PHYS 1441 – Section 001
Lecture #24

Wednesday, Dec. 6, 2017

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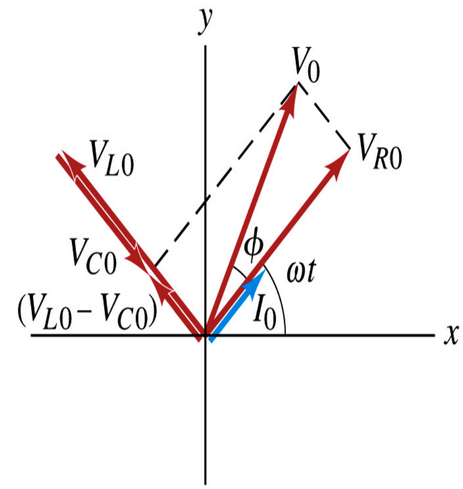
- Chapter 30: Inductance
  - AC Circuit W/ LRC
- Chapter 31: Maxwell's Equations
  - Expansion of Ampere's Law
  - Gauss' Law for Magnetism
  - Production of EM Waves
  - Light as EM Waves

Announcements

- Reading Assignments
  - CH30.10 and CH30.11
- Final exam
  - Date and time: 11am – 12:30pm, Monday, Dec. 11 in SH101
  - Comprehensive exam: covers CH21.1 through what we finish today. Wednesday, Dec. 6
  - Bring your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants
  - No derivations, word definitions or solutions of any kind!
  - No additional formulae or values of constants will be provided!

AC Circuit w/ LRC

- Since the sum of the projections of the three vectors on the y axis is equal to the projection of their sum.
  - The sum of the projections represents the instantaneous voltage across the whole circuit which is the source voltage
  - So we can use the sum of all vectors as the representation of the peak source voltage V_0.



- V_0 forms an angle φ to V_{R0} and rotates together with the other vectors as a function of time, $V = V_0 \sin(\varpi t + \phi)$
- We determine the total impedance Z of the circuit defined by the relationship $V_{rms} = I_{rms} Z$ or $V_0 = I_0 Z$
- From Pythagorean theorem, we obtain

$$V_0 = \sqrt{V_{R0}^2 + \left(V_{L0} - V_{C0}\right)^2} = \sqrt{I_0^2 R^2 + I_0^2 \left(X_L - X_C\right)^2} = I_0 \sqrt{R^2 + \left(X_L - X_C\right)^2} = I_0 Z$$

- Thus the total impedance is $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(\varpi L - \dfrac{1}{\varpi C}\right)^2}$

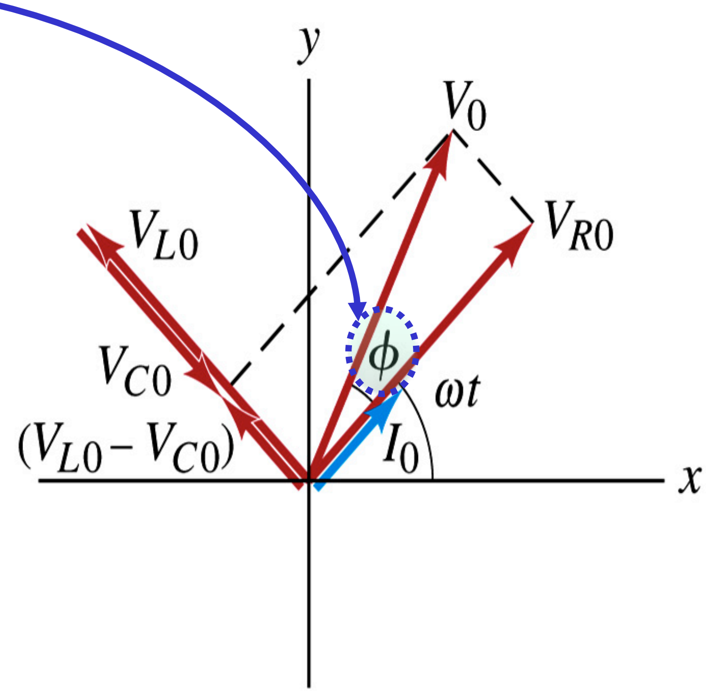
AC Circuit w/ LRC

- The phase angle ϕ is

$$\tan\phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0\left(X_L - X_C\right)}{I_0 R} = \frac{\left(X_L - X_C\right)}{R}$$

- or

$$\cos\phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$



- What is the power dissipated in the circuit?
  - Which element dissipates the power?
  - Only the resistor
- The average power is $\bar{P} = I_{rms}^2 R$
  - Since R=Zcosϕ
  - We obtain $$\bar{P} = I_{rms}^2 Z\cos\phi = I_{rms} V_{rms} \cos\phi$$
  - The factor cosϕ is referred as the power factor of the circuit
  - For a pure resistor, cosϕ=1 and $\bar{P} = I_{rms} V_{rms}$
  - For a capacitor or inductor alone ϕ=-90° or +90°, so cosϕ=0 and $\bar{P} = 0.$

Maxwell's Equations

- The development of EM theory by Oersted, Ampere and others was not done in terms of EM fields
  - The idea of fields was introduced somewhat by Faraday
- Scottish physicist James C. Maxwell unified all the phenomena of electricity and magnetism in one theory with only four equations (Maxwell's Equations) using the concept of fields
  - This theory provided the prediction of EM waves
  - As important as Newton's law since it provides dynamics of electromagnetism
  - This theory is also in agreement with Einstein's special relativity
- The biggest achievement of 19^{th} century electromagnetic theory is the prediction and experimental verifications that the electromagnetic waves can travel through the empty space
  - What do you think this accomplishment did?
    - Open a new world of communication
    - It also yielded the prediction that the light is an EM wave
- Since all of Electromagnetism is contained in the four Maxwell's equations, this is considered as one of the greatest achievements of human intellect

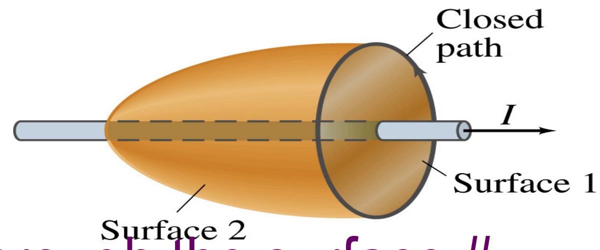
Ampere's Law

- Do you remember the mathematical expression of Oersted discovery of a magnetic field produced by an electric current, given by Ampere?

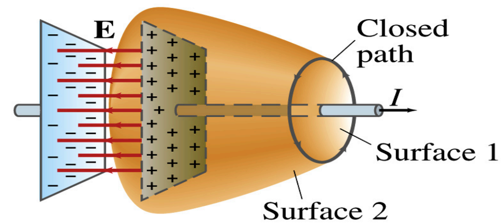
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- We've learned that a varying magnetic field produces an electric field
- Then can the reverse phenomena, that a changing electric field producing a magnetic field, possible?
  - If this is the case, it would demonstrate a beautiful symmetry in nature between electricity and magnetism

Expanding Ampere's Law



- Let's consider a wire carrying current I
  - The current that is enclosed in the loop passes through the surface # 1 in the figure
  - We could imagine a different surface # 2 that shares the same enclosed path but cuts through the wire in a different location. What is the current that passes through the surface?
    - Still I.
  - So the Ampere's law still works



- We could then consider a capacitor being charged up or being discharged.
  - The current I enclosed in the loop passes through the surface #1
  - However the surface #2 that shares the same closed loop do not have any current passing through it.
    - There is magnetic field present since there is current ➔ In other words there is a changing electric field in between the plates
    - Maxwell resolved this by adding an additional term to Ampere's law involving the changing electric field

Modifying Ampere's Law

- To determine what the extra term should be, we first have to figure out what the electric field between the two plates is
  - The charge Q on the capacitor with capacitance C is Q=CV
    - Where V is the potential difference between the plates
  - Since V=Ed
    - Where E is the uniform field between the plates, and d is the separation of the plates
  - And for parallel plate capacitor $C=\varepsilon_0 A/d$
  - We obtain

$$Q = CV = \left(\varepsilon_0 \frac{A}{d}\right) Ed = \varepsilon_0 AE$$

Modifying Ampere's Law

– If the charge on the plate changes with time, we can write

$$\frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

– Using the relationship between the current and charge we obtain

$$I = \frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d\left(AE\right)}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Where $\Phi_E = EA$ is the electric flux through the surface between the plates

– So in order to make Ampere's law work for the surface 2 in the figure, we must write it in the following form

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Extra term by Maxwell

– This equation represents the general form of Ampere's law

- This means that a magnetic field can be caused not only by an ordinary electric current but also by a changing electric flux

Example 31 – 1

Charging capacitor. A 30-pF air-gap capacitor has circular plates of area A=100cm^2. It is charged by a 70-V battery through a 2.0-Ω resistor. At the instance the battery is connected, the electric field between the plates is changing most rapidly. At this instance, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Since this is an RC circuit, the charge on the plates is: $Q = CV_0\left(1 - e^{-t/RC}\right)$

For the initial current (t=0), we differentiate the charge with respect to time.

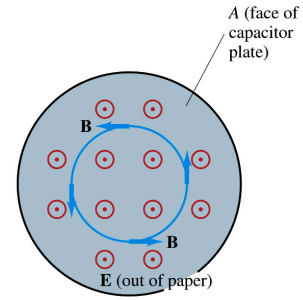
$$I_0 = \left.\frac{dQ}{dt}\right|_{t=0} = \left.\frac{CV_0}{RC} e^{-t/RC}\right|_{t=0} = \frac{V_0}{R} = \frac{70V}{2.0\Omega \cdot} = 35A$$

The electric field is $E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0}$

Change of the electric field is $\frac{dE}{dt} = \frac{dQ/dt}{A\varepsilon_0} = \frac{35A}{\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\cdot\left(1.0\times10^{-2}\,m^2\right)} = 4.0\times10^{14}\,V/m\cdot s$

Example 31 – 1

(c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.



The magnetic field lines generated by changing electric field is perpendicular to E and is circular due to symmetry

Whose law can we use to determine B?

Extended Ampere's Law w/ $I_{encl}=0$!

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

We choose a circular path of radius r, centered at the center of the plane, following the B.

For $r<r_{plate}$, the electric flux is $\Phi_E = EA = E\pi r^2$ since E is uniform throughout the plate

So from Ampere's law, we obtain $B\cdot(2\pi r) = \mu_0\varepsilon_0 \frac{d\left(E\pi r^2\right)}{dt} = \mu_0\varepsilon_0\pi r^2 \frac{dE}{dt}$

Solving for B $B = \mu_0\varepsilon_0 \frac{r}{2}\frac{dE}{dt}$ For $r<r_{plate}$

Since we assume E=0 for $r>r_{plate}$, the electric flux beyond the plate is fully contained inside the surface. $\Phi_E = EA = E\pi r_{plate}^2$

So from Ampere's law, we obtain $B\cdot(2\pi r) = \mu_0\varepsilon_0 \frac{d\left(E\pi r_{plate}^2\right)}{dt} = \mu_0\varepsilon_0\pi r_{plate}^2 \frac{dE}{dt}$

Solving for B $B = \frac{\mu_0\varepsilon_0 r_{plate}^2}{2r}\frac{dE}{dt}$ For $r>r_{plate}$

Displacement Current

- Maxwell interpreted the second term in the generalized Ampere's law equivalent to an electric current
  - He called this term as the displacement current, I_D
  - While the other term is called as the conduction current, I
- Ampere's law then can be written as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{encl} + I_D \right)$$

  - Where

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

  - While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow itself

Gauss' Law for Magnetism

- If there is a symmetry between electricity and magnetism, there must be an equivalent law in magnetism as the Gauss' Law in electricity
- For a magnetic field B, the magnetic flux Φ_B through the surface is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

  - Where the integration is over the area of either an open or a closed surface
- The magnetic flux through a closed surface which completely encloses a volume is

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

- What was the Gauss' law in the electric case?
  - The electric flux through a closed surface is equal to the total net charge Q enclosed by the surface divided by ε_0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

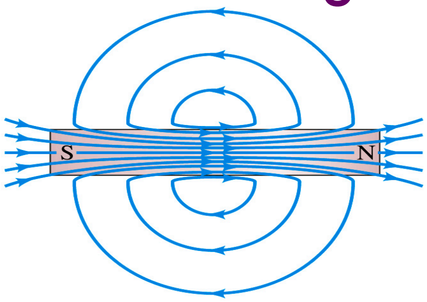
- Why is result of the integral zero?
  - There is no isolated magnetic poles, the magnetic equivalent of single electric charges

Gauss' Law for Magnetism

- What does the Gauss' law in magnetism mean physically?

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- – There are as many magnetic flux lines that enter the enclosed volume as leave it
- – If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
  - Electricity do have the source and the sink



- – Magnetic field lines must be continuous
- – Even for bar magnets, the field lines exist both insides and outside of the magnet

Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampére's Law

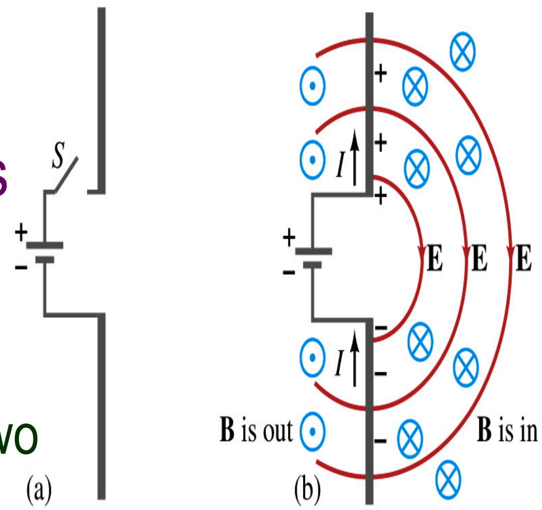
A magnetic field is produced by an electric current or by a changing electric field

Maxwell's Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
  - He then took this concept one step further and concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces the magnetic field that also changes.
    - This changing magnetic field then in turn produces the electric field that changes.
    - This process continues.
  - With the manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space

Production of EM Waves

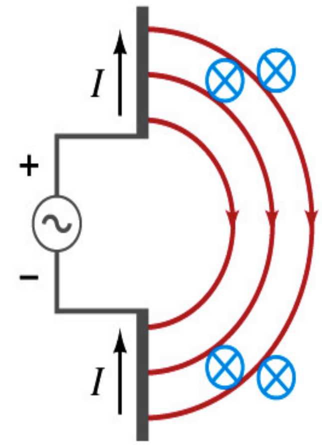
- Consider two conducting rods that will serve as an antenna are connected to a DC power source
  - What do you think will happen when the switch is closed?
    - The rod connected to the positive terminal is charged positive and the other negatively
    - Then the electric field will be generated between the two rods
    - Since there is current that flows through, the rods generates a magnetic field around them

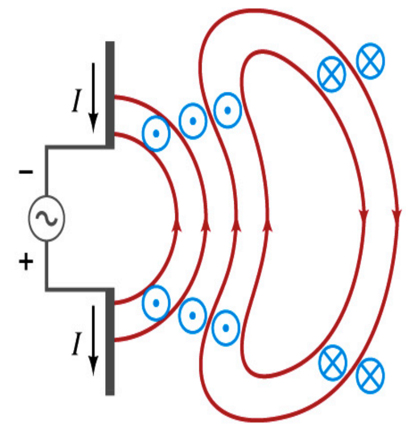


- How far would the electric and magnetic fields extend?
  - In static case, the field extends indefinitely
  - When the switch is closed, the fields are formed nearby the rods quickly but
  - The stored energy in the fields won't propagate w/ infinite speed

Production of EM Waves

- What happens if the antenna is connected to an ac power source?
  - When the connection was initially made, the rods are charging up quickly w/ the current flowing in one direction as shown in the figure
    - The field lines form as in the dc case
    - The field lines propagate away from the antenna
  - Then the direction of the voltage reverses
    - The new field lines with the opposite direction forms
    - While the original field lines still propagates away from the rod reaching out far
      - Since the original field propagates through an empty space, the field lines must form a closed loop (no charge exist)
    - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward is self supporting and do not need antenna with flowing charge
  - The fields far from the antenna is called the **radiation field**
  - Both electric and magnetic fields form closed loops perpendicular to each other





Properties of Radiation Fields

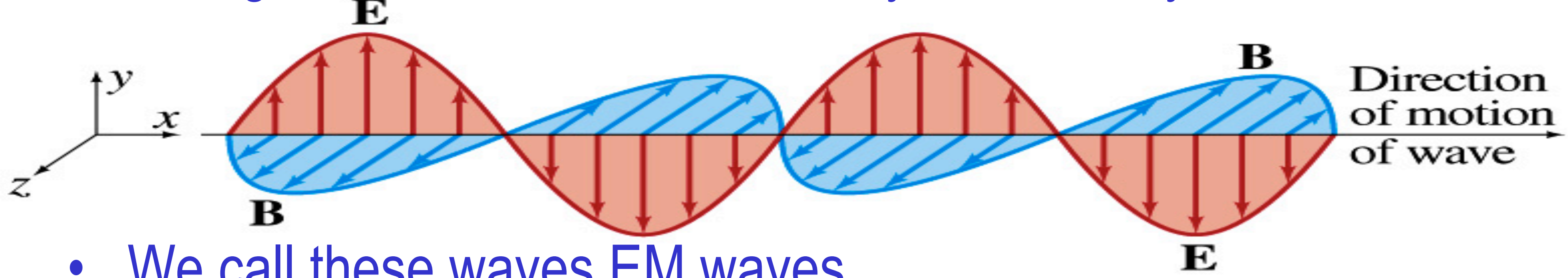
- The fields travel on the other side of the antenna as well
- The field strength are the greatest in the direction perpendicular to the oscillating charge while along the direction is 0
- The magnitude of E and B in the radiation field decrease with distance as 1/r
- The energy carried by the EM wave is proportional to the square of the amplitude, E^2 or B^2
  - So the intensity of wave decreases as $1/r^2$

Properties of Radiation Fields

- The electric and magnetic fields at any point are perpendicular to each other and to the direction of motion
- The fields alternate in direction
  - The field strengths vary from maximum in one direction, to 0 and to max in the opposite direction
- The electric and magnetic fields are in phase
- Very far from the antenna, the field lines are pretty flat over a reasonably large area
  - Called plane waves

EM Waves

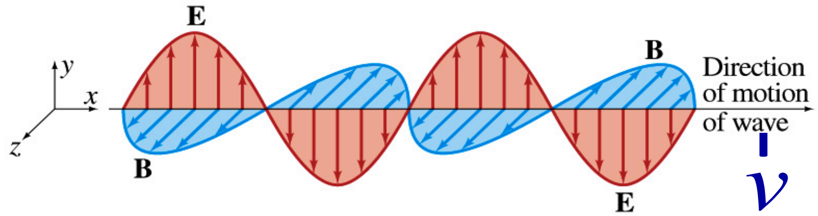
- If the voltage of the source varies sinusoidally, the field strengths of the radiation field vary sinusoidally



- We call these waves EM waves
- They are transverse waves
- EM waves are always waves of fields
  – Since these are fields, they can propagate through an empty space
- In general accelerating electric charges give rise to electromagnetic waves
- This prediction from Maxwell's equations was experimentally by Heinrich Hertz through the discovery of radio waves

EM Waves and Their Speeds

- Let's consider a region of free space. What's a free space?
  - An area of space where there is no charges or conduction currents
  - In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
  - What are these flat waves called?
    - Plane waves
    - At any instance **E** and **B** are uniform over a large plane perpendicular to the direction of propagation
  - So we can also assume that the wave is traveling in the x-direction w/ velocity, **v**=v**i**, and that **E** is parallel to y axis and **B** is parallel to z axis



$\vec{v}$

Maxwell's Equations w/ Q=I=0

- In this region of free space, Q=0 and I=0, thus the four Maxwell's equations become

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$ → **Q_{encl}=0** → $$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$ → **No Changes** → $$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ → **No Changes** → $$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ → **I_{encl}=0** → $$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!

EM Waves from Maxwell's Equations

- If the wave is sinusoidal w/ wavelength λ and frequency *f*, such traveling wave can be written as

$$E = E_y = E_0 \sin\left(kx - \omega t\right)$$
$$B = B_z = B_0 \sin\left(kx - \omega t\right)$$

  – Where

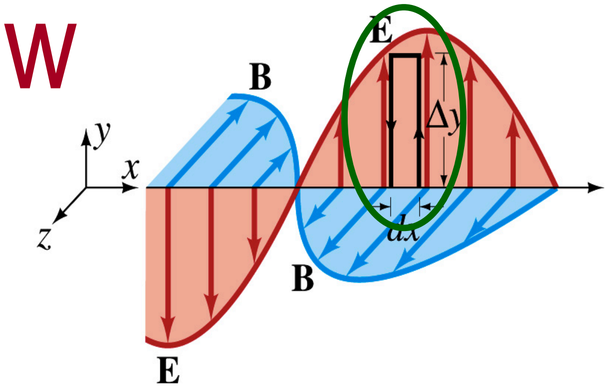
$$k = \frac{2\pi}{\lambda} \qquad \varpi = 2\pi f$$

Thus

$$f\lambda = \frac{\omega}{k} = v$$

  – What is v?
    - It is the speed of the traveling wave
  – What are E_0 and B_0?
    - The amplitudes of the EM wave. Maximum values of **E** and **B** field strengths.

From Faraday's Law



- Let's apply Faraday's law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

  – to the rectangular loop of height Δy and width dx

- $\vec{E}\cdot d\vec{l}$ along the top and bottom of the loop is 0. Why?
  – Since **E** is perpendicular to d*l*.
  – So the result of the integral through the loop counterclockwise becomes $\oint \vec{E}\cdot d\vec{l} = \vec{E}\cdot d\vec{x} + \left(\vec{E}+d\vec{E}\right)\cdot\Delta\vec{y} + \vec{E}\cdot d\vec{x} + \vec{E}\cdot\Delta\vec{y} =$

$$= 0 \;+ \left(E + dE\right)\Delta y \;-0 \;-E\Delta y = dE\Delta y$$

  – For the right-hand side of Faraday's law, the magnetic flux through the loop changes as

$$-\frac{d\Phi_B}{dt} = \frac{dB}{dt}dx\Delta y$$

Thus

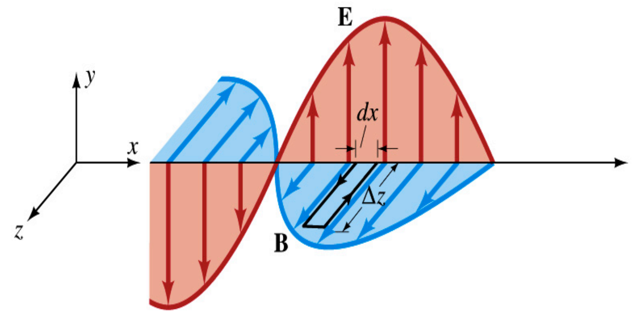
$$dE\Delta y = -\frac{dB}{dt}dx\Delta y$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Since E and B depend on x and t

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

From Modified Ampére's Law



- Let's apply Maxwell's 4th equation

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

  - to the rectangular loop of length Δz and width dx
- $\vec{B}\cdot d\vec{l}$ along the x-axis of the loop is 0
  - Since **B** is perpendicular to d*l*.
  - So the result of the integral through the loop counterclockwise becomes

$$\oint \vec{B}\cdot d\vec{l} = B\Delta z - \left(B + dB\right)\Delta z = -dB\Delta z$$

  - For the right-hand side of the equation is

$$\mu_0\varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0\varepsilon_0 \frac{dE}{dt} dx\Delta z$$

Thus

$$-dB\Delta z = \mu_0\varepsilon_0 \frac{dE}{dt} dx\Delta z$$

  - $$\frac{dB}{dx} = -\mu_0\varepsilon_0 \frac{dE}{dt}$$

Since E and B depend on x and t

$$\frac{\partial B}{\partial x} = -\mu_0\varepsilon_0 \frac{\partial E}{\partial t}$$

Relationship between **E**, **B** and **v**

- Let's now use the relationship from Faraday's law $\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial E}{\partial x}=\frac{\partial}{\partial x}\left(E_0\sin\left(kx-\omega t\right)\right)=kE_0\cos\left(kx-\omega t\right)$$

$$\frac{\partial B}{\partial t}=\frac{\partial}{\partial t}\left(B_0\sin\left(kx-\omega t\right)\right)=-\omega B_0\cos\left(kx-\omega t\right)$$

Since $\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$ **We obtain** $kE_0\cos\left(kx-\omega t\right)=\omega B_0\cos\left(kx-\omega t\right)$

Thus $\frac{E_0}{B_0}=\frac{\omega}{k}=\mathrm{v}$

– Since E and B are in phase, we can write $E/B=\mathrm{v}$
  - This is valid at any point and time in space. What is v?
    – The velocity of the wave

Speed of EM Waves

- Let's now use the relationship from Apmere's law $\frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial B}{\partial x} = \frac{\partial}{\partial x}\left(B_0 \sin\left(kx - \omega t\right)\right) = kB_0 \cos\left(kx - \omega t\right)$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t}\left(E_0 \sin\left(kx - \omega t\right)\right) = -\omega E_0 \cos\left(kx - \omega t\right)$$

Since $\frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$ **We obtain** $kB_0 \cos\left(kx - \omega t\right) = \varepsilon_0 \mu_0 \omega E_0 \cos\left(kx - \omega t\right)$

Thus $\frac{B_0}{E_0} = \frac{\varepsilon_0 \mu_0 \omega}{k} = \varepsilon_0 \mu_0 v$

- However, from the previous page we obtain $E_0 / B_0 = \mathrm{v} = \frac{1}{\varepsilon_0 \mu_0 \mathrm{v}}$
- Thus $\mathrm{v}^2 = \frac{1}{\varepsilon_0 \mu_0}$

$$\mathrm{v} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{\left(8.85\times10^{-12}\, C^2 / N \cdot m^2\right)\cdot\left(4\pi\times10^{-7}\, T \cdot m / A\right)}} = 3.00\times10^8\, m/s$$

The speed of EM waves is the same as the speed of light. EM waves behaves like the light.

Speed of Light w/o Sinusoidal Wave Forms

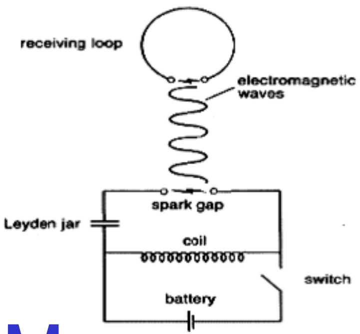
- Taking the time derivative on the relationship from Ampere's laws, we obtain $\frac{\partial^2 B}{\partial x \partial t} = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$
- By the same token, we take position derivative on the relationship from Faraday's law $\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$
- From these, we obtain

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2} \quad \textbf{and} \quad \frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2}$$

- Since the equation for traveling wave is $\frac{\partial^2 x}{\partial t^2} = \mathrm{v}^2 \frac{\partial^2 x}{\partial x^2}$
- By correspondence, we obtain $\mathrm{v}^2 = \frac{1}{\varepsilon_0 \mu_0}$
- A natural outcome of Maxwell's equations is that E and B obey the wave equation for waves traveling w/ speed $\mathrm{v} = 1/\sqrt{\varepsilon_0 \mu_0}$
  - Maxwell predicted the existence of EM waves based on this

Light as EM Wave

- People knew some 60 years before Maxwell that light behaves like a wave, but …
  - They did not know what kind of waves they are.
    - Most importantly what is it that oscillates in light?

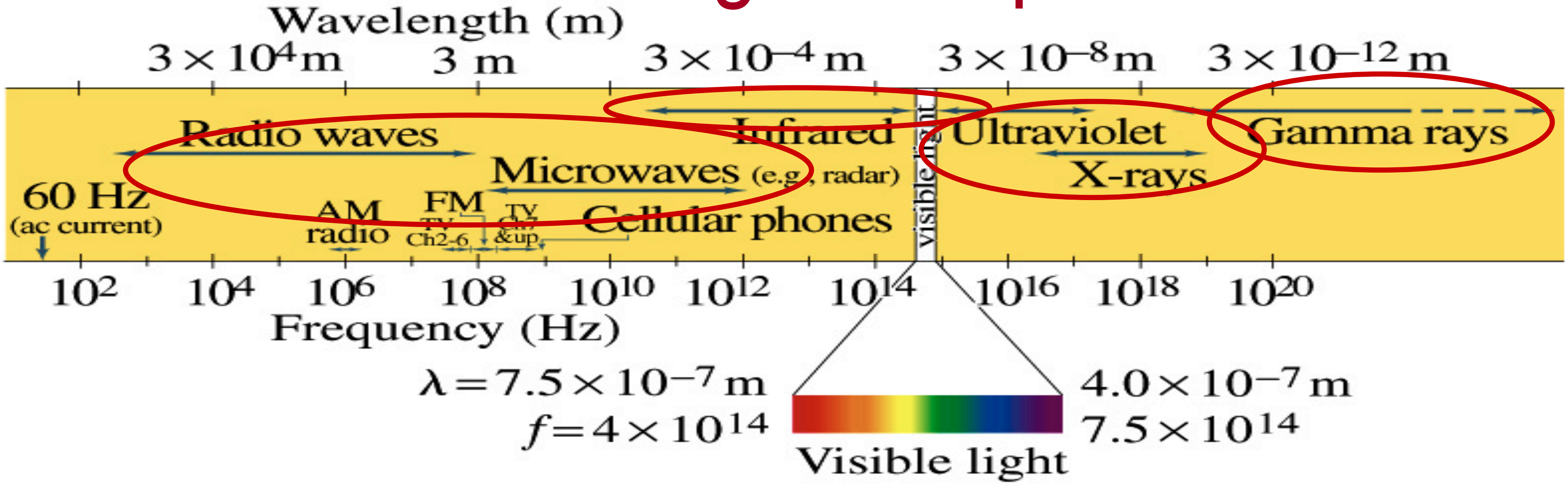


- Heinrich Hertz first generated and detected EM waves experimentally in 1887 using a spark gap apparatus
  - Charge was rushed back and forth in a short period of time, generating waves with frequency about 10^9Hz (these are called radio waves)
  - He detected using a loop of wire in which an emf was produced when a changing magnetic field passed through
  - These waves were later shown to travel at the speed of light and behave exactly like the light just not visible

Light as EM Wave

- The wavelengths of visible light were measured in the first decade of the 19th century
  - The visible light wave length were found to be between 4.0×10^{-7}m (400nm) and 7.5×10^{-7}m (750nm)
  - The frequency of visible light is $f\lambda=c$
    - Where f and λ are the frequency and the wavelength of the wave
      - What is the range of visible light frequency?
      - 4.0×10^{14}Hz to 7.5×10^{14}Hz
    - c is 3×10^{8}m/s, the speed of light
- EM Waves, or EM radiation, are categorized using EM spectrum

Electromagnetic Spectrum



- Low frequency waves, such as radio waves or microwaves can be easily produced using electronic devices
- Higher frequency waves are produced natural processes, such as emission from atoms, molecules or nuclei
- Or they can be produced from acceleration of charged particles
- Infrared radiation (IR) is mainly responsible for the heating effect of the Sun
  - The Sun emits visible lights, IR and UV
    - The molecules of our skin resonate at infrared frequencies so IR is preferentially absorbed and thus warm up

Example 31 – 3

Wavelength of EM waves. Calculate the wavelength (a) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave and (c) of a beam of visible red light from a laser at frequency 4.74×10^{14}Hz.

What is the relationship between speed of light, frequency and the wavelength? $c = f\lambda$

Thus, we obtain $\lambda = \dfrac{c}{f}$

For f=60Hz
$$\lambda = \frac{3\times10^{8}\ m/s}{60s^{-1}} = 5\times10^{6}\,m$$

For f=93.3MHz
$$\lambda = \frac{3\times10^{8}\ m/s}{93.3\times10^{6}\,s^{-1}} = 3.22m$$

For f=4.74×10^{14}Hz
$$\lambda = \frac{3\times10^{8}\ m/s}{4.74\times10^{14}\,s^{-1}} = 6.33\times10^{-7}\,m$$

EM Wave in the Transmission Lines

- Can EM waves travel through a wire?
  - Can it not just travel through the empty space?
  - Nope. It sure can travel through a wire.
- When a source of emf is connected to a transmission line, the electric field within the wire does not set up immediately at all points along the line
  - When two wires are separated via air, the EM wave travel through the air at the speed of light, c.
  - However, through medium w/ permittivity ε and permeability μ, the speed of the EM wave is given $v = 1/\sqrt{\varepsilon\mu} \quad < c$
    - Is this faster than c? **Nope! It is slower.**

Example 31 – 5

Phone call time lag. You make a telephone call from New York to London. Estimate the time the electrical signal to travel to reach London (a) carried on a 5000km telephone cable under the Atlantic Ocean and (b) sent via satellite 36,000km above the ocean. Would this cause a noticeable delay in either case?

Time delay via the cable: $t = \frac{d}{c} = \frac{5\times10^6}{3\times10^8} = 0.017s$

Delay via satellite $t = \frac{2d_s}{c} = \frac{2\times3.6\times10^7}{3.0\times10^8} = 0.24s$

So in case of satellite, the delay is likely noticeable!!

Energy in EM Waves

- Since B=E/c and $c = 1/\sqrt{\varepsilon_0 \mu_0}$, we can rewrite the energy density

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{\varepsilon_0 \mu_0 E^2}{\mu_0} = \varepsilon_0 E^2$$

$$u = \varepsilon_0 E^2$$

  - Note that the energy density associate with B field is the same as that associate with E
  - So each field contribute half to the total energy
- By rewriting in B field only, we obtain

$$u = \frac{1}{2}\varepsilon_0 \frac{B^2}{\varepsilon_0 \mu_0} + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{B^2}{\mu_0}$$

$$u = \frac{B^2}{\mu_0}$$

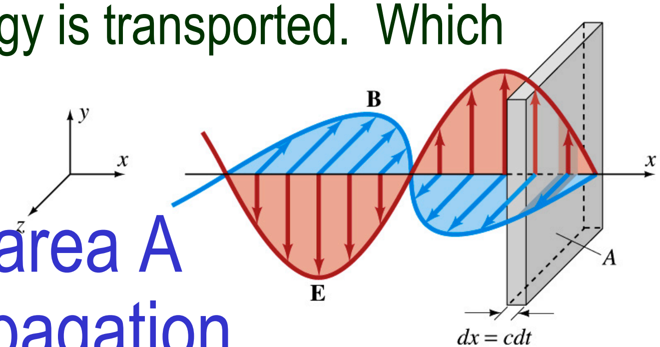
- We can also rewrite to contain both E and B

$$u = \varepsilon_0 E^2 = \varepsilon_0 EcB = \frac{\varepsilon_0 EB}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{\varepsilon_0}{\mu_0}} EB$$

$$u = \sqrt{\frac{\varepsilon_0}{\mu_0}} EB$$

Energy Transport

- What is the energy the wave transport per unit time per unit area?
  - This is given by the vector **S**, the Poynting vector
    - The unit of **S** is **W/m²**.
    - The direction of **S** is the direction in which the energy is transported. Which direction is this?
      - The direction the wave is moving



- Let's consider a wave passing through an area A perpendicular to the x-axis, the axis of propagation
  - How much does the wave move in time dt?
    - dx=cdt
  - The energy that passes through A in time dt is the energy that occupies the volume dV, $dV = Adx = Acdt$
  - Since the energy density is $u=\varepsilon_0 E^2$, the total energy, dU, contained in the volume V is $dU = udV = \varepsilon_0 E^2 Acdt$

Energy Transport

- Thus, the energy crossing the area A per time dt is

$$S = \frac{1}{A}\frac{dU}{dt} = \varepsilon_0 c E^2$$

- Since E=cB and $c = 1/\sqrt{\varepsilon_0 \mu_0}$, we can also rewrite

$$S = \varepsilon_0 c E^2 = \frac{cB^2}{\mu_0} = \frac{EB}{\mu_0}$$

- Since the direction of S is along **v**, perpendicular to **E** and **B**, the Poynting vector **S** can be written

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$$

What is the unit? W/m^2

  - This gives the energy transported per unit area per unit time at any instant

Average Energy Transport

- The average energy transport in an extended period of time is useful since sometimes we do not detect the rapid variation with respect to time because the frequency is so high.
- If E and B are sinusoidal, $\overline{E^2} = E_0^2 / 2$
- Thus we can write the magnitude of the average Poynting vector as

$$\bar{S} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0}B_0^2 = \frac{E_0 B_0}{2\mu_0}$$

  – This time averaged value of S is the intensity, defined as the average power transferred across unit area. E_0 and B_0 are maximum values.
- We can also write

$$\bar{S} = \frac{E_{rms} B_{rms}}{\mu_0}$$

  – Where E_{rms} and B_{rms} are the rms values ($E_{rms} = \sqrt{\overline{E^2}}$, $B_{rms} = \sqrt{\overline{B^2}}$)

Example 31 – 6

E and B from the Sun. Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about 1350W/m^2. Assume that this is a single EM wave and calculate the maximum values of E and B.

What is given in the problem? The average S!!

$$\bar{S} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0}B_0^2 = \frac{E_0 B_0}{2\mu_0}$$

For E_0, $$E_0 = \sqrt{\frac{2\bar{S}}{\varepsilon_0 c}} = \sqrt{\frac{2 \cdot 1350\, W/m^2}{\left(8.85\times10^{-12}\, C^2/N\cdot m^2\right)\cdot\left(3.00\times10^8\, m/s\right)}} = 1.01\times10^3\, V/m$$

For B_0 $$B_0 = \frac{E_0}{c} = \frac{1.01\times10^3\, V/m}{3\times10^8\, m/s} = 3.37\times10^{-6}\, T$$