PHYS 1441 – Section 002
Lecture #17

Wednesday, Nov. 7, 2018

Dr. Jaehoon Yu

- Chapter 27: Magnetism & Magnetic Field
  - Magnetic Force on a Moving Charge
  - About Magnetic Field
  - Charged Particle Path in a Magnetic Field
  - The cyclotron frequency
  - Magnetic dipole Moment
  - The Hall Effect

Announcements

- Reading Assignments: CH27.6, 27.8 and 27.9
- Bring out your special project #4!
- Reminder:2nd non-comprehensive term exam
  - In class Wednesday, Nov. 14
  - Covers CH25.5 to what we learn Monday, Nov. 12
  - Bring your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants
  - No derivations, word definitions or solutions of any kind!
  - No additional formulae or values of constants will be provided
- No class on Thanksgiving Wednesday, Nov. 21
- The final exam will be in class Wednesday, Dec. 5
- Colloquium at 4pm today in SH100
  - Dr. E. Gramellini of Fermilab

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Total Hadronic Cross Sections at the LArIAT Experiment

Elena Gramellini
Fermilab

Wednesday November 7, 2018
4:00 p.m. Room 100 Science Hall

Abstract

The Liquid Argon Time Projection Chamber (LArTPC) represents one of the most advanced experimental technologies for neutrino detection at neutrino beams due to its full 3D-imaging, excellent particle identification and precise calorimetric energy reconstruction. By deploying a LArTPC in a dedicated calibration test beam line at Fermilab, LArIAT (Liquid Argon In A Testbeam) aims to experimentally calibrate this technology in a controlled environment and to provide physics results key to the neutrino oscillation physics and proton decay searches of the Short Baseline Neutrino (SBND, MicroBooNE, ICARUS) and Long Baseline Neutrino programs (DUNE). LArIAT's physics program entails a vast set of topics with a particular focus on the study of nuclear effects such as pion and kaon characteristic interaction modes. In this talk, I will presents LArIAT's first two physics measurements: the negative pion and the positive kaon total hadronic cross section on argon. The core concept of both the analyses is a novel technique to measure cross sections — the "thin slice method" — only possible thanks to the combination of the tracking and calorimetry capability of the LArTPC technology. The negative pion and the positive kaon total hadronic cross section measurements will be the basis for LArIAT's future measurements in the pion and kaon exclusive channels. The outcome of these measurements will ultimately enable to quantify and reduce the systematic associated with the hadronic interaction models in neutrino-argon interactions and proton decay searches.

Refreshments will be served at 3:30 in the Physics lounge.

Reminder: Special Project #5

- Make a list of the power consumption and the resistance of all electric and electronic devices at your home and compile them in a table. (10 points total for the first 10 items and 0.5 points each additional item.)
- Estimate the cost of electricity for each of the items on the table using your own electric cost per kWh (if you don’t find your own, use \$0.12/kWh) and put them in the relevant column. (5 points total for the first 10 items and 0.2 points each additional items)
- Estimate the the total amount of energy in Joules and the total electricity cost per day, per month and per year for your home. (8 points)
- Due: Beginning of the class Wednesday, Nov. 28

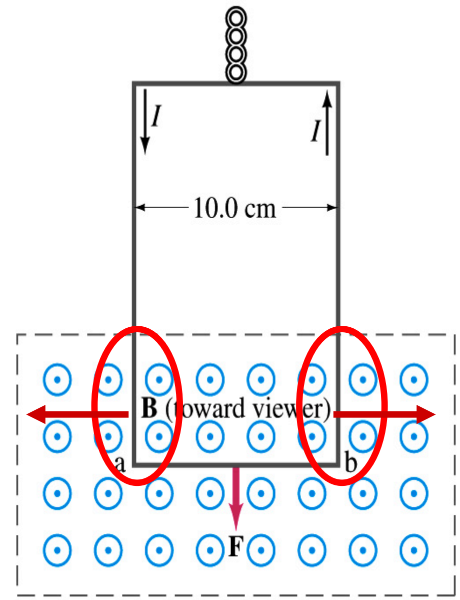
Item Name	Rated power (W)	Number of devices	Number of Hours per day	Daily Power Consumption (kWh)	Energy Cost per kWh (cents)	Daily Energy Consumption (J).	Daily Energy Cost ($)	Monthly Energy Consumption (J)	Monthly Energy Cost (S)	Yearly Energy Consumption (J)	Yearly Energy Cost ($)
Light Bulbs	30	4									
	40	6									
	60	15									
Heaters	1000	2									
	1500	1									
	2000	1									
Home Appliances (Fans, vacuum cleaners, hair dryers, pool pumps, etc)											
Air Conditioners											
Kitchen Appliances (Fridges, freeezers, cook tops, microwave ovens,											
Computing devices (desktop, laptop, ipad, mobile phones, printers, chargers, etc))											
Tools (power tools, electric mower, electric cutter, etc)											
Medical Devices (blood pressure machine, thermometer, etc)											
Total				0		0	0	0	0	0	0

Recap: Magnetic Force on Electric Current

- Magnetic field strength B can be defined using the previous proportionality relationship w/ the constant 1: $F = IlB\sin\theta$
- if θ=90°, $F_{max} = IlB$ and if θ=0° $F_{min} = 0$
- So the magnitude of the magnetic field B can be defined as
  - $B = F_{max}/Il$ where F_{max} is the magnitude of the force on a straight length l of the wire carrying the current I when the wire is perpendicular to **B**
- The relationship between F, B and I can be written in a vector formula: $\vec{F} = I\vec{l} \times \vec{B}$
  - l is the vector whose magnitude is the length of the wire and its direction is along the wire in the direction of the conventional current
  - This formula works if **B** is uniform.
- If B is not uniform or l does not form the same angle with B everywhere, the infinitesimal force acting on a differential length dl is $d\vec{F} = Id\vec{l} \times \vec{B}$

Example 27 – 2

Measuring a magnetic field. A rectangular loop of wire hangs vertically as shown in the figure. A magnetic field **B** is directed horizontally perpendicular to the wire, and points out of the page. The magnetic field **B** is very nearly uniform along the horizontal portion of wire *ab* (length ℓ=10.0cm) which is near the center of a large magnet producing the field. The top portion of the wire loop is free of the field. The loop hangs from a balance which measures a downward force (in addition to the gravitational force) of F=3.48x10^{-2}N when the wire carries a current I=0.245A. What is the magnitude of the magnetic field B at the center of the magnet?



Magnetic force exerted on the wire due to the uniform field is $\vec{F} = I\vec{l} \times \vec{B}$

Since $\vec{B} \perp \vec{l}$ Magnitude of the force is $F = IlB$

Solving for B

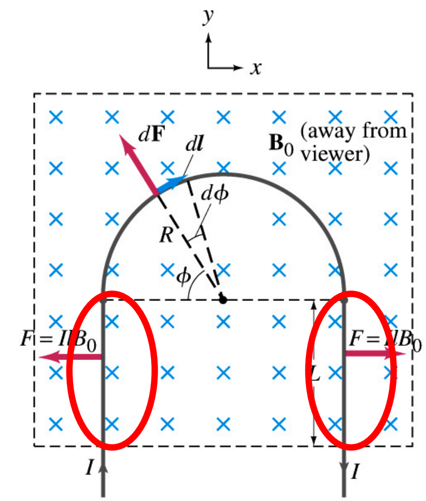
$$B = \frac{F}{Il} = \frac{3.48 \times 10^{-2} N}{0.245A \cdot 0.10m} = 1.42T$$

Something is not right! What happened to the forces on the loop on the side?

The two forces cancel out since they are in opposite direction with the same magnitude.

Example 27 – 3

Magnetic force on a semi-circular wire. A rigid wire, carrying the current I, consists of a semicircle of radius R and two straight portions as shown in the figure. The wire lies in a plane perpendicular to the uniform magnetic field $\mathbf{B_0}$. The straight portions each have length ℓ within the field. Determine the net force on the wire due to the magnetic field $\mathbf{B_0}$.



As in the previous example, the forces on the straight sections of the wire is equal and in opposite direction. Thus they cancel.

What do we use to figure out the net force on the semicircle? $d\vec{F} = Id\vec{l} \times \vec{B}$

We divide the semicircle into infinitesimal straight sections. $dl = Rd\phi$

What is the net x component of the force exerting on the circular section? 0 **Why?**

Because the forces on left and the right-hand sides of the semicircle balance.

Since $\vec{B}_0 \perp d\vec{l}$ Y-component of the force dF is $dF_y = d\left(F\sin\phi\right) = IRB_0 d\phi$

Integrating over φ=0 - π

$$F = \int_0^{\pi} d\left(F\sin\phi\right) = IB_0R\int_0^{\pi}\sin\phi d\phi = -IB_0R\left[\cos\phi\right]_0^{\pi} = 2RIB_0$$

Which direction?

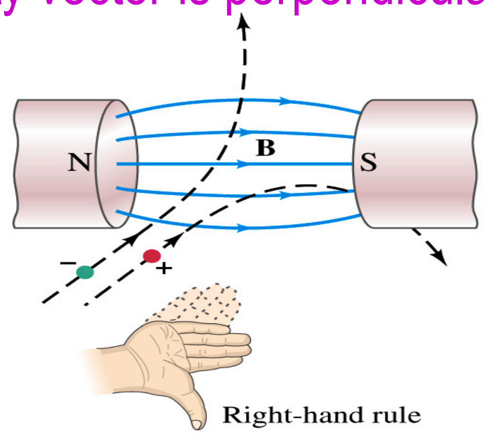
Vertically upward direction. The wire will be pulled deeper into the field.

Magnetic Forces on a Moving Charge

- Will moving charge in a magnetic field experience force?
  - Yes
  - Why?
  - Since the wire carrying a current (moving charge) experiences force in a magnetic field, a free moving charge must feel the same kind of force…☺
- OK, then how much force would it experience?
  - Let's consider N moving particles with charge q each, and they pass by a given point in a time interval t.
    - What is the current? $I = Nq/t$
  - Let t be the time for the charge q to travel a distance L in a magnetic field **B**
    - Then, the length vector *l* becomes $\vec{l} = \vec{v}t$
    - Where **v** is the velocity of the particle
- Thus the force on N particles by the field is $\vec{F} = I\vec{l} \times \vec{B} = Nq\vec{v} \times \vec{B}$
- The force on one particle with charge q, $\vec{F} = q\vec{v} \times \vec{B}$

Magnetic Forces on a Moving Charge

- This can be an alternative way of defining the magnetic field.
  - How?
  - The magnitude of the force on a particle with charge q moving with a velocity **v** in a field **B** is
    - $F = qvB\sin\theta$
    - What is θ?
      - The angle between the magnetic field and the direction of particle's movement
    - When is the force maximum?
      - When the angle between the field and the velocity vector is perpendicular.
    - $F_{\max} = qvB$ ➔ $B = \dfrac{F_{\max}}{qv}$
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field



Example 27 – 5

Magnetic force on a proton. A proton having a speed of 5×10^6m/s in a magnetic field feels a force of $F=8.0 \times 10^{-14}$N toward West when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. What is the magnitude and the direction of the magnetic field in this region?

What is the charge of a proton? $q_p = +e = 1.6 \times 10^{-19} C$

What does the fact that the proton does not feel any force in a northerly direction tell you about the magnetic field?

The field is along the north-south direction. Why?

Because the particle does not feel any magnetic force when it is moving along the direction of the field.

Since the particle feels force toward West, the field should be pointing to …. North

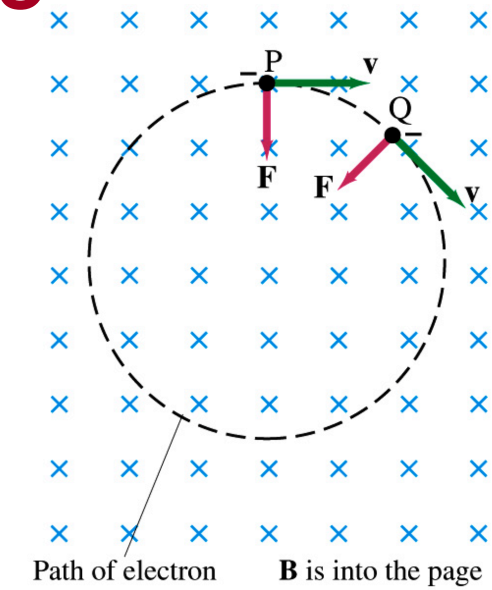
Using the formula for the magnitude of the field B, we obtain

$$B = \frac{F}{qv} = \frac{8.0 \times 10^{-14} N}{1.6 \times 10^{-19} C \cdot 5.0 \times 10^6 m/s} = 0.10T$$

We can use magnetic field to measure the momentum of a particle. How?

Charged Particle's Path in Magnetic Field

- What shape do you think is the path of a charged particle on a plane perpendicular to a uniform magnetic field?
  - Circle!! Why?
  - An electron moving to right at the point P in the figure will be pulled downward



  - At a later time, the force is still perpendicular to the velocity
  - Since the force is always perpendicular to the velocity, the magnitude of the velocity is constant
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
  - Thus, the electron moves on a circular path with a centripetal force F.

Example 27 – 7

Electron's path in a uniform magnetic field. An electron travels at the speed of 2.0×10^{7}m/s in a plane perpendicular to a 0.010-T magnetic field. What is the radius of the electron's path?

What is formula for the centripetal force? $F = ma = m\dfrac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is

$$F = evB$$

Since the magnetic force provides the centripetal force, we can establish an equation with the two forces

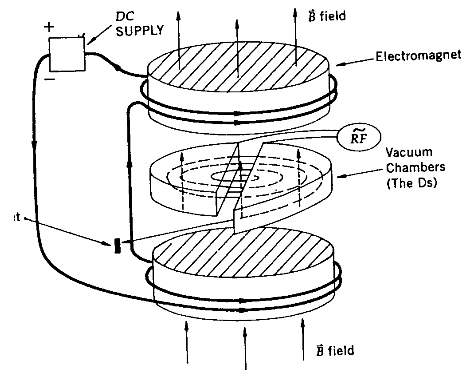
$$F = evB = m\frac{v^2}{r}$$

Solving for r

$$r = \frac{mv}{eB} = \frac{\left(9.1\times10^{-31}\,kg\right)\cdot\left(2.0\times10^{7}\,m/s\right)}{\left(1.6\times10^{-19}\,C\right)\cdot\left(0.010T\right)} = 1.1\times10^{-2}\,m$$

Cyclotron Frequency

- The time required for a particle of charge q moving w/ constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



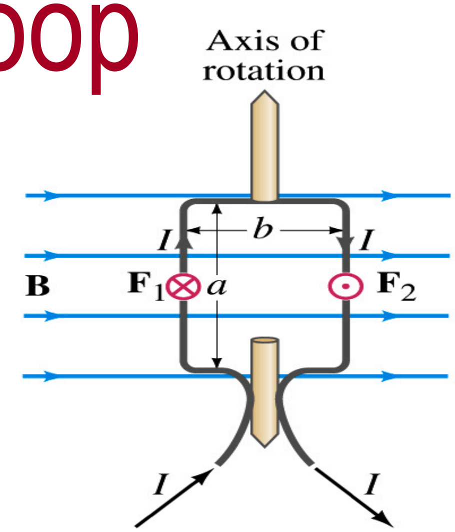
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

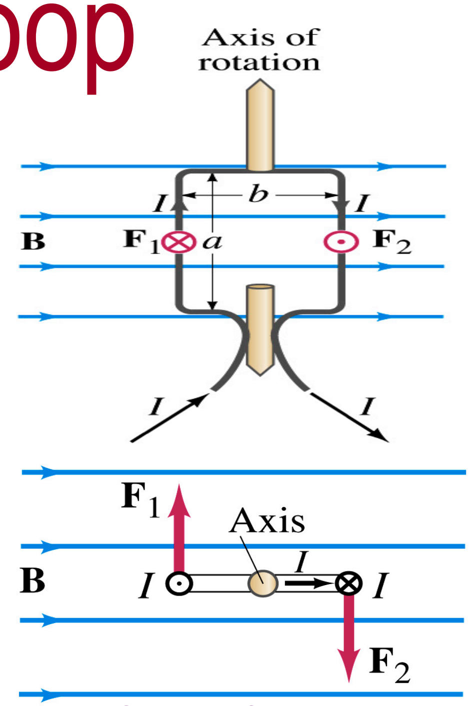
- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.

Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts a force on both vertical sections of wire.
  - Where is this principle used in?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque to the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
  - It will not turn unless the direction of the current changes

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length *a*?
    - $F_a = IaB$
    - The moment arm of the coil is $b/2$
  - So the total torque is the sum of the torques by each of the forces

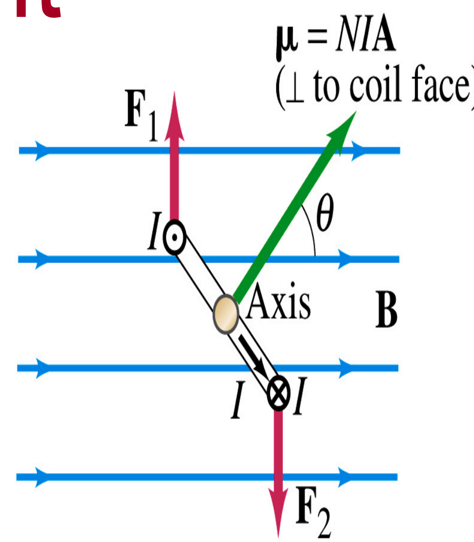
$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

    - Where $A=ab$ is the area of the coil loop
  - What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

  - If the coil makes an angle θ w/ the field $\tau = NIAB\sin\theta$

Magnetic Dipole Moment



- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity $NI\mathcal{A}$ is called the **magnetic dipole moment of the coil**
  - It is considered a vector $\vec{\mu} = NI\overrightarrow{A}$
    - Its direction is the same as that of the area vector **A** and is perpendicular to the plane of the coil consistent with the right-hand rule
      - Your thumb points to the direction of the magnetic moment when your finer cups around the loop in the direction of the current
  - Using the definition of magnetic moment, the torque can be written in vector form $\vec{\tau} = NI\overrightarrow{A} \times \vec{B} = \vec{\mu} \times \vec{B}$

Magnetic Dipole Potential Energy

- Where else did you see the same form of the torque?
  - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
  - The potential energy of the electric dipole is
  - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
  - The work done by the torque is
  - $U = \int \tau d\theta = \int NIAB \sin\theta \, d\theta = -\mu B \cos\theta + C$
  - If we chose U=0 at θ=π/2, then C=0
  - Thus the potential energy is $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$
    - Very similar to the electric dipole

Example 27 – 12

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius 0.529×10^{-10}m.

What provides the centripetal force? The Coulomb force

So we can obtain the speed of the electron from $F = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}$

$25 \cdot \ln(0.1)$

Solving for v $v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}} = \sqrt{\frac{\left(8.99\times10^9\, N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(9.1\times10^{-31}\,kg\right)\cdot\left(0.529\times10^{-10}\,m\right)}} = 2.19\times10^6\, m/s$

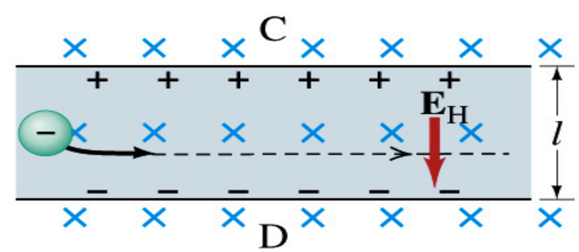
Since the electric current is the charge that passes through the given point per unit time, we can obtain the current $I = \frac{e}{T} = \frac{ev}{2\pi r}$

Since the area of the orbit is $A=\pi r^2$, we obtain the hydrogen magnetic moment

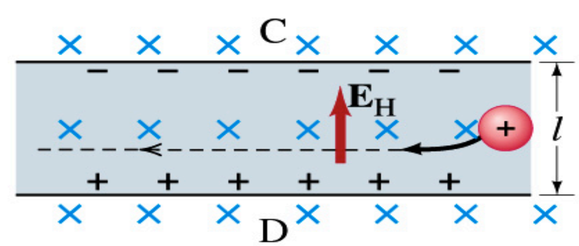
$$\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2} = \frac{er}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}} = \frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m_e}}$$

The Hall Effect

- What do you think will happen to the electrons flowing through a conductor immersed in a magnetic field?
  - Magnetic force will push the electrons toward one side of the conductor. Then what happens?
    - $\vec{F}_B = -e\vec{v}_d \times \vec{B}$
  - A potential difference will be created due to continued accumulation of electrons on one side. Till when? Forever?
  - Nope. Till the electric force inside the conductor is equal and opposite to the magnetic force



(a)

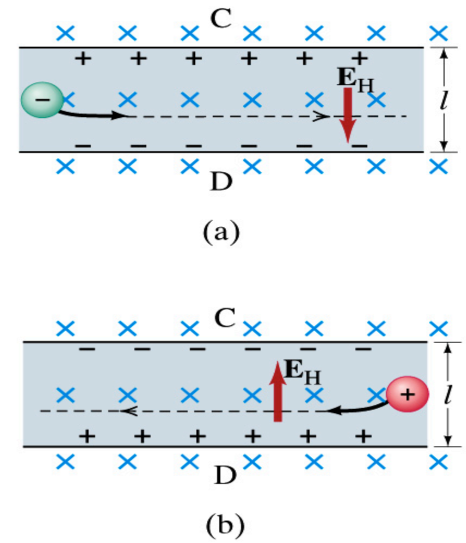


(b)

- This is called the **Hall Effect**
  - The potential difference produced is called
    - The Hall emf
  - The electric field due to the separation of charge is called the Hall field, $\mathbf{E}_H$, and it points to the direction opposite to the magnetic force

The Hall Effect

- In the equilibrium, the force due to Hall field is balanced by the magnetic force $ev_d B$, so we obtain



- $eE_H = ev_d B$ and $E_H = v_d B$
- The Hall emf is then $\varepsilon_H = E_H l = v_d Bl$
  – Where l is the width of the conductor
- What do we use the Hall effect for?
  – The current of negative charge moving to right is equivalent to the positive charge moving to the left
  – The Hall effect can distinguish these since the direction of the Hall field or direction of the Hall emf is opposite
  – Since the magnitude of the Hall emf is proportional to the magnetic field strength ➔ can measure the B-field strength
    - Hall probe