torque exerting on each of the charges with respect to the rotational axis at the center is
    - $\tau_{+Q} = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta = \left|\left(\frac{l}{2}\right)(QE)\sin\theta\right| = \frac{l}{2}QE\sin\theta$
    - $\tau_{-Q} = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta = \left|\left(-\frac{l}{2}\right)(-QE)\sin\theta\right| = \frac{l}{2}QE\sin\theta$
  - Thus, the total torque is
    - $\tau_{Total} = \tau_{+Q} + \tau_{-Q} = \frac{l}{2}QE\sin\theta + \frac{l}{2}QE\sin\theta = lQE\sin\theta = pE\sin\theta$
  - So the torque on a dipole in vector notation is $\vec{\tau} = \vec{p} \times \vec{E}$
- The effect of the torque is to try to turn the dipole so that the dipole moment is parallel to **E**. Which direction?

Potential Energy of a Dipole in an External Field

- What is the work done on the dipole by the electric field to change the angle from θ_1 to θ_2?

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} \vec{\tau} \cdot d\vec{\theta} = \int_{\theta_1}^{\theta_2} -\tau d\theta$$

Why negative?

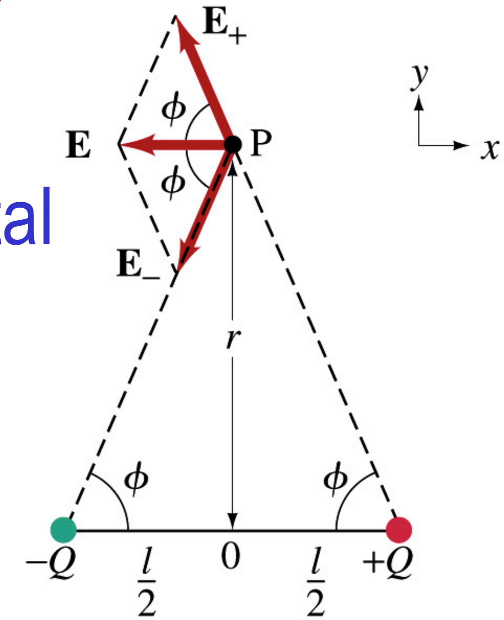
Because τ and θ are opposite directions to each other.

- The torque is $\tau = pE\sin\theta$.
- Thus the work done on the dipole by the field is

$$W = \int_{\theta_1}^{\theta_2} -pE\sin\theta d\theta = pE\left[\cos\theta\right]_{\theta_1}^{\theta_2} = pE\left(\cos\theta_2 - \cos\theta_1\right)$$

- What happens to the dipole's potential energy, U, when a positive work is done on it by the field?
  - It decreases.
- We choose U=0 when θ_1=90 degrees, then the potential energy at $\theta_2=\theta$ becomes $U = -W = -pE\cos\theta = -\vec{p}\cdot\vec{E}$

Electric Field by a Dipole



- Let's consider the case in the picture.
- There are fields by both the charges. So the total electric field by the dipole is $\vec{E}_{Tot} = \vec{E}_{+Q} + \vec{E}_{-Q}$
- The magnitudes of the two fields are equal

$$E_{+Q} = E_{-Q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\sqrt{r^2+(l/2)^2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2+(l/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2+l^2/4}$$

- Now we must work out the x and y components of the total field.
  - Sum of the two y components is
    - Zero since they are the same but in opposite direction
  - So the magnitude of the total field is the same as the sum of the two x-components:

$$E = 2E_+\cos\phi = \frac{1}{2\pi\varepsilon_0}\frac{Q}{r^2+l^2/4}\frac{l}{2\sqrt{r^2+l^2/4}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left(r^2+l^2/4\right)^{3/2}}$$

Dipole Electric Field from Afar

- What happens when r>>l?.

$$E_D = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left(r^2 + l^2/4\right)^{3/2}} \approx \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \quad \left(\text{when } r \gg l\right)$$

- Why does this make sense?
  - Since from a long distance, the two charges are very close so that the overall charge gets close to 0!!
  - This dependence works for the point not on the bisecting line as well

Example 21 – 17

- **Dipole in a field**. The dipole moment of a water molecule is 6.1×10^{-30}C-m. A water molecule is placed in a uniform electric field with magnitude 2.0×10^{5}N/C. (a) What is the magnitude of the maximum torque that the field can exert on the molecule? (b) What is the potential energy when the torque is at its maximum? (c) In what position will the potential energy take on its greatest value? Why is this different than the position where the torque is maximized?

(a) The torque is maximized when θ=90 degrees. Thus the magnitude of the maximum torque is

$$\tau = pE\sin\theta = pE =$$

$$=\left(6.1\times10^{-30}\,C\cdot m\right)\left(2.5\times10^{5}\,N/C\right)=1.2\times10^{-24}\,N\cdot m$$

What is the distance between a hydrogen atom and the oxygen atom?

Example 21 – 17

(b) What is the potential energy when the torque is at its maximum?

Since the dipole potential energy is $U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$

And τ is at its maximum at θ=90 degrees, the potential energy, U, is

$$U = -pE\cos\theta = -pE\cos\left(90^{\circ}\right) = 0$$

Is the potential energy at its minimum at θ=90 degrees? No

Why not? Because U will become negative as θ increases.

(c) In what position will the potential energy take on its greatest value?

The potential energy is maximum when cosθ= -1, θ=180 degrees.

Why is this different than the position where the torque is maximized?

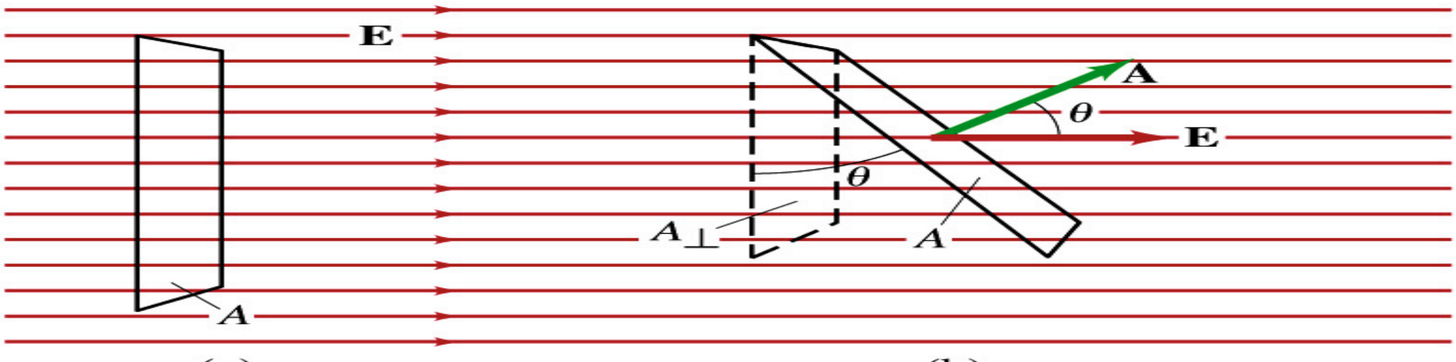
The potential energy is maximized when the dipole is oriented so that it has to rotate through the largest angle against the direction of the field, to reach the equilibrium position at θ=0.

Torque is maximized when the field is perpendicular to the dipole, θ=90.

Gauss' Law

- Gauss' law states the relationship between electric charge and the electric field.
  - More generalized and elegant form of Coulomb's law.
- The electric field by the distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions.
- Gauss' law, however, gives an additional insight into the nature of electrostatic field and a more general relationship between the charge and the field

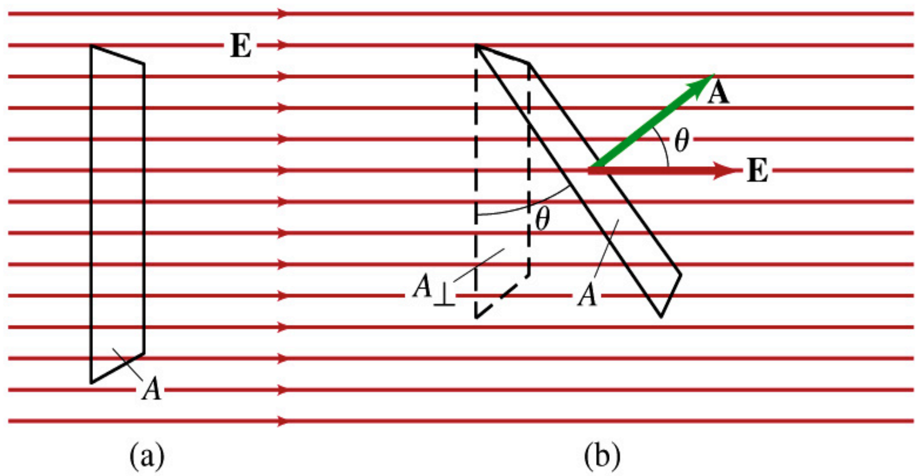
Electric Flux



- Let's imagine a surface of area A through which a uniform electric field E passes
- The electric flux Φ_E is defined as
  - Φ_E=EA, if the field is perpendicular to the surface
  - Φ_E=EAcosθ, if the field makes an angle θ to the surface
- So the electric flux is defined as $\Phi_E = \vec{E}\cdot\vec{A}.$
- How would you define the electric flux in words?
  - The total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_\perp = \Phi_E$

Example 22 – 1

- **Electric flux.** (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux in figure if the angle is 30 degrees?



The electric flux is defined as

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

So when (a) θ=0, we obtain

$$\Phi_E = EA\cos\theta = EA = (200N/C)\cdot\left(0.1\times0.2m^2\right) = 4.0\,\mathrm{N}\cdot\mathrm{m}^2/C$$

And when (b) θ=30 degrees, we obtain

$$\Phi_E = EA\cos 30^\circ = (200N/C)\cdot\left(0.1\times0.2m^2\right)\cos 30^\circ = 3.5\,\mathrm{N}\cdot\mathrm{m}^2/C$$

Generalization of the Electric Flux

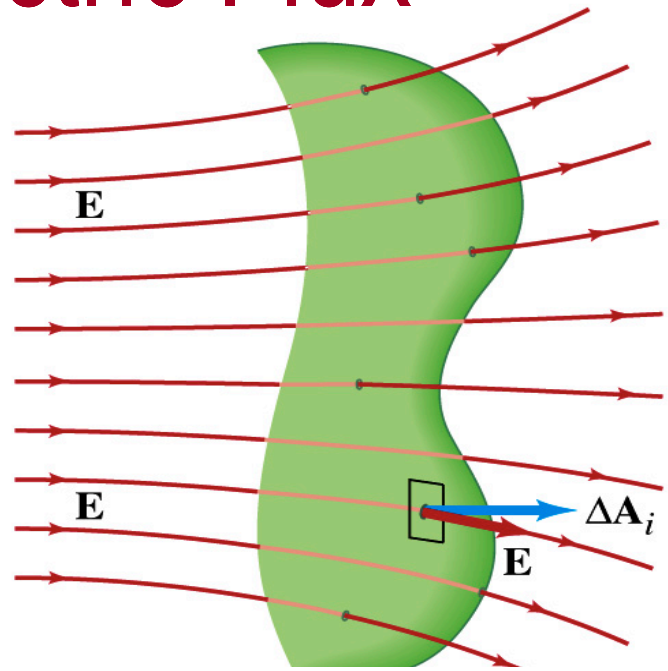
- Let's consider a surface of area A that is not a square or flat but in some random shape, and that the field is not uniform.
- The surface can be divided up into infinitesimally small areas of $\Delta\mathbf{A}_i$ that can be considered flat.
- And the electric field through this area can be considered uniform since the area is very small.
- Then the electric flux through the entire surface is approximately

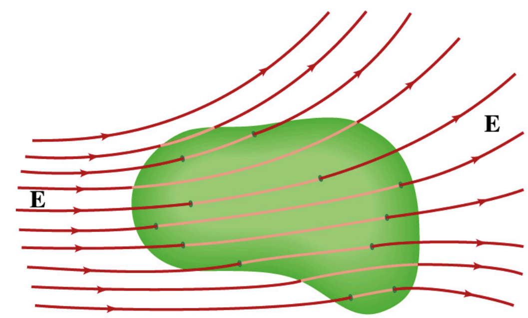
$$\Phi_E \approx \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i$$

- In the limit where $\Delta\mathbf{A}_i \rightarrow 0$, the discrete summation becomes an integral.

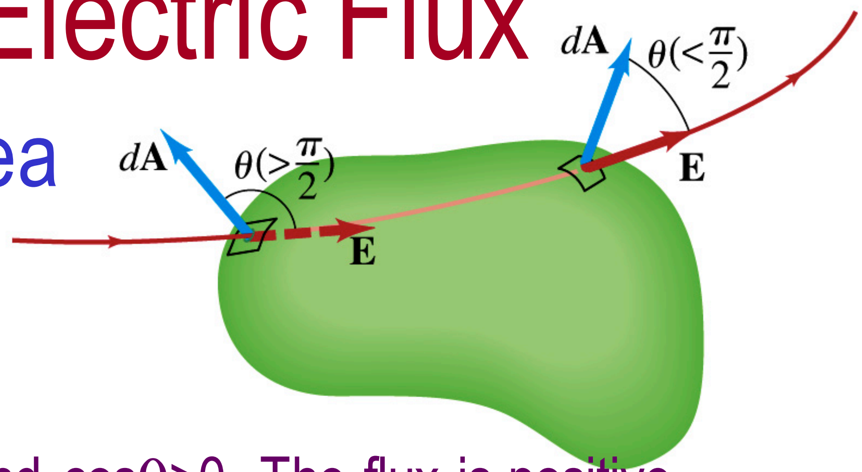
$$\Phi_E = \int \vec{E}_i \cdot d\vec{A}$$ open surface

$$\Phi_E = \oint \vec{E}_i \cdot d\vec{A}$$ enclosed surface

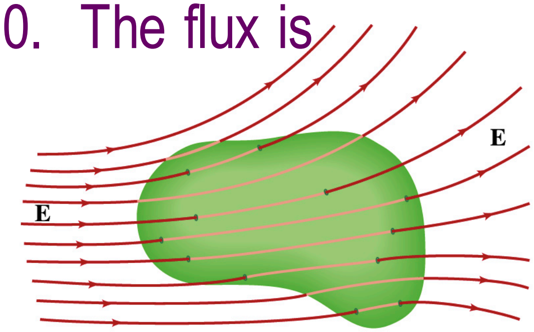




Generalization of the Electric Flux



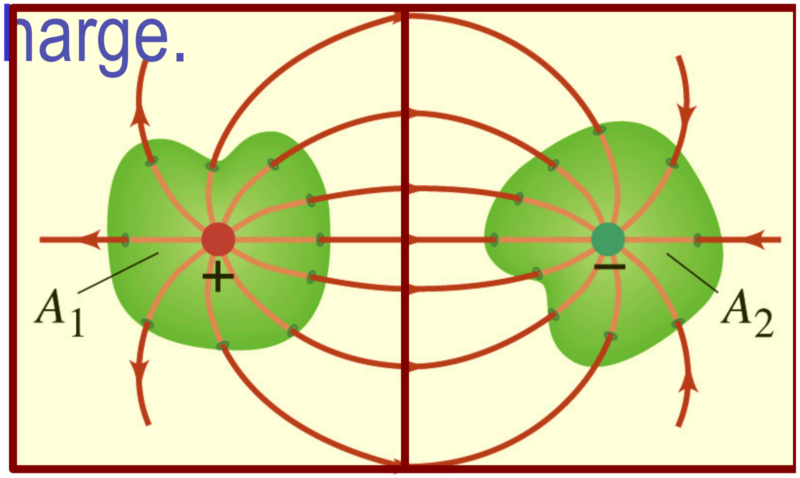
- We arbitrarily define that the area vector points outward from the enclosed volume.
  - For the line leaving the volume, $\theta<\pi/2$ and $\cos\theta>0$. The flux is positive.
  - For the line coming into the volume, $\theta>\pi/2$ and $\cos\theta<0$. The flux is negative.
  - If $\Phi_E>0$, there is net flux out of the volume.
  - If $\Phi_E<0$, there is flux into the volume.



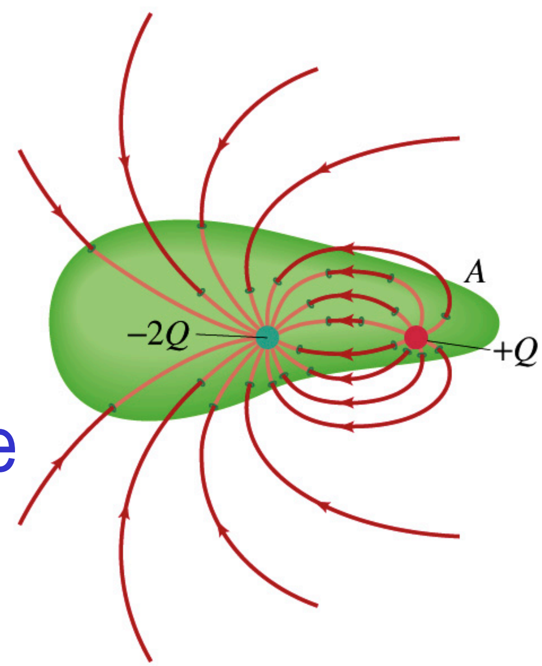
- In the above figures, each field that enters the volume also leaves the volume, so $\Phi_E = \oint \vec{E} \cdot d\vec{A} = 0.$
- The flux is non-zero only if one or more lines start or end inside the surface.

Generalization of the Electric Flux

- The field line starts or ends only on a charge.
- Sign of the net flux on the surface A_1?
  - The net outward flux (positive flux)
- How about A_2?
  - Net inward flux (negative flux)

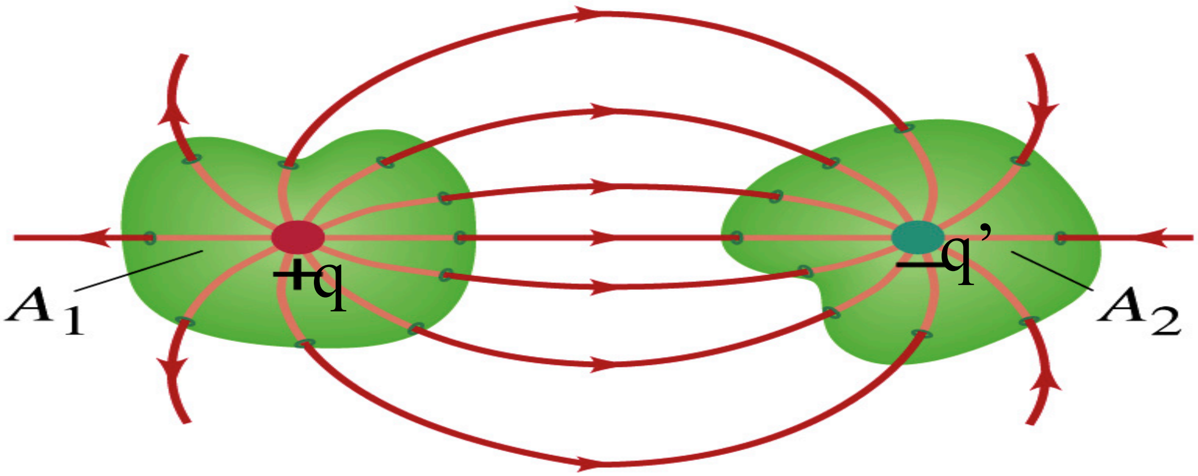


- What is the flux in the bottom figure?
  - There should be a net inward flux (negative flux) since the total charge inside the volume is negative.



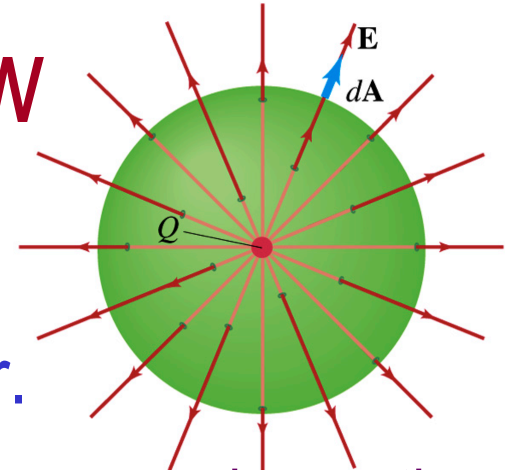
- The net flux that crosses an enclosed surface is proportional to the total charge inside the surface. ➔ This is the crux of Gauss' law.

Gauss' Law



- Let's consider the case in the above figure.
- What are the results of the closed integral of the Gaussian surfaces A_1 and A_2?
  - For A_1 $\oint \vec{E} \cdot d\vec{A} = \frac{+q}{\varepsilon_0}$
  - For A_2 $\oint \vec{E} \cdot d\vec{A} = \frac{-q'}{\varepsilon_0}$

Coulomb's Law from Gauss' Law



- Let's consider a charge Q enclosed inside our imaginary Gaussian surface of sphere of radius r.
  - Since we can choose any surface enclosing the charge, we choose the simplest possible one! ☺
- The surface is symmetric about the charge.
  - What does this tell us about the field E?
    - Must have the same magnitude (uniform) at any point on the surface
    - Points radially outward parallel to the surface vector d**A**.
- The Gaussian integral can be written as

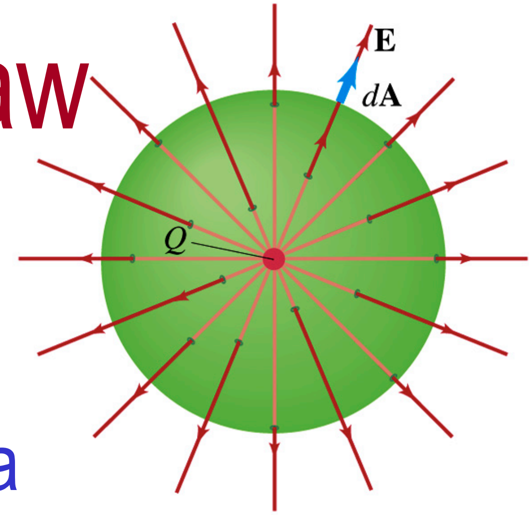
$$\oint \vec{E} \cdot d\vec{A} = \oint E\, dA = E \oint dA = E\left(4\pi r^2\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Solve for E

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Electric Field of Coulomb's Law

Gauss' Law from Coulomb's Law



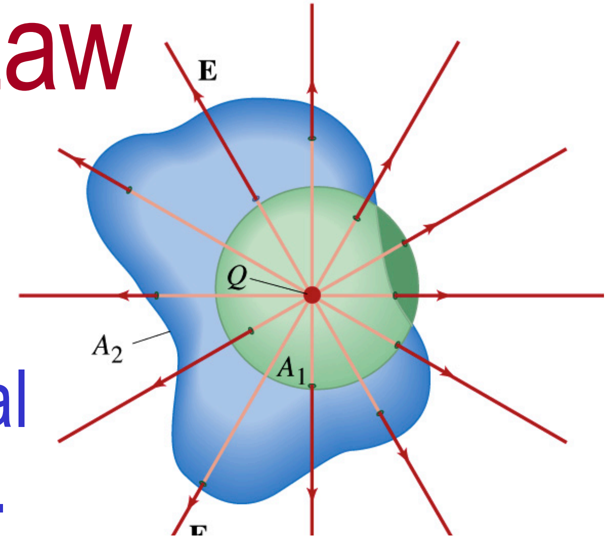
- Let's consider a single static point charge Q surrounded by an imaginary spherical surface.
- Coulomb's law tells us that the electric field at a spherical surface of radius r is $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
- Performing a closed integral over the surface, we obtain

$$\oint \vec{E}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dA$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\oint dA = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

Gauss' Law

Gauss' Law from Coulomb's Law Irregular Surface



- Let's consider the same single static point charge Q surrounded by a symmetric spherical surface A_1 and a randomly shaped surface A_2.
- What is the difference in the total number of field lines due to the charge Q, passing through the two surfaces?
  - None. What does this mean?
    - The total number of field lines passing through the surface is the same no matter what the shape of the enclosed surface is.
  - So we can write: $\oint_{A_1} \vec{E} \cdot d\vec{A} = \oint_{A_2} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$
  - What does this mean?
    - The flux due to the given enclosed charge is the same no matter what the shape of the surface enclosing it is. ➔ Gauss' law, $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, is valid for any surface surrounding a single point charge Q.

Gauss' Law w/ more than one charge

- Let's consider several charges inside a closed surface.
- For each charge, Q_i inside the chosen closed surface,

$$\oint \vec{E}_i \cdot d\vec{A} = \frac{Q_i}{\varepsilon_0}$$

What is $\vec{E}_i$?

The electric field produced by Q_i alone!

- Since electric fields can be added vectorially, following the superposition principle, the total field **E** is equal to the sum of the fields due to each charge $\vec{E} = \sum \vec{E}_i$ and any external fields. So

$$\oint \vec{E} \cdot d\vec{A} = \oint \left(\vec{E}_{ext} + \sum \vec{E}_i\right) \cdot d\vec{A} = \frac{\sum Q_i}{\varepsilon_0} = \frac{Q_{encl}}{\varepsilon_0}$$

What is Q_{encl}?

The total enclosed charge!

- The value of the flux depends only on the charge enclosed in the surface!! ➔ Gauss' law.

So what is Gauss' Law good for?

- Derivation of Gauss' law from Coulomb's law is only valid for **static electric charge**.
- Electric field can also be produced by changing magnetic fields.
  - Coulomb's law cannot describe this field while Gauss' law is still valid
- Gauss' law is more general than Coulomb's law.
  - Can be used to obtain electric field, forces or obtain charges

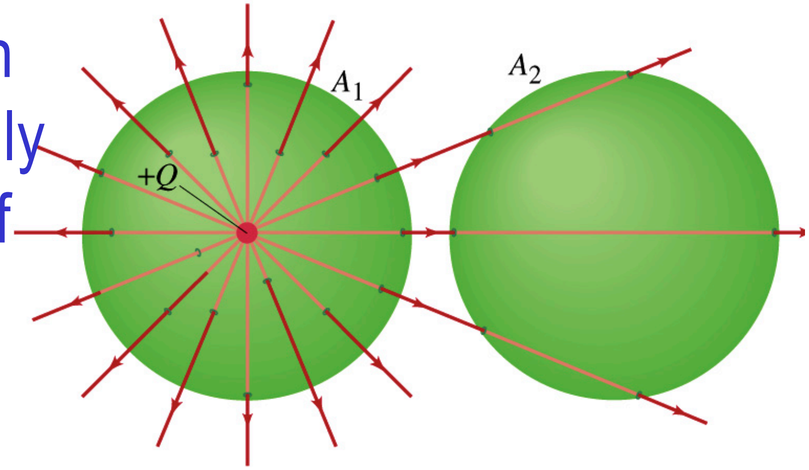
Gauss' Law: Any **differences** between the input and output flux of the electric field over any enclosed surface is due to the charge inside that surface!!!

Solving problems with Gauss' Law

- Identify the symmetry of the charge distributions
- Draw an appropriate Gaussian surface, making sure it passes through the point you want to know the electric field
- Use the symmetry of charge distribution to determine the direction of E at the point of the Gaussian surface
- Evaluate the flux
- Calculate the enclosed charge by the Gaussian surface
  - Ignore all the charges outside the Gaussian surface
- Equate the flux to the enclosed charge and solve for E

Example 22 – 2

Flux from Gauss' Law: Consider two Gaussian surfaces, A_1 and A_2, shown in the figure. The only charge present is the charge +Q at the center of surface A_1. What is the net flux through each surface A_1 and A_2?



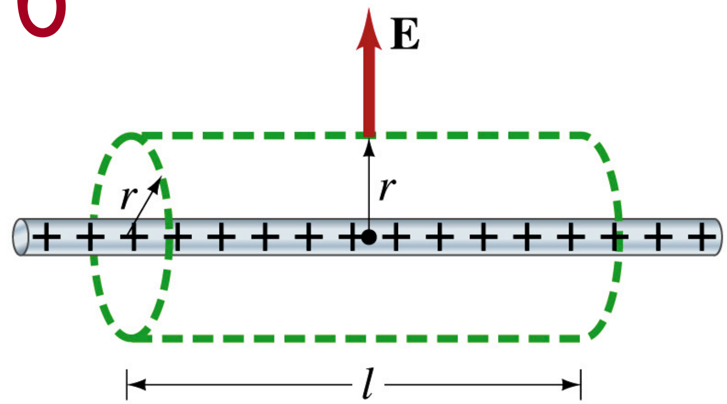
- The surface A_1 encloses the charge +Q, so from Gauss' law we obtain the total net flux

$$\oint \vec{E} \cdot d\vec{A} = \frac{+Q}{\varepsilon_0}$$

- The surface A_2 the charge, +Q, is outside the surface, so the total net flux is 0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{0}{\varepsilon_0} = 0$$

Example 22 – 6



Long uniform line of charge: A very long straight wire possesses a uniform positive charge per unit length, λ. Calculate the electric field at points near but outside the wire, far from the ends.

- Which direction do you think the field due to the charge on the wire is?
  - Radially outward from the wire, the direction of radial vector **r**.
- Due to cylindrical symmetry, the field is the same on the Gaussian surface of a cylinder surrounding the wire.
  - The end surfaces do not contribute to the flux at all. Why?
    - Because the field vector **E** is perpendicular to the surface vector d**A**.
- From Gauss' law

$$\oint \vec{E} \cdot d\vec{A} = E \oint dA = E(2\pi r l) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

Solving for E

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$