PHYS 1441 – Section 002
Lecture #21

Wednesday, Nov. 28, 2018

Dr. Jaehoon Yu

- Chapter 29:EM Induction & Faraday's Law
  - Electric Field Due to Changing Magnetic Flux
- Chapter 30: Inductance
  - Inductance
  - Mutual and Self Inductance
  - Energy Stored in the Magnetic Field
  - LR Circuit
  - LC circuit and EM Oscillation

Announcements – II

- Final comprehensive exam
  - In class 1:00 – 2:20pm, Wed. Dec. 5
  - Covers CH21.1 through what we finish next Monday, Dec. 3
  - Bring your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants
  - No derivations, word definitions or solutions of any kind!
  - No additional formulae or values of constants will be provided!
- Planetarium extra credit
  - Due next Wednesday, Dec. 5
  - Tape one end of the ticket stub on a sheet of paper with your name on it
- Colloquium today
  - 4pm in SH100
  - Dr. C. Li, MD Anderson

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Multi-Functional Nanoparticles for Image-Guided Photothermal Therapy

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Professor Chun Li earned his doctorate in chemistry at Rutgers-The State University of New Jersey and his undergraduate degree from Peking University, Beijing, China. He spent 3 years as an Instructor at Sichuan University before studying oversea. Dr. Li has more than 150 papers published in peer-reviewed journals (H-index 60 based on Google Scholar), 28 patents (4 of which have been licensed), 1 edited book, and 14 book chapters. Dr. Li has served as an adjunct professor in Texas A&M University (College Station, Texas, USA), Rice University (Houston, Texas, USA), University of Houston, and Peking University Cancer Hospital (Beijing, China).

Wednesday November 28, 2018
4:00 p.m. Room 100 Science Hall

Abstract

The development of biocompatible nanoparticles for molecular imaging and targeted cancer therapy is an area of intense research across a number of disciplines. The premise is that nanoparticles possess unique structural and functional properties that are not available from either small-molecular-weight molecules or bulk materials. However, successful delivery of nanomaterials to the tumor sites requires overcoming many biological barriers, including evasion of the mononuclear phagocytic system, extravazation from tumor vasculature and dispersion of nanoparticles from perivascular area. In my presentation, I will discuss different nanoparticle platforms developed in the lab aimed at enhancing therapeutic efficacy with reduced systemic toxicity. The obstacles and promises of multifunctional nanoparticles in image-guided, multimodality anticancer therapy will also be discussed.

Refreshments will be served at 3:30 in physics lounge

Electric Field due to Magnetic Flux Change

- When the electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
  - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This result applies not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- Recall the relationship between the electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field
- $\varepsilon = \int_a^b \vec{E} \cdot d\vec{l}$
- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

- The integral is taken around a path enclosing the area through which the magnetic flux Φ_B is changing.

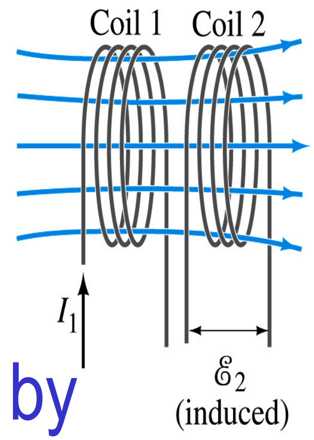
Inductance

- A changing magnetic flux through a circuit induces an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit → Mutual inductance
  - Or induce an emf in itself → Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2, in coil 2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil 2 created by coil1 and N_2 is the number of closely packed loops in coil 2, then $N_2\Phi_{21}$ is the total flux passing through coil 2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21} I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined as $M_{21} = N_2\Phi_{21}/I_1$.
- The emf induced in coil 2 due to the changing current in coil 1 is

$$\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d\left(N_2\Phi_{21}\right)}{dt} = -M_{21}\frac{dI_1}{dt}$$



Mutual Inductance

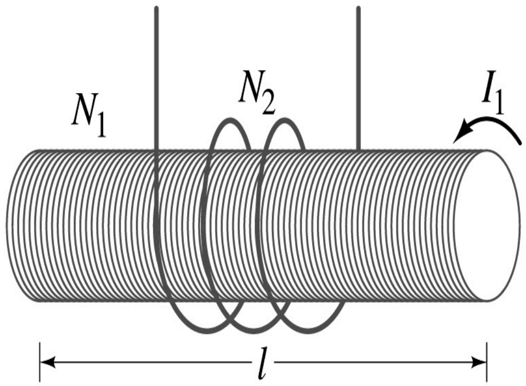
- The mutual induction of coil 2 with respect to coil 1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

    What? Does this make sense?

    - The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
  - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil 2 will induce an emf in coil 1
- $$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain $$\varepsilon_1 = -M\frac{dI_2}{dt} \text{ and } \varepsilon_2 = -M\frac{dI_1}{dt}$$
  - SI unit for mutual inductance is Henry (H) $$1H = 1V\cdot s/A = 1\Omega\cdot s$$

Example 30 – 1

Solenoid and coil. A long thin solenoid of length ℓ and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $B = \dfrac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil 2 is

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this?
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impedes its increase, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = L I$
- We define self-inductance, $\mathcal{L}$: $$L = \frac{N\Phi_B}{I}$$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V\cdot s/A = 1\Omega\cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

- It is an impediment onto the electrical current due to the existence of changing magnetic flux
- So what?
- In other words, it behaves like a resistance to the varying current, such as AC, that causes the constant change of magnetic flux
- But it also provides means to store energy, just like the capacitance

Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  - Precision resisters are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a “non-inductive winding”
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

Example 30 – 3

Solenoid inductance. (a) Determine the formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing N turns of wire in its length ℓ and whose cross-sectional area is A. (b) Calculate the value of $\mathcal{L}$ if N=100, ℓ=5.0cm, A=0.30cm^2 and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with $\mu=4000\mu_0$.

What is the magnetic field inside a solenoid? $B = \mu_0 nI = \mu_0 NI/l$

The flux is, therefore, $\Phi_B = BA = \mu_0 NIA/l$

Using the formula for self inductance: $L = \dfrac{N\Phi_B}{I} = \dfrac{N \cdot \mu_0 NIA/l}{I} = \dfrac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\left(4\pi \times 10^{-7}\, T \cdot m/A\right) 100^2 \left(0.30 \times 10^{-4} m^2\right)}{5.0 \times 10^{-2} m} = 7.5 \mu H$$

(c) The magnetic field with an iron core solenoid is $B = \mu NI/l$

$$L = \frac{\mu N^2 A}{l} = \frac{4000\left(4\pi \times 10^{-7}\, T \cdot m/A\right) 100^2 \left(0.30 \times 10^{-4} m^2\right)}{5.0 \times 10^{-2} m} = 0.030 H = 30 mH$$

Energy Stored in the Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $$U = \frac{1}{2} L I^2$$

    Energy Stored in a magnetic field inside an inductor

  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V: $U = \frac{1}{2} C V^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without a fringe effect

$$L = \mu_0 N^2 A / l$$

  - The magnetic field in a solenoid is $B = \mu_0 NI / l$
  - Thus the energy stored in an inductor is

$$U = \frac{1}{2} LI^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \left(\frac{Bl}{\mu_0 N} \right)^2 = \frac{1}{2} \frac{B^2}{\mu_0} Al$$

What is this?

$$U = \frac{1}{2} \frac{B^2}{\mu_0} Al$$

E

  - Thus the energy density is

$$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2} \frac{B^2}{\mu_0}$$

Volume V

$$u = \frac{1}{2} \frac{B^2}{\mu_0}$$

E density

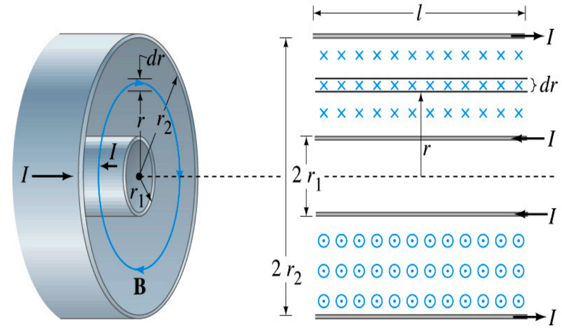
  - This formula is valid in any region of space
  - If a ferromagnetic material is present, μ_0 becomes μ.

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I? (b) Where is the energy density highest?



(a) (b)

(a) The total flux through l of the cable is $\Phi_B = \int Bl\,dr = \frac{\mu_0 Il}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\mu_0 Il}{2\pi}\ln\frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}$

Thus the energy stored per unit length is

$$\frac{U}{l} = \frac{1}{2}\frac{LI^2}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{r_2}{r_1}$$

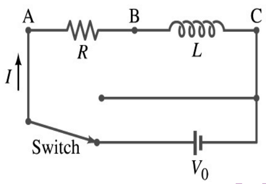
(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2}\frac{B^2}{\mu_0}$

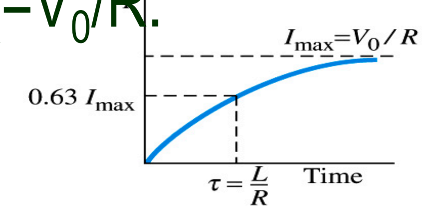
The energy density is highest where B is highest. Since B is highest close to $r=r_1$, near the surface of the inner conductor.

LR Circuits

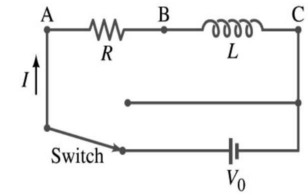
- What happens when an emf is applied to an inductor?
  - An inductor has some resistance, however negligible



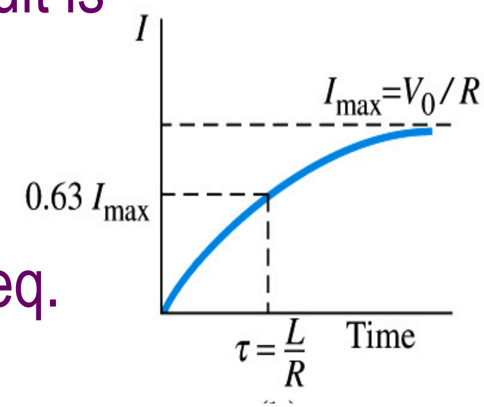
    - So an inductor can be drawn as a circuit of separate resistance and coil. What is the name this kind of circuit? **LR Circuit**
  - What happens at the instance the switch is thrown to apply emf to the circuit?
    - The current starts to flow, gradually increasing from 0
    - This change is opposed by the induced emf in the inductor ➔ the emf at point B is higher than point C
    - However there is a voltage drop at the resistance which reduces the voltage across inductance
    - Thus the current increases less rapidly
    - The overall behavior of the current is a gradual increase, reaching to the maximum current $I_{max}=V_0/R$.



LR Circuits



- This can be shown w/ Kirchhoff loop rules
  - The emfs in the circuit are the battery voltage V_0 and the emf $\varepsilon=-\mathcal{L}(dI/dt)$ in the inductor opposing the current increase
  - The sum of the potential changes through the circuit is



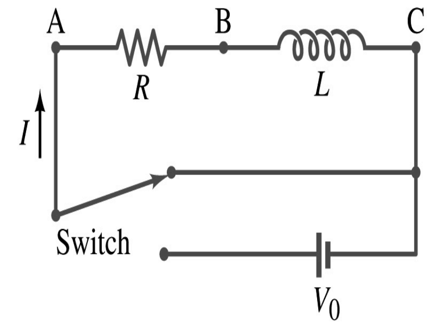
$$V_0+\varepsilon-IR=V_0-L\,dI/dt-IR=0$$

  - Where I is the current at any instance
  - By rearranging the terms, we obtain a differential eq.
  - $L\,dI/dt+IR=V_0$
  - We can integrate just as in RC circuit $\int_{I=0}^{I}\frac{dI}{V_0-IR}=\int_{t=0}^{t}\frac{dt}{L}$
  - So the solution is $-\frac{1}{R}\ln\left(\frac{V_0-IR}{V_0}\right)=\frac{t}{L}$

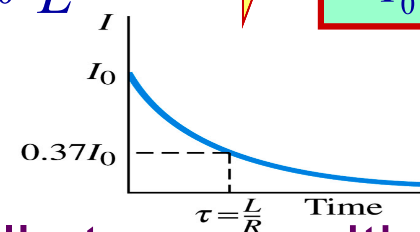
$$I=V_0\left(1-e^{-t/\tau}\right)/R=I_{max}\left(1-e^{-t/\tau}\right)$$

  - Where τ=L/R
    - This is the time constant τ of the LR circuit and is the time required for the current I to reach 0.63 of the maximum

Discharge of LR Circuits

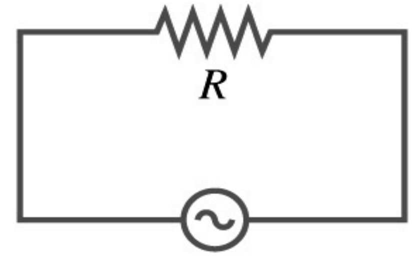


- If the switch is flipped away from the battery
  - The differential equation becomes
  - $$L\,dI/dt + IR = 0$$
  - So the integration is $\int_{I_0}^{I} \frac{dI}{IR} = \int_{t=0}^{t} \frac{dt}{L}$ ⇒ $\ln \frac{I}{I_0} = -\frac{R}{L}t$
  - Which results in the solution
  - $$I = I_0 e^{-\frac{R}{L}t} = I_0 e^{-t/\tau}$$



  - The current decays exponentially to zero with the time constant τ=L/R
  - So there always is a reaction time when a system with a coil, such as an electromagnet, is turned on or off.
  - The current in LR circuit behaves almost the same as that in RC circuit but the time constant is inversely proportional to R in LR circuit unlike the RC circuit

AC Circuit w/ Resistance only



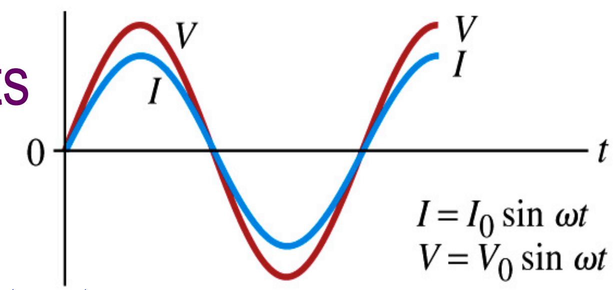
- What do you think will happen when an AC source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain

$$V - IR = 0$$

- Thus

$$V = I_0 R \sin \varpi t = V_0 \sin \varpi t$$

  – where $V_0 = I_0 R$
- What does this mean?
  – Current is 0 when voltage is 0 and current is in its peak when voltage is in its peak.
  – Current and voltage are "in phase"

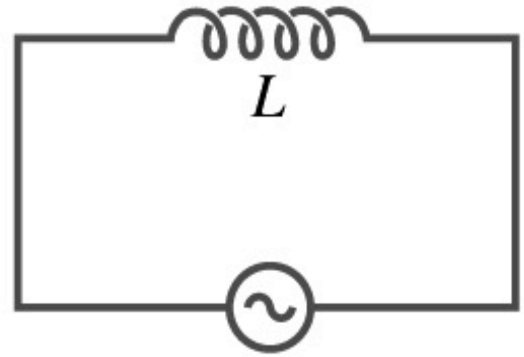


- Energy is lost via the transformation into heat at an average rate $\bar{P} = \bar{I}\,\bar{V} = I_{rms}^2 R = V_{rms}^2 / R$

AC Circuit w/ Inductance only

- From Kirchhoff's loop rule, we obtain

$$V - L\frac{dI}{dt} = 0$$



- Thus

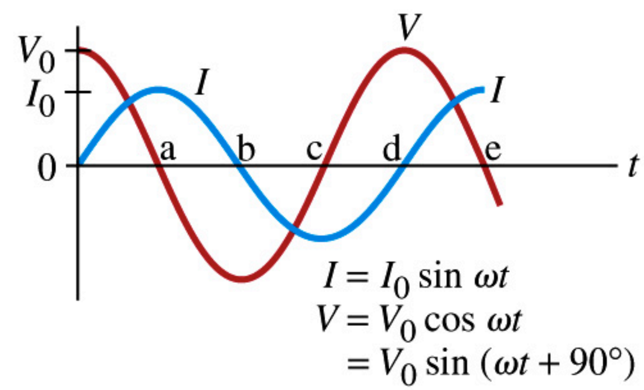
$$V = L\frac{dI}{dt} = L\frac{d\left(I_0 \sin \varpi t\right)}{dt} = \varpi L I_0 \cos \varpi t$$

  – Using the identity $\cos\theta = \sin\left(\theta + 90^\circ\right)$

- $$V = \varpi L I_0 \sin\left(\varpi t + 90^\circ\right) = V_0 \sin\left(\varpi t + 90^\circ\right)$$

  – where $V_0 = \varpi L I_0$

- What does this mean?
  – Current and voltage are "out of phase by π/2 or 90°". In other words the current reaches its peak ¼ cycle after the voltage
- What happens to the energy?
  – No energy is dissipated
  – The average power is 0 at all times
  – The energy is stored temporarily in the magnetic field
  – Then released back to the source



AC Circuit w/ Inductance only

- How are the resistor and inductor different in terms of energy?
  - Inductor — Stores the energy temporarily in the magnetic field and then releases it back to the emf source
  - Resistor — Does not store energy but transforms it to thermal energy, losing it to the environment
- How are they the same?
  - They both impede the flow of charge
  - For a resistance R, the peak voltage and current are related to $V_0 = I_0 R$
  - Similarly, for an inductor we may write $V_0 = I_0 X_L$
    - Where X_L is the inductive reactance of the inductor $X_L = \varpi L$ **0 when ω=0.**
    - What do you think is the unit of the reactance? Ω
    - The relationship $V_0 = I_0 X_L$ is not valid at a particular instance. Why not?
      - Since V_0 and I_0 do not occur at the same time

$V_{rms} = I_{rms} X_L$ is valid!

Example 30 – 9

Reactance of a coil. A coil has a resistance R=1.00Ω and an inductance of 0.300H. Determine the current in the coil if (a) 120 V DC is applied to it; (b) 120 V AC (rms) at 60.0Hz is applied.

Is there a reactance for DC? Nope. Why not? Since ω=0, $X_L = \varpi L = 0$

So for DC power, the current is from Kirchhoff's rule $V - IR = 0$

$$I_0 = \frac{V_0}{R} = \frac{120V}{1.00\Omega} = 120A$$

For an AC power with f=60Hz, the reactance is

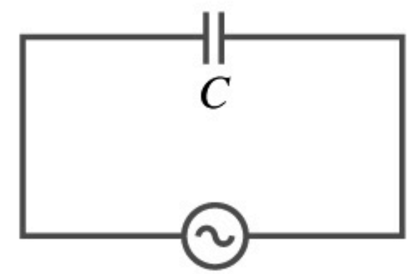
$$X_L = \varpi L = 2\pi fL = 2\pi \cdot \left(60.0s^{-1}\right) \cdot 0.300H = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is

$$I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120V}{113\Omega} = 1.06A$$

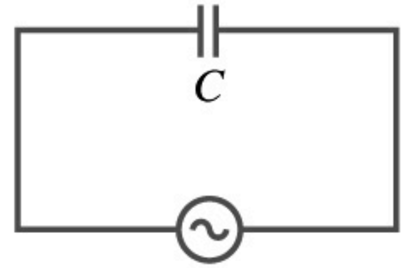
AC Circuit w/ Capacitance only

- What happens when a capacitor is connected to a DC power source?
  - The capacitor quickly charges up.
  - There is no steady current flow in the circuit
    - Since the capacitor prevents the flow of the DC current



- What do you think will happen if it is connected to an AC power source?
  - The current flows continuously. Why?
  - When the AC power turns on, charge begins to flow one direction, charging up the plates
  - When the direction of the power reverses, the charge flows in the opposite direction

AC Circuit w/ Capacitance only



- From Kirchhoff's loop rule, we obtain

$$V = \frac{Q}{C}$$

- The current at any instance is $I = \frac{dQ}{dt} = I_0 \sin \varpi t$

- The charge Q on the plate at any instance is

$$Q = \int_{Q=0}^{Q} dQ = \int_{t=0}^{t} I_0 \sin \varpi t dt = -\frac{I_0}{\varpi} \cos \varpi t$$

- Thus the voltage across the capacitor is

$$V = \frac{Q}{C} = -I_0 \frac{1}{\varpi C} \cos \varpi t$$

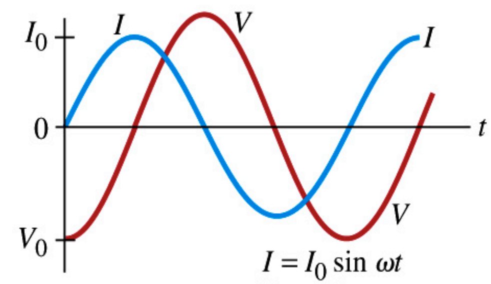
  - Using the identity $\cos\theta = -\sin\left(\theta - 90^\circ\right)$

$$V = I_0 \frac{1}{\varpi C} \sin\left(\varpi t - 90^\circ\right) = V_0 \sin\left(\varpi t - 90^\circ\right)$$

  - Where
  - $V_0 = \frac{I_0}{\varpi C}$

AC Circuit w/ Capacitance only

- So the voltage is $V = V_0 \sin\left(\varpi t - 90^\circ\right)$
- What does this mean?
  – Current and voltage are "out of phase by π/2 or 90º" but in this case, the voltage reaches its peak ¼ cycle after the current
- What happens to the energy?
  – No energy is dissipated
  – The average power is 0 at all times
  – The energy is stored temporarily in the electric field
  – Then released back to the source



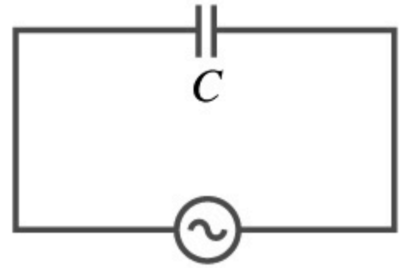
- Applied voltage and the current in the capacitor can be written as $V_0 = I_0 X_C$
  – Where the capacitive reactance X_C is defined as $X_C = \dfrac{1}{\varpi C}$
  – Again, this relationship is only valid for rms quantities

Infinite when ω=0.

$$V_{rms} = I_{rms} X_C$$

Example 30 – 10

Capacitor reactance. What are the peak and rms current in the circuit in the figure if C=1.0μF and V_{rms}=120V? Calculate for (a) f=60Hz, and then for (b) f=6.0x10^{5}Hz.



The peak voltage is $V_0 = \sqrt{2}V_{rms} = 120V \cdot \sqrt{2} = 170V$

The capacitance reactance is

$$X_C = \frac{1}{\varpi C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \cdot \left(60s^{-1}\right) \cdot 1.0 \times 10^{-6} F} = 2.7k\Omega$$

Thus the peak current is

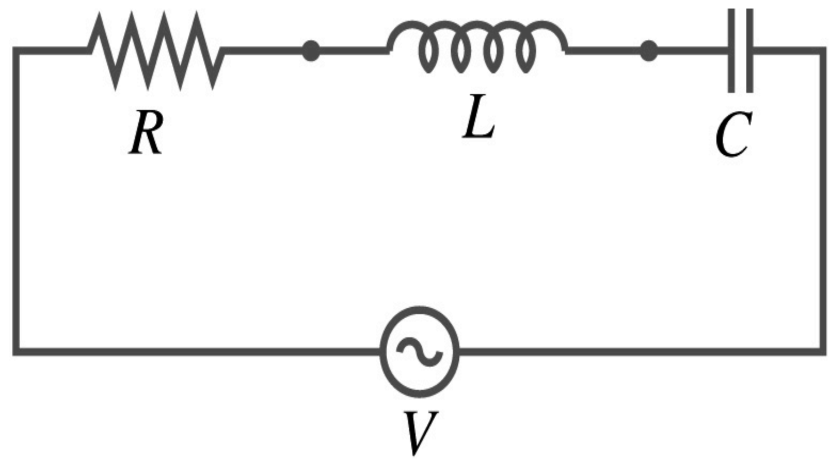
$$I_0 = \frac{V_0}{X_C} = \frac{170V}{2.7k\Omega} = 63mA$$

The rms current is

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{120V}{2.7k\Omega} = 44mA$$

AC Circuit w/ LRC

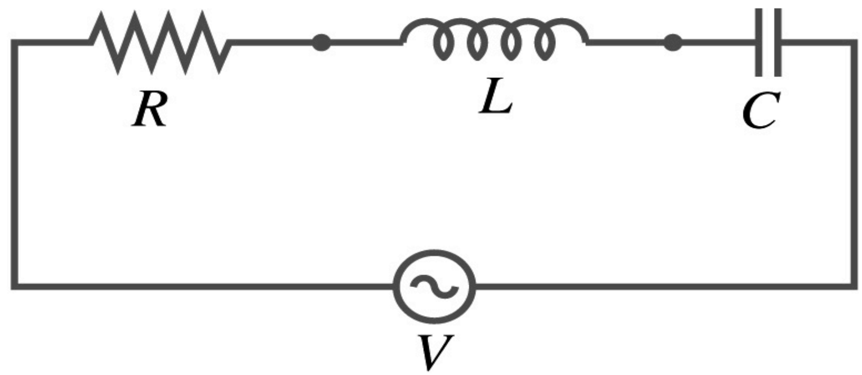
- The voltage across each element is
  - V_R is in phase with the current
  - V_L leads the current by 90°
  - V_C lags the current by 90°
- From Kirchhoff's loop rule



- $V=V_R+V_L+V_C$
  - However since they do not reach the peak voltage at the same time, the peak voltage of the source V_0 will not equal $V_{R0}+V_{L0}+V_{C0}$
  - The rms voltage also will not be the simple sum of the three
- Let's try to find the total impedance, peak current I_0 and the phase difference between I_0 and V_0.

AC Circuit w/ LRC

- The current at any instance is the same at all point in the circuit
  - The currents in each elements are in phase
  - Why?
    - Since the elements are in series
  - How about the voltage?
    - They are not in phase.



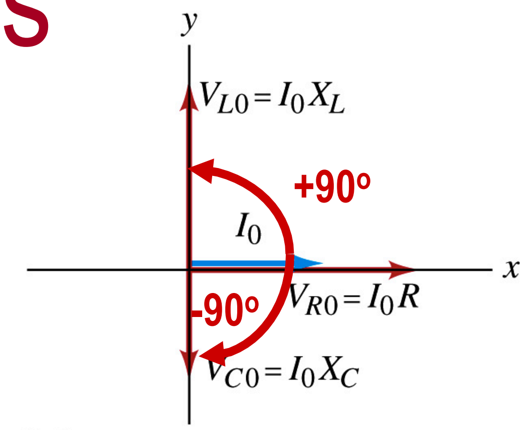
- The current at any given time is

$$I = I_0 \sin \varpi t$$

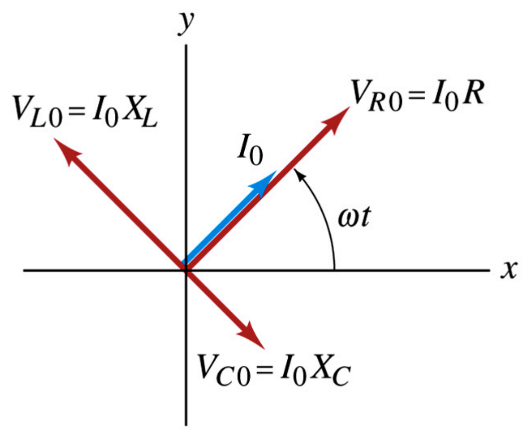
- The analysis of LRC circuit is done using the "phasor" diagram in which arrows are drawn in an xy plane to represent the amplitude of each voltage, just like vectors
  - The lengths of the arrows represent the magnitudes of the peak voltages across each element; $V_{R0}=I_0R$, $V_{L0}=I_0X_L$ and $V_{C0}=I_0X_C$
  - The angle of each arrow represents the phase of each voltage relative to the current, and the arrows rotate at the angular frequency ω to take into account the time dependence.
    - The projection of each arrow on y axis represents voltage across each element at any given time

Phasor Diagrams

- At t=0, I=0.
  - Thus $V_{R0}=0$, $V_{L0}=I_0X_L$, $V_{C0}=I_0X_C$
- At t=t, $I = I_0 \sin \varpi t$



(a)



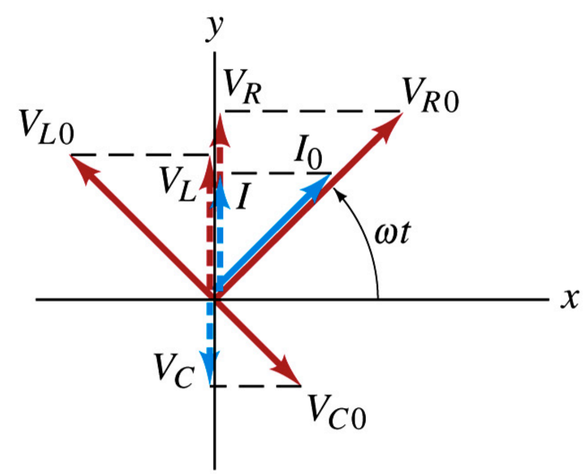
(b)

- Thus, the voltages (y-projections) are

$$V_R = V_{R0} \sin \varpi t$$

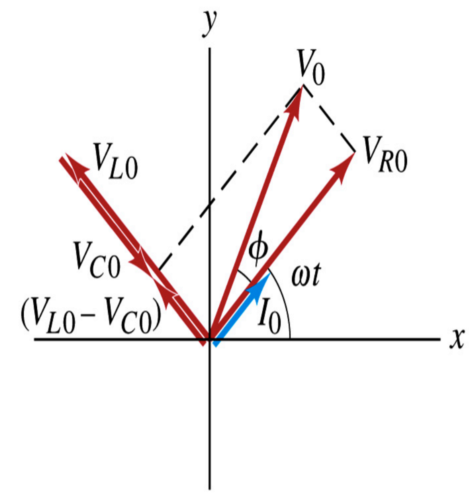
$$V_L = V_{L0} \sin\left(\varpi t + 90^\circ\right)$$

$$V_C = V_{C0} \sin\left(\varpi t - 90^\circ\right)$$



(c)

AC Circuit w/ LRC



- Since the sum of the projections of the three vectors on the y axis is equal to the projection of their sum.
  - The sum of the projections represents the instantaneous voltage across the whole circuit which is the source voltage
  - So we can use the sum of all vectors as the representation of the peak source voltage V_0.
- V_0 forms an angle ϕ to V_{R0} and rotates together with the other vectors as a function of time, $V = V_0 \sin(\varpi t + \phi)$
- We determine the total impedance Z of the circuit defined by the relationship $V_{rms} = I_{rms} Z$ or $V_0 = I_0 Z$
- From Pythagorean theorem, we obtain

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = \sqrt{I_0^2 R^2 + I_0^2 (X_L - X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} = I_0 Z$$

- Thus the total impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\varpi L - \frac{1}{\varpi C}\right)^2}$

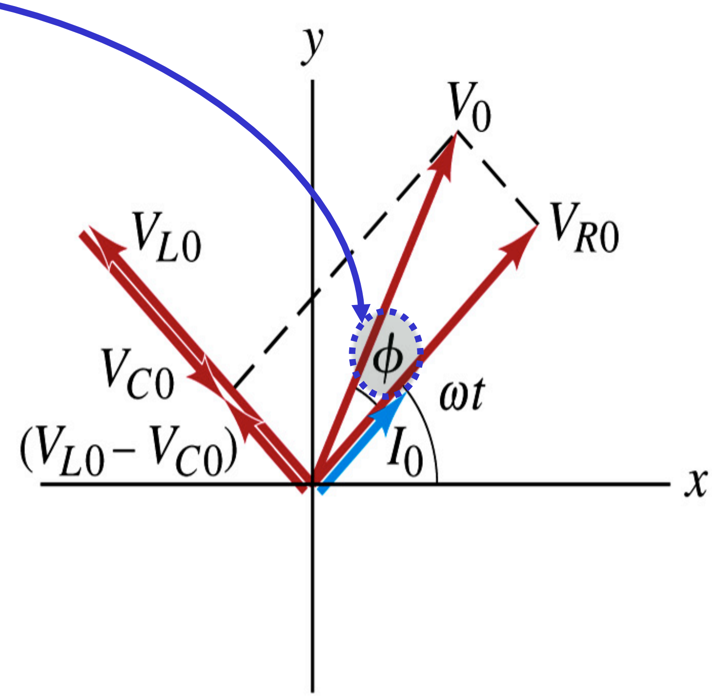
AC Circuit w/ LRC

- The phase angle ϕ is

$$\tan\phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0\left(X_L - X_C\right)}{I_0 R} = \frac{\left(X_L - X_C\right)}{R}$$

- or

$$\cos\phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$



- What is the power dissipated in the circuit?
  - Which element dissipates the power?
  - Only the resistor
- The average power is $\bar{P} = I_{rms}^2 R$
  - Since R=Zcosϕ
  - We obtain

$$\bar{P} = I_{rms}^2 Z\cos\phi = I_{rms} V_{rms} \cos\phi$$

  - The factor cosϕ is referred as the power factor of the circuit
  - For a pure resistor, cosϕ=1 and $\bar{P} = I_{rms} V_{rms}$
  - For a capacitor or inductor alone ϕ=-90° or +90°, so cosϕ=0 and $\bar{P} = 0.$