PHYS 1441 – Section 001
Lecture #23

Monday, Dec. 4, 2017

Dr. Jaehoon Yu

- Chapter 30: Inductance
  - Mutual and Self Inductance
  - Energy Stored in Magnetic Field
  - Alternating Current and AC Circuits
  - AC Circuit W/ LRC
- Chapter 31: Maxwell's Equations
  - Expansion of Ampere's Law

Announcements

- Reading Assignments
  - CH30.10 and CH30.11
- Final exam
  - Date and time: 11am – 12:30pm, Monday, Dec. 11 in SH101
  - Comprehensive exam: covers CH21.1 through what we finish this Wednesday, Dec. 6
  - Bring your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants
  - No derivations, word definitions or solutions of any kind!
  - No additional formulae or values of constants will be provided!

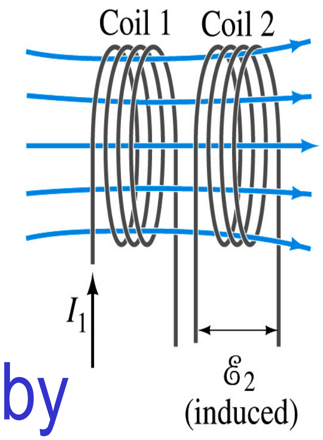
Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - Or induce an emf in itself ➔ Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2, in coil2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21} I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined as $\boxed{M_{21} = N_2\Phi_{21}/I_1}$.
- The emf induced in coil2 due to the changing current in coil1 is

$$\boxed{\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d\left(N_2\Phi_{21}\right)}{dt} = -M_{21}\frac{dI_1}{dt}}$$



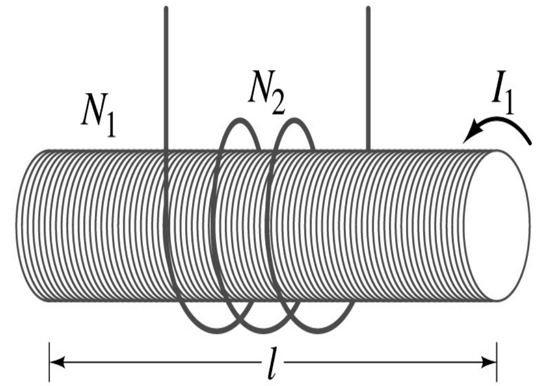
Mutual Inductance

- The mutual induction of coil2 with respect to coil1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

    What? Does this make sense?

    - The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
  - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain $$\varepsilon_1 = -M\frac{dI_2}{dt} \text{ and } \varepsilon_2 = -M\frac{dI_1}{dt}$$
  - SI unit for mutual inductance is henry (H) $$1H = 1V \cdot s/A = 1\Omega \cdot s$$

Example 30 – 1



Solenoid and coil. A long thin solenoid of length ℓ and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.

First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $B = \dfrac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil 2 is

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this?
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impedes its increase, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = L I$
- We define self-inductance, $\mathcal{L}$: $$\boxed{L = \frac{N\Phi_B}{I}}$$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V \cdot s/A = 1\Omega \cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

- It is an impediment onto the electrical current due to the existence of changing flux
- So what?
- In other words, it behaves like a resistance to the varying current, such as AC, that causes the constant change of flux
- But it also provides means to store energy, just like the capacitance

Self Inductance

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Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = LI$
- We define self-inductance, $\mathcal{L}$: $\boxed{L = \frac{N\Phi_B}{I}}$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
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Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  - Precision resisters are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a "non-inductive winding"
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

Example 30 – 3

Solenoid inductance. (a) Determine the formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing N turns of wire in its length ℓ and whose cross-sectional area is A. (b) Calculate the value of $\mathcal{L}$ if N=100, ℓ=5.0cm, A=0.30cm^2 and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with $\mu=4000\mu_0$.

What is the magnetic field inside a solenoid? $B = \mu_0 nI = \mu_0 NI/l$

The flux is, therefore, $\Phi_B = BA = \mu_0 NIA/l$

Using the formula for self inductance: $L = \frac{N\Phi_B}{I} = \frac{N \cdot \mu_0 NIA/l}{I} = \frac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\left(4\pi \times 10^{-7}\, T\cdot m/A\right)100^2\left(0.30\times 10^{-4}\, m^2\right)}{5.0\times 10^{-2}\, m} = 7.5\mu H$$

(c) The magnetic field with an iron core solenoid is $B = \mu NI/l$

$$L = \frac{\mu N^2 A}{l} = \frac{4000\left(4\pi \times 10^{-7}\, T\cdot m/A\right)100^2\left(0.30\times 10^{-4}\, m^2\right)}{5.0\times 10^{-2}\, m} = 0.030H = 30mH$$

Energy Stored in the Magnetic Field

- When an inductor of inductance $\mathcal{L}$ is carrying current I which is changing at a rate dI/dt, energy is supplied to the inductor at a rate
  - $P = I\varepsilon = IL\frac{dI}{dt}$
- What is the work needed to increase the current in an inductor from 0 to I?
  - The work, dW, done in time dt is $dW = Pdt = LIdI$
  - Thus the total work needed to bring the current from 0 to I in an inductor is

$$W = \int dW = \int_0^I LIdI = L\left[\frac{1}{2}I^2\right]_0^I = \frac{1}{2}LI^2$$

Energy Stored in the Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $\boxed{U = \frac{1}{2}LI^2}$

    Energy Stored in a magnetic field inside an inductor

  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V: $U = \frac{1}{2}CV^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without a fringe effect

$$L = \mu_0 N^2 A / l$$

  - The magnetic field in a solenoid is $B = \mu_0 NI / l$
  - Thus the energy stored in an inductor is

$$U = \frac{1}{2} LI^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \left(\frac{Bl}{\mu_0 N} \right)^2 = \frac{1}{2} \frac{B^2}{\mu_0} Al$$

What is this?

$$\boxed{U = \frac{1}{2} \frac{B^2}{\mu_0} Al}$$

E

  - Thus the energy density is

$$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2} \frac{B^2}{\mu_0}$$

Volume V

$$\boxed{u = \frac{1}{2} \frac{B^2}{\mu_0}}$$

E density

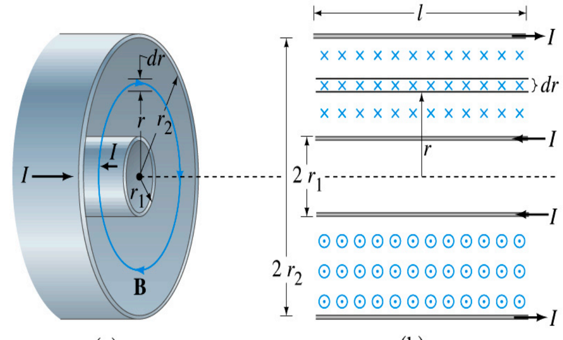
  - This formula is valid in any region of space
  - If a ferromagnetic material is present, μ_0 becomes μ.

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I? (b) Where is the energy density highest?



(a) (b)

(a) The total flux through l of the cable is $\Phi_B = \int Bl\,dr = \frac{\mu_0 Il}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\mu_0 Il}{2\pi}\ln\frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}$

Thus the energy stored per unit length is

$$\frac{U}{l} = \frac{1}{2}\frac{LI^2}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{r_2}{r_1}$$

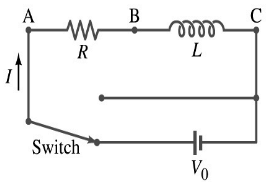
(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2}\frac{B^2}{\mu_0}$

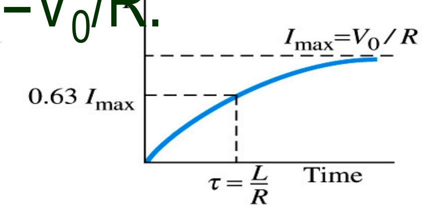
The energy density is highest where B is highest. Since B is highest close to $r=r_1$, near the surface of the inner conductor.

LR Circuits

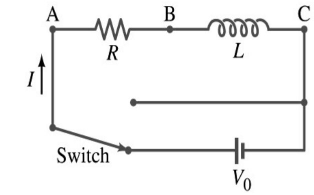
- What happens when an emf is applied to an inductor?
  - An inductor has some resistance, however negligible



    - So an inductor can be drawn as a circuit of separate resistance and coil. What is the name this kind of circuit? **LR Circuit**
  - What happens at the instance the switch is thrown to apply emf to the circuit?
    - The current starts to flow, gradually increasing from 0
    - This change is opposed by the induced emf in the inductor ➔ the emf at point B is higher than point C
    - However there is a voltage drop at the resistance which reduces the voltage across inductance
    - Thus the current increases less rapidly
    - The overall behavior of the current is a gradual increase, reaching to the maximum current $I_{max}=V_0/R$.

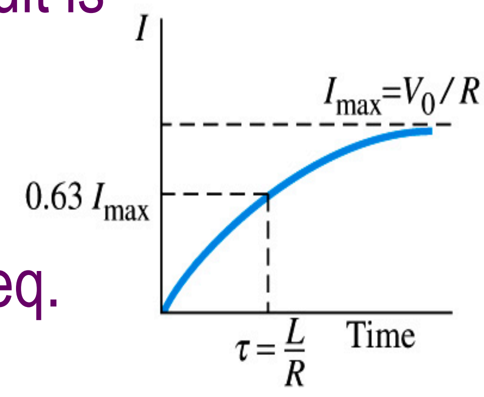


LR Circuits



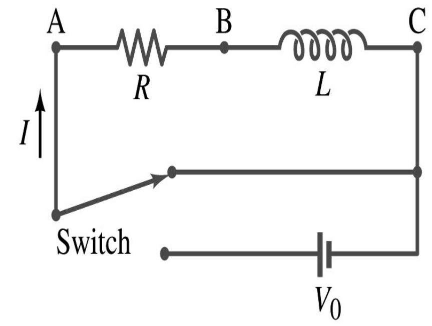
- This can be shown w/ Kirchhoff loop rules
  - The emfs in the circuit are the battery voltage V_0 and the emf $\varepsilon = -\mathcal{L}(dI/dt)$ in the inductor opposing the current increase
  - The sum of the potential changes through the circuit is

$$V_0 + \varepsilon - IR = V_0 - L\,dI/dt - IR = 0$$

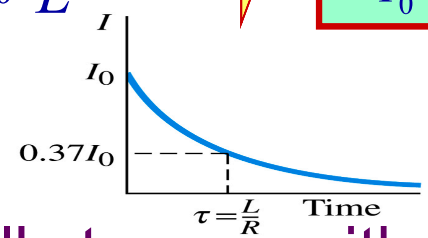


  - Where I is the current at any instance
  - By rearranging the terms, we obtain a differential eq.
  - $L\,dI/dt + IR = V_0$
  - We can integrate just as in RC circuit $\int_{I=0}^{I} \frac{dI}{V_0 - IR} = \int_{t=0}^{t} \frac{dt}{L}$
  - So the solution is $-\frac{1}{R}\ln\left(\frac{V_0 - IR}{V_0}\right) = \frac{t}{L}$ $\boxed{I = V_0\left(1 - e^{-t/\tau}\right)/R = I_{max}\left(1 - e^{-t/\tau}\right)}$
  - Where τ=L/R
    - This is the time constant τ of the LR circuit and is the time required for the current I to reach 0.63 of the maximum

Discharge of LR Circuits

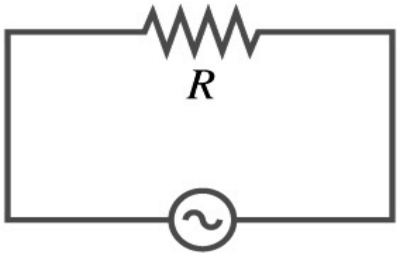


- If the switch is flipped away from the battery
  - The differential equation becomes
  - $L\,dI/dt + IR = 0$
  - So the integration is $\int_{I_0}^{I} \frac{dI}{IR} = \int_{t=0}^{t} \frac{dt}{L}$ $\Rightarrow$ $\boxed{\ln \frac{I}{I_0} = -\frac{R}{L}t}$
  - Which results in the solution
  - $\boxed{I = I_0 e^{-\frac{R}{L}t} = I_0 e^{-t/\tau}}$



  - The current decays exponentially to zero with the time constant τ=L/R
  - So there always is a reaction time when a system with a coil, such as an electromagnet, is turned on or off.
  - The current in LR circuit behaves almost the same as that in RC circuit but the time constant is inversely proportional to R in LR circuit unlike the RC circuit

AC Circuit w/ Resistance only



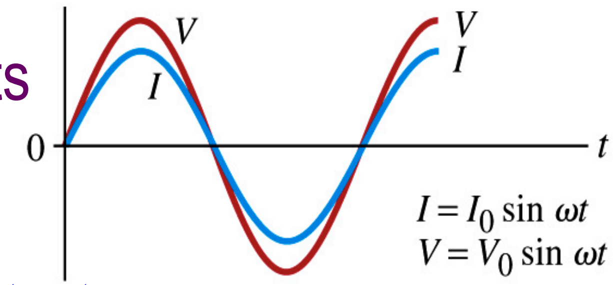
- What do you think will happen when an AC source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain

$$V - IR = 0$$

- Thus

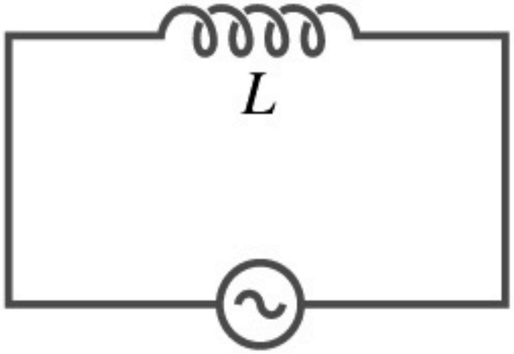
$$V = I_0 R \sin \varpi t = V_0 \sin \varpi t$$

  - where $V_0 = I_0 R$
- What does this mean?
  - Current is 0 when voltage is 0 and current is in its peak when voltage is in its peak.
  - Current and voltage are "in phase"



- Energy is lost via the transformation into heat at an average rate $\overline{P} = \overline{I}\,\overline{V} = I_{rms}^2 R = V_{rms}^2 / R$

AC Circuit w/ Inductance only



- From Kirchhoff's loop rule, we obtain

$$V - L\frac{dI}{dt} = 0$$

- Thus

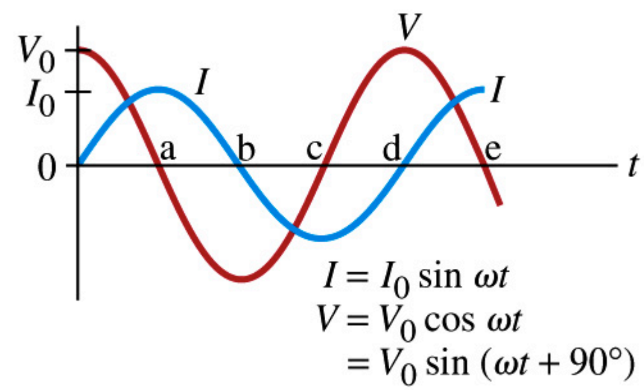
$$V = L\frac{dI}{dt} = L\frac{d\left(I_0 \sin \varpi t\right)}{dt} = \varpi L I_0 \cos \varpi t$$

  – Using the identity $\cos\theta = \sin\left(\theta + 90^\circ\right)$

- $$V = \varpi L I_0 \sin\left(\varpi t + 90^\circ\right) = V_0 \sin\left(\varpi t + 90^\circ\right)$$

  – where $V_0 = \varpi L I_0$

- What does this mean?
  – Current and voltage are "out of phase by π/2 or 90°". In other words the current reaches its peak ¼ cycle after the voltage
- What happens to the energy?
  – No energy is dissipated
  – The average power is 0 at all times
  – The energy is stored temporarily in the magnetic field
  – Then released back to the source



AC Circuit w/ Inductance only

- How are the resistor and inductor different in terms of energy?
  - Inductor Stores the energy temporarily in the magnetic field and then releases it back to the emf source
  - Resistor Does not store energy but transforms it to thermal energy, losing it to the environment
- How are they the same?
  - They both impede the flow of charge
  - For a resistance R, the peak voltage and current are related to $V_0 = I_0 R$
  - Similarly, for an inductor we may write $\boxed{V_0 = I_0 X_L}$
    - Where X_L is the inductive reactance of the inductor $\boxed{X_L = \varpi L}$ $\boxed{\text{0 when } \omega\text{=0.}}$
    - What do you think is the unit of the reactance? Ω
    - The relationship $V_0 = I_0 X_L$ is not valid at a particular instance. Why not?
      - Since V_0 and I_0 do not occur at the same time

$\boxed{V_{rms} = I_{rms} X_L}$ $\boxed{\text{is valid!}}$

Example 30 – 9

Reactance of a coil. A coil has a resistance R=1.00Ω and an inductance of 0.300H. Determine the current in the coil if (a) 120 V DC is applied to it; (b) 120 V AC (rms) at 60.0Hz is applied.

Is there a reactance for DC? Nope. Why not? Since ω=0,

$$X_L = \varpi L = 0$$

So for DC power, the current is from Kirchhoff's rule

$$V - IR = 0$$

$$I_0 = \frac{V_0}{R} = \frac{120V}{1.00\Omega} = 120A$$

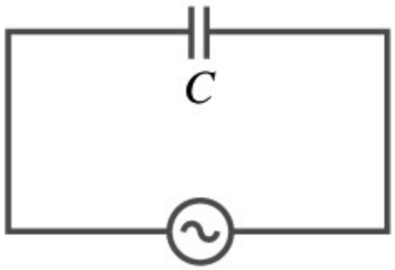
For an AC power with f=60Hz, the reactance is

$$X_L = \varpi L = 2\pi fL = 2\pi \cdot \left(60.0s^{-1}\right) \cdot 0.300H = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is

$$I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120V}{113\Omega} = 1.06A$$

AC Circuit w/ Capacitance only

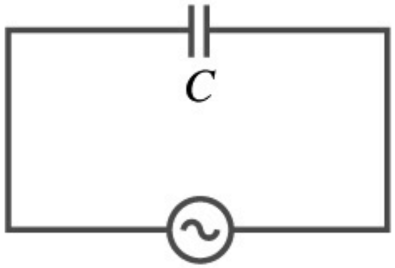


- What happens when a capacitor is connected to a DC power source?
  - The capacitor quickly charges up.
  - There is no steady current flow in the circuit
    - Since the capacitor prevents the flow of the DC current
- What do you think will happen if it is connected to an AC power source?
  - The current flows continuously. Why?
  - When the AC power turns on, charge begins to flow one direction, charging up the plates
  - When the direction of the power reverses, the charge flows in the opposite direction

AC Circuit w/ Capacitance only

- From Kirchhoff's loop rule, we obtain

$$V=\frac{Q}{C}$$



- The current at any instance is $I=\frac{dQ}{dt}=I_0\sin\varpi t$
- The charge Q on the plate at any instance is

$$Q=\int_{Q=0}^{Q}dQ=\int_{t=0}^{t}I_0\sin\varpi t\,dt=-\frac{I_0}{\varpi}\cos\varpi t$$

- Thus the voltage across the capacitor is

$$V=\frac{Q}{C}=-I_0\frac{1}{\varpi C}\cos\varpi t$$

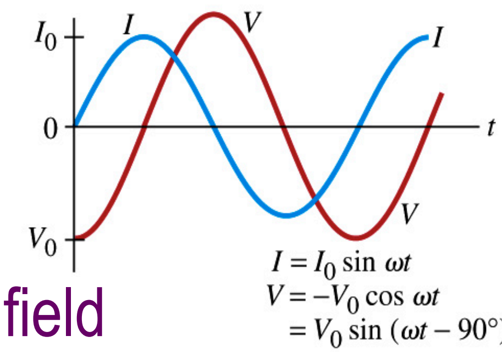
  - Using the identity $\cos\theta=-\sin\left(\theta-90^\circ\right)$

$$V=I_0\frac{1}{\varpi C}\sin\left(\varpi t-90^\circ\right)=V_0\sin\left(\varpi t-90^\circ\right)$$

  - Where
  - $V_0=\frac{I_0}{\varpi C}$

AC Circuit w/ Capacitance only

- So the voltage is $V = V_0 \sin\left(\varpi t - 90^o\right)$
- What does this mean?
  - Current and voltage are "out of phase by π/2 or 90º" but in this case, the voltage reaches its peak ¼ cycle after the current
- What happens to the energy?
  - No energy is dissipated
  - The average power is 0 at all times
  - The energy is stored temporarily in the electric field
  - Then released back to the source

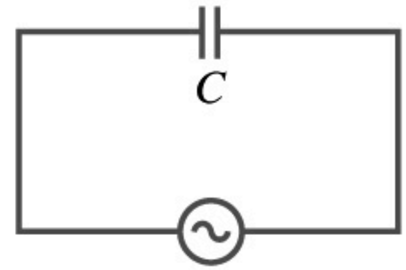


- Applied voltage and the current in the capacitor can be written as $\boxed{V_0 = I_0 X_C}$
  - Where the capacitive reactance X_C is defined as $\boxed{X_C = \frac{1}{\varpi C}}$
  - Again, this relationship is only valid for rms quantities $\boxed{V_{rms} = I_{rms} X_C}$

Infinite when ω=0.

Example 30 – 10

Capacitor reactance. What are the peak and rms current in the circuit in the figure if C=1.0μF and V_{rms}=120V? Calculate for (a) f=60Hz, and then for (b) f=6.0x10^{5}Hz.



The peak voltage is $V_0 = \sqrt{2}V_{rms} = 120V \cdot \sqrt{2} = 170V$

The capacitance reactance is

$$X_C = \frac{1}{\varpi C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \cdot \left(60s^{-1}\right) \cdot 1.0 \times 10^{-6} F} = 2.7k\Omega$$

Thus the peak current is

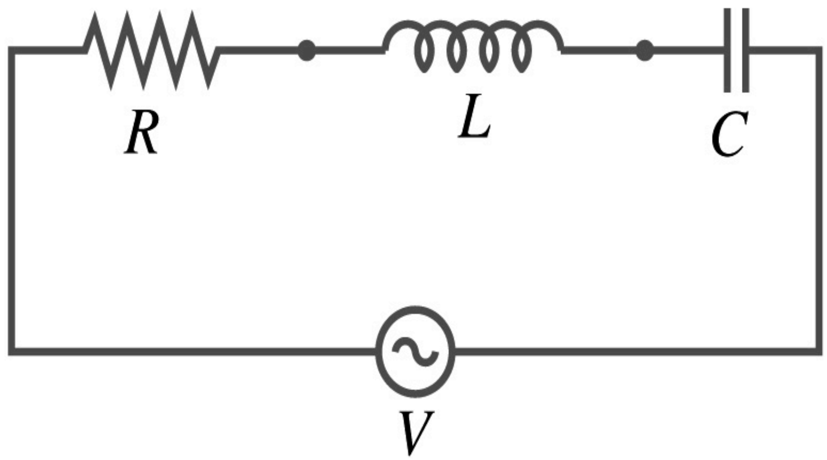
$$I_0 = \frac{V_0}{X_C} = \frac{170V}{2.7k\Omega} = 63mA$$

The rms current is

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{120V}{2.7k\Omega} = 44mA$$

AC Circuit w/ LRC

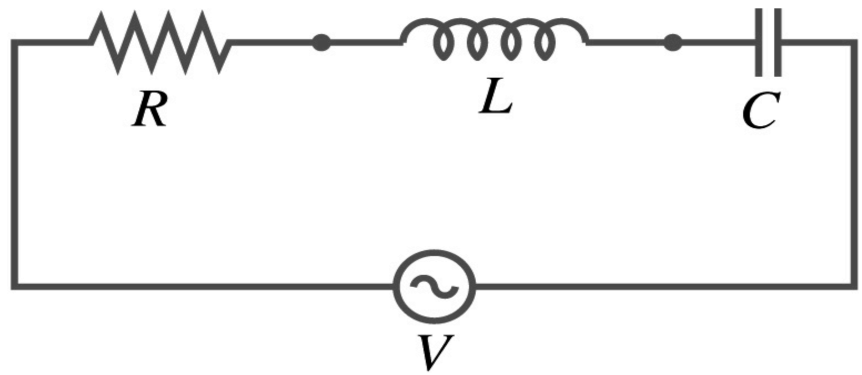
- The voltage across each element is
  – V_R is in phase with the current
  – V_L leads the current by 90°
  – V_C lags the current by 90°



- From Kirchhoff's loop rule
- $V=V_R+V_L+V_C$
  – However since they do not reach the peak voltage at the same time, the peak voltage of the source V_0 will not equal $V_{R0}+V_{L0}+V_{C0}$
  – The rms voltage also will not be the simple sum of the three
- Let's try to find the total impedance, peak current I_0 and the phase difference between I_0 and V_0.

AC Circuit w/ LRC

- The current at any instance is the same at all point in the circuit
  - The currents in each elements are in phase
  - Why?
    - Since the elements are in series
  - How about the voltage?
    - They are not in phase.



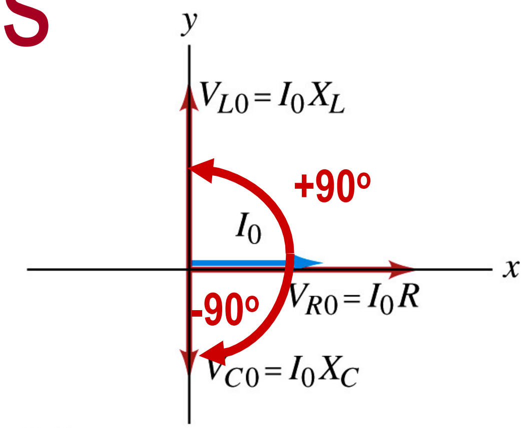
- The current at any given time is

$$I = I_0 \sin \varpi t$$

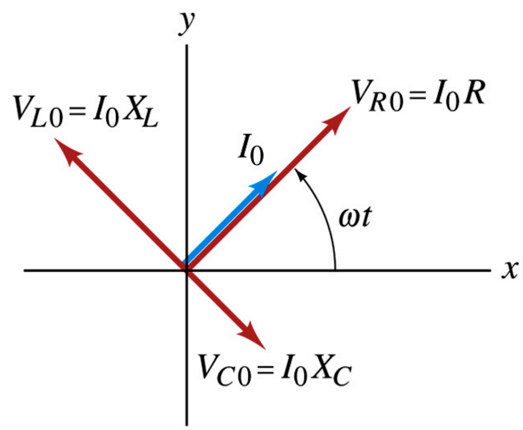
- The analysis of LRC circuit is done using the "phasor" diagram in which arrows are drawn in an xy plane to represent the amplitude of each voltage, just like vectors
  - The lengths of the arrows represent the magnitudes of the peak voltages across each element; $V_{R0}=I_0R$, $V_{L0}=I_0X_L$ and $V_{C0}=I_0X_C$
  - The angle of each arrow represents the phase of each voltage relative to the current, and the arrows rotate at the angular frequency ω to take into account the time dependence.
    - The projection of each arrow on y axis represents voltage across each element at any given time

Phasor Diagrams

- At t=0, I=0.
  - Thus $V_{R0}=0$, $V_{L0}=I_0X_L$, $V_{C0}=I_0X_C$
- At t=t, $I = I_0 \sin \varpi t$



(a)



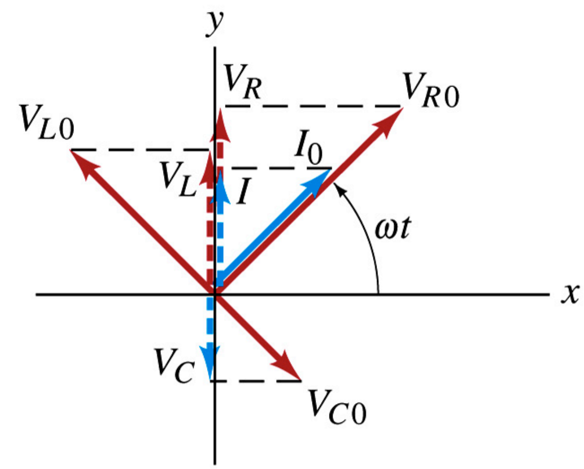
(b)

- Thus, the voltages (y-projections) are

$$V_R = V_{R0} \sin \varpi t$$

$$V_L = V_{L0} \sin\left(\varpi t + 90^\circ\right)$$

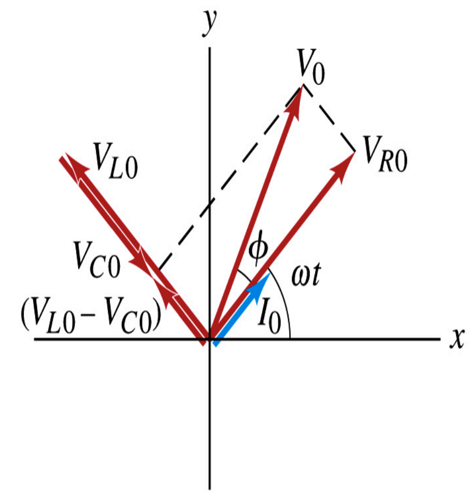
$$V_C = V_{C0} \sin\left(\varpi t - 90^\circ\right)$$



(c)

AC Circuit w/ LRC

- Since the sum of the projections of the three vectors on the y axis is equal to the projection of their sum.
  - The sum of the projections represents the instantaneous voltage across the whole circuit which is the source voltage
  - So we can use the sum of all vectors as the representation of the peak source voltage V_0.



- V_0 forms an angle ϕ to V_{R0} and rotates together with the other vectors as a function of time, $V = V_0 \sin(\varpi t + \phi)$
- We determine the total impedance Z of the circuit defined by the relationship $V_{rms} = I_{rms} Z$ or $V_0 = I_0 Z$
- From Pythagorean theorem, we obtain

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = \sqrt{I_0^2 R^2 + I_0^2 (X_L - X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} = I_0 Z$$

- Thus the total impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\varpi L - \frac{1}{\varpi C}\right)^2}$

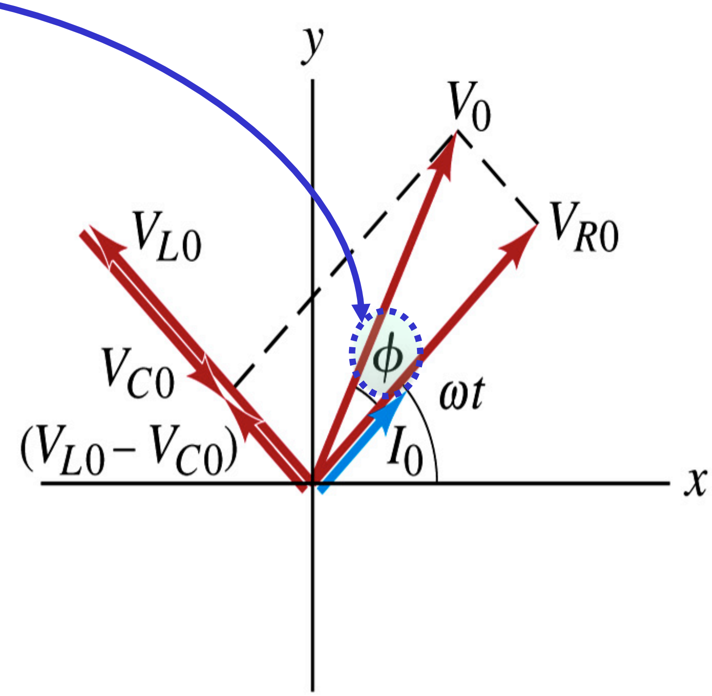
AC Circuit w/ LRC

- The phase angle ϕ is

$$\tan\phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0\left(X_L - X_C\right)}{I_0 R} = \frac{\left(X_L - X_C\right)}{R}$$

- or

$$\cos\phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$



- What is the power dissipated in the circuit?
  - Which element dissipates the power?
  - Only the resistor
- The average power is $\overline{P} = I_{rms}^2 R$
  - Since R=Zcosϕ
  - We obtain $$\overline{P} = I_{rms}^2 Z\cos\phi = I_{rms} V_{rms} \cos\phi$$
  - The factor cosϕ is referred as the power factor of the circuit
  - For a pure resistor, cosϕ=1 and $\overline{P} = I_{rms} V_{rms}$
  - For a capacitor or inductor alone ϕ=-90° or +90°, so cosϕ=0 and $\overline{P} = 0.$