PHYS 1444 – Section 002
Lecture #5

Monday, Sept. 9, 2019

Dr. Jaehoon Yu

- Ch 21
  - The Electric Field & Field Lines
  - Electric Fields and Conductors
  - Motion of a Charged Particle in an E Field
  - Electric Dipole and Dipole Moment

Today's homework is homework #4, due 11pm, Monday, Sept. 16!!

Announcements

- Reading assignments
  – CH21.11, CH21.12 and CH21.13
- 1st Term Exam
  – In class, Wednesday, Sept. 18: DO NOT MISS THE EXAM!
  – CH21.1 to what we learn on Monday, Sept. 16 + Appendices A1 – A8
  – You can bring your calculator but it must not have any relevant formula pre-input
  – BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  – No derivations, word definitions, or solutions of any problems !
  – No additional formulae or values of constants will be provided!

Reminder: Special Project #2 – Angels & Demons

- Compute the total possible energy released from an annihilation of xx-grams of anti-matter and the same quantity of matter, where xx is the last two digits of your SS#. (20 points)
  - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in yy ns, where yy is the first two digits of your SS#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of world electricity usage (3.6GJ/mo) this energy corresponds to. (5 points)
- Due at the beginning of the class Monday, Sept. 23

Direction of the Electric Field

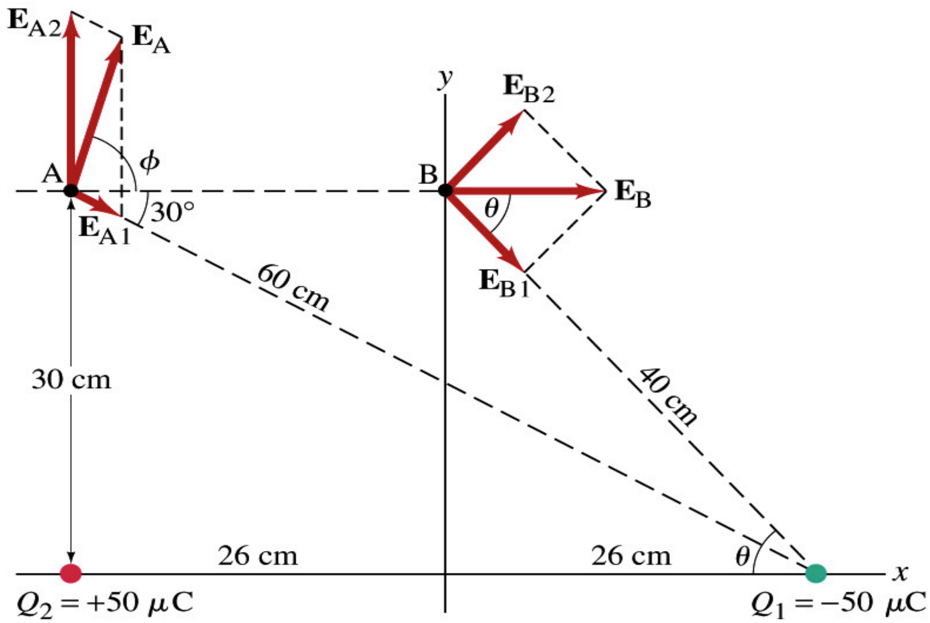
- If there are more than one charge present, the individual fields due to each charge are added vectorially to obtain the total field at any point in space

$$\vec{E}_{Tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + ...$$

- This superposition principle of electric field has been verified by experiments.
- For a given electric field **E** at a given point in space, we can calculate the force **F** on any charge q, **F**=q**E**.
  - What is the direction of the force on the charge placed in the field and the field depending on the sign of the charge q?
  - The **F and E** are in the same directions if the sign of q is positive
  - The **F and E** are in the opposite directions if the sign of q is negative

Example 21 – 8

- **E above two point charges**: Calculate the total electric field (a) at point A and (b) at point B in the figure on the right due to both the charges Q_1 and Q_2.



How do we solve this problem?

First, compute the magnitude of fields at each point due to each of the two charges.

Then add them at each point vectorially!

First, the electric field at point A by Q_1 and then Q_2.

$$\left|E_{A1}\right| = k\frac{Q_1}{r_{A1}^2} = \frac{\left(9.0\times10^9\ N\cdot m^2/C^2\right)\cdot\left(50\times10^{-6}C\right)}{\left(0.60m\right)^2} = 1.25\times10^6\ N/C$$

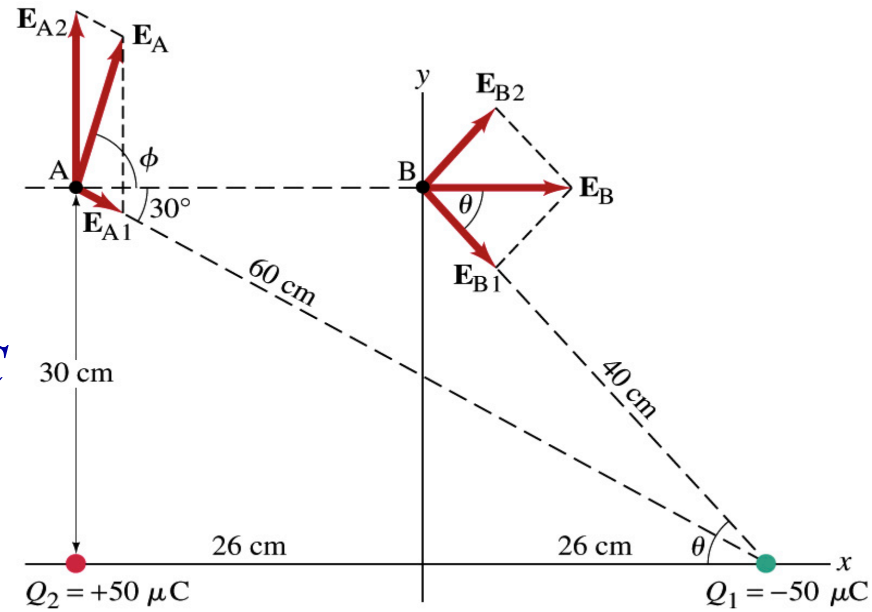
$$\left|E_{A2}\right| = k\frac{Q_2}{r_{A2}} = \frac{\left(9.0\times10^9\ N\cdot m^2/C^2\right)\cdot\left(50\times10^{-6}C\right)}{\left(0.30m\right)^2} = 5.0\times10^6\ N/C$$

Example 21 – 8, cnt'd

Now the components of the electric field vectors by the two charges at point A.

$$E_{Ax} = E_{A1}\cos 30 = 1.1\times 10^{6}\ N/C$$

$$E_{Ay} = E_{A2} - E_{A1}\sin 30 = 4.4\times 10^{6}\ N/C$$



So the electric field at point A is

$$\vec{E}_A = E_{Ax}\vec{i} + E_{Ay}\vec{j} = \left(1.1\vec{i} + 4.4\vec{j}\right)\times 10^{6}\ N/C$$

The magnitude of the electric field at point A is

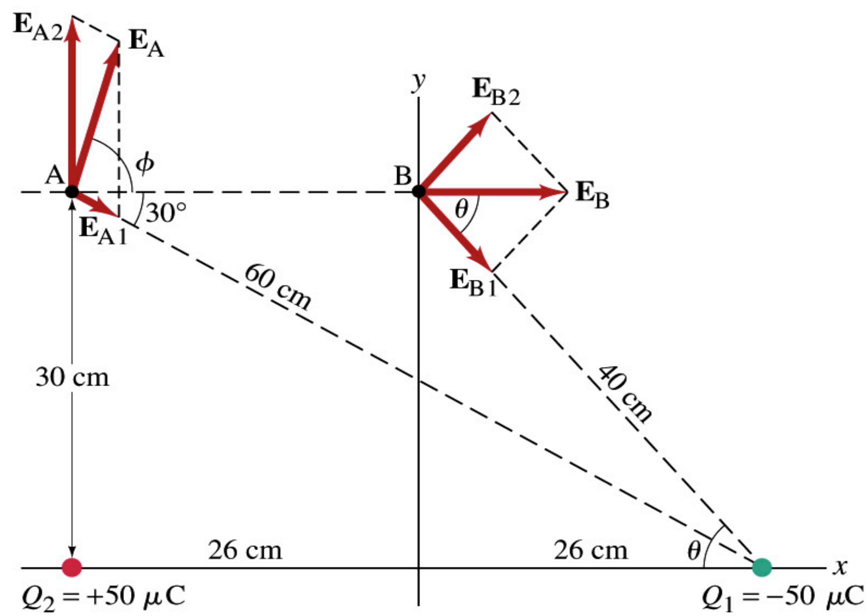
$$\left|E_A\right| = \sqrt{E_{Ax}^2 + E_{Ay}^2} = \sqrt{(1.1)^2 + (4.4)^2}\times 10^{6}\ N/C = 4.5\times 10^{6}\ N/C$$

Now onto the electric field at point B

Example 21 – 8, cnt'd

Electric field at point B is easier due to symmetry!

Since the magnitude of the charges are the same and the distance to point B from the two charges are the same, the magnitude of the electric field by the two charges at point B are the same!!



$$\left|E_{B1}\right| = k\frac{Q_1}{r_{B1}} = \left|E_{B2}\right| = k\frac{Q_2}{r_{B2}} =$$

$$= \frac{\left(9.0\times10^{9}\ N\cdot m^2/C^2\right)\cdot\left(50\times10^{-6}\ C\right)}{\left(0.40m\right)^2} = 2.8\times10^{6}\ N/C$$

Now the components! First, the y-component! $E_{By} = E_{B2}\sin\theta - E_{B1}\sin\theta = 0$

Now, the x-component! $\cos\theta = 0.26/0.40 = 0.65$

$$E_{Bx} = 2E_{B1}\cos\theta = 2\cdot2.8\times10^{6}\cdot0.65 = 3.6\times10^{6}\ N/C$$

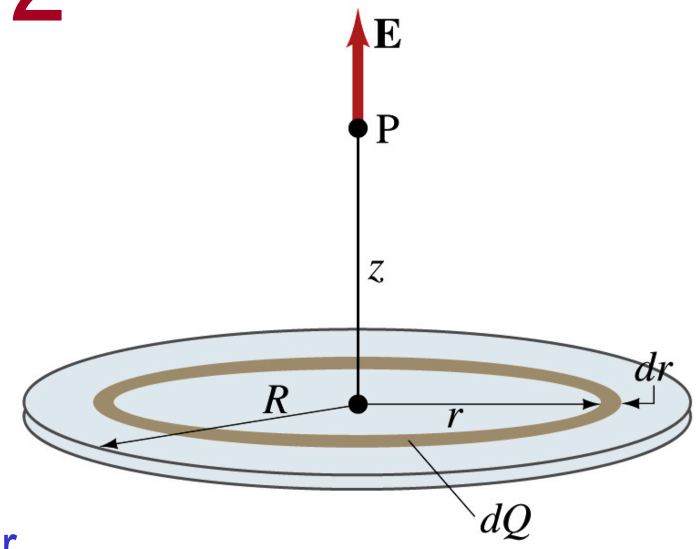
So the electric field at point B is

$$\vec{E}_B = E_{Bx}\vec{i} + E_{By}\vec{j} = \left(3.6\vec{i} + 0\vec{j}\right)\times10^{6}\ N/C = 3.6\times10^{6}\vec{i}\ N/C$$

The magnitude of the electric field at point B

$$\left|E_B\right| = E_{Bx} = 2E_{B1}\cos\theta = 2\cdot2.8\times10^{6}\cdot0.65 = 3.6\times10^{6}\ N/C$$

Example 21 – 12



- **Uniformly charged disk**: Charge is distributed uniformly over a thin circular disk of radius R. The charge per unit area (C/m^2) is σ. Calculate he electric field at a point P on the axis of the disk, a distance z above its center.

How do we solve this problem?

First, compute the magnitude of the field (dE) at point P due to the charge (dQ) on the ring of infinitesimal width dr.

From the result of example 21 – 11 (please do this problem yourself)

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{zdQ}{\left(z^2+r^2\right)^{3/2}}$$

Since the surface charge density is constant, σ, and the ring has an area of $2\pi rdr$, the infinitesimal charge of dQ is

$$dQ = 2\pi\sigma rdr$$

So the infinitesimal field dE can be written

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{zdQ}{\left(z^2+r^2\right)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{2\pi z\sigma}{\left(z^2+r^2\right)^{3/2}}rdr = \frac{\sigma z}{2\varepsilon_0}\frac{r}{\left(z^2+r^2\right)^{3/2}}dr$$

Example 21 – 12 cnt'd

Now integrating dE over 0 through R, we get

$$E = \int dE = \int_0^R \frac{1}{4\pi\varepsilon_0} \frac{2\pi z\sigma}{\left(z^2+r^2\right)^{3/2}} r\,dr = \frac{z\sigma}{2\varepsilon_0} \int_0^R \frac{r}{\left(z^2+r^2\right)^{3/2}} dr$$

$$= \frac{\sigma}{2\varepsilon_0}\left[-\frac{z}{\left(z^2+r^2\right)^{1/2}}\right]_0^R = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{1}{\left(z^2+R^2\right)^{1/2}}\right]$$

What happens if the disk has infinitely large area?

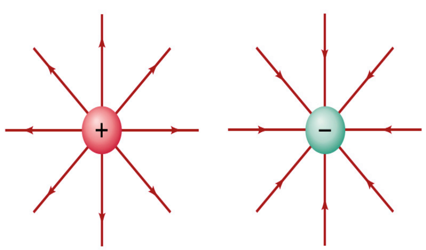
$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{1}{\left(z^2+R^2\right)^{1/2}}\right] \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

So the electric field due to an evenly distributed surface charge with density, σ, is

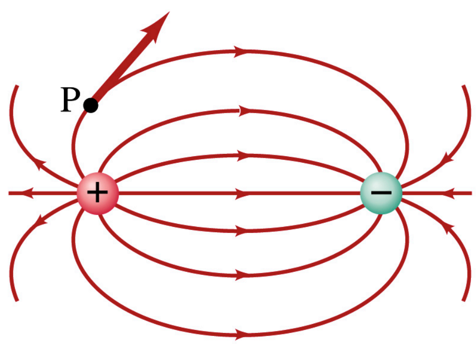
$$E = \frac{\sigma}{2\varepsilon_0}$$

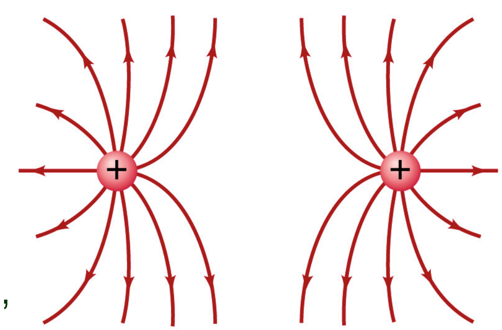
Field Lines

- The electric field is a vector quantity. Thus, its magnitude can be expressed by the length of an arrow and the direction by the direction the arrowhead points.
- Since the field permeates through the entire space, drawing vector arrows is not a good way of expressing the field.
- Electric field lines are drawn to indicate the direction of the force due to the given field on a **positive test charge**.
  - Number of lines crossing unit area perpendicular to E is proportional to the magnitude of the electric field.
  - The closer the lines are together, the stronger the electric field in that region.
  - Starts on a positive charge and ends on a negative charge.

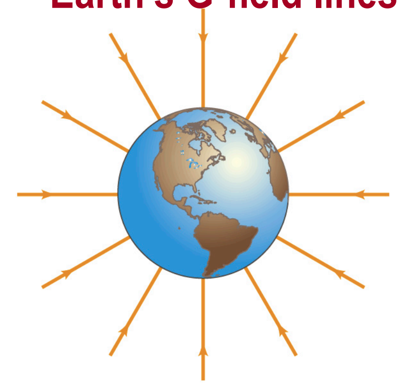


(a) (b)



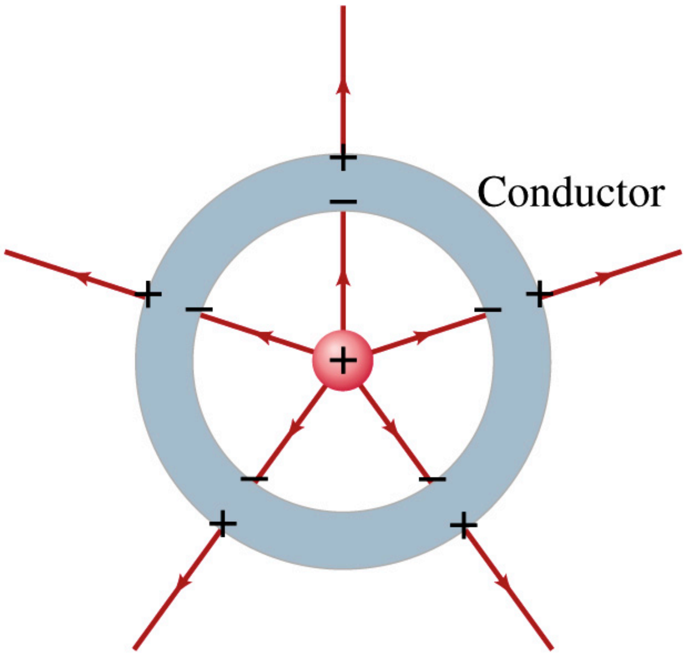


Earth's G-field lines



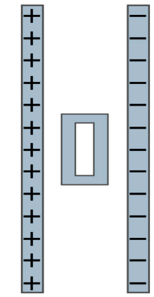
Electric Fields and Conductors

- The electric field inside a conductor is **ZERO** in a static situation. (If the charge is at rest.) Why?
  - If there were an electric field within a conductor, there would be a force on its free electrons.
  - The electrons will move until they reach the position where the electric field becomes zero.
  - Electric field, however, can exist inside a non-conductor.
- Consequences of the above
  - Any net charge on a conductor distributes itself on its surface.
  - Although no field exists inside a conductor, the fields can exist outside the conductor due to induced charges on either surface
  - The electric field is always perpendicular to the surface outside of a conductor.

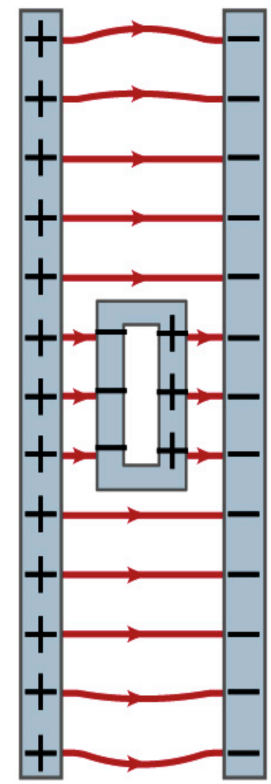


Example 21-13

- **Shielding, and safety in a storm.** A hollow metal box is placed between two parallel charged plates. What is the field like in the box?

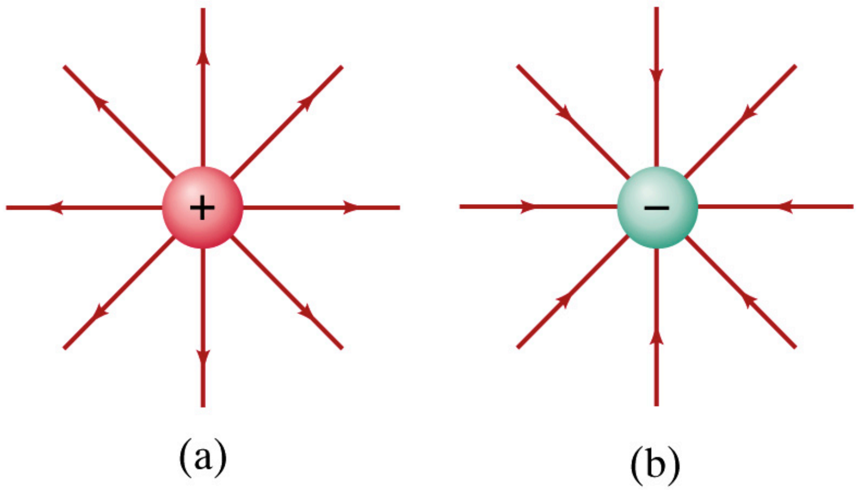


- If the metal box were solid
  - The free electrons in the box would redistribute themselves along the surface so that the field lines would not penetrate into the metal.
- The free electrons do the same in hollow metal boxes just as well as it did in a solid metal box.
- Thus a conducting hollow box is an effective device for shielding. ➔ The Faraday cage
- So what do you think will happen if you were inside a car when the car was struck by a lightening?



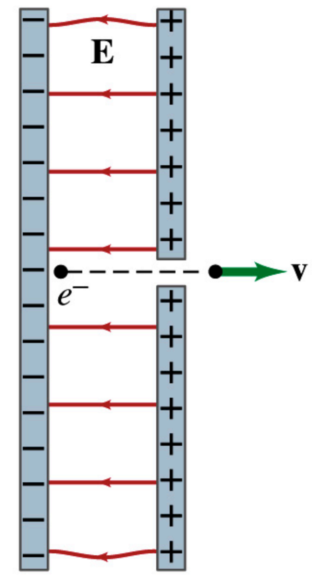
Motion of a Charged Particle in an Electric Field

- If an object with an electric charge q is at a point in space where electric field is **E**, the force exerting on the object is $\vec{F} = q\vec{E}$.
- What do you think will happen to the charge?
  - Let's think about the cases like these on the right.
  - The object will move along the field line…Which way?
    - Depends on the sign of the charge
  - The charge gets accelerated under an electric field.



Example 21 – 14

- **Electron accelerated by electric field.** An electron (mass m = 9.1×10^{-31}kg) is accelerated in a uniform field E ($E=2.0 \times 10^4$N/C) between two parallel charged plates. The separation of the plates is 1.5cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate. (a) With what speed does it leave the hole? (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.



The magnitude of the force on the electron is F=qE and is directed to the right. The equation to solve this problem is

$$F = qE = ma$$

The magnitude of the electron's acceleration is $a = \frac{F}{m} = \frac{qE}{m}$

Between the plates the field **E** is uniform, thus the electron undergoes a uniform acceleration

$$a = \frac{eE}{m_e} = \frac{\left(1.6 \times 10^{-19} C\right)\left(2.0 \times 10^4 N/C\right)}{\left(9.1 \times 10^{-31} kg\right)} = 3.5 \times 10^{15} \, m/s^2$$

Example 21 – 14

Since the travel distance is 1.5x10^{-2}m, using one of the kinetic eq. of motions,

$$v^2 = v_0^2 + 2ax \;\; \therefore v = \sqrt{2ax} = \sqrt{2 \cdot 3.5 \times 10^{15} \cdot 1.5 \times 10^{-2}} = 1.0 \times 10^{7}\, m/s$$

Since there is no electric field outside the conductor, the electron continues moving with this speed after passing through the hole.

- (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.

The magnitude of the electric force on the electron is

$$F_e = qE = eE = \left(1.6 \times 10^{-19} C\right)\left(2.0 \times 10^{4} N/C\right) = 3.2 \times 10^{-15} N$$

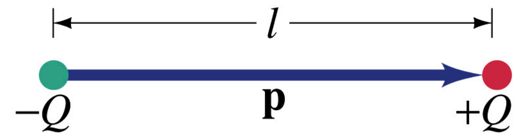
The magnitude of the gravitational force on the electron is

$$F_G = mg = 9.8 m/s^2 \times \left(9.1 \times 10^{-31} kg\right) = 8.9 \times 10^{-30} N$$

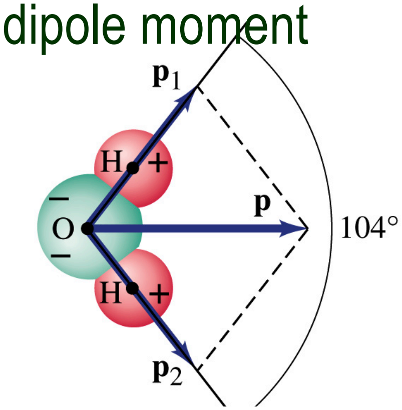
Thus the gravitational force on the electron is negligible compared to the electromagnetic force.

Electric Dipoles

- An electric dipole is the combination of two equal charges of opposite signs, +Q and –Q, separated by a distance *l*, which behaves as one entity.
- The quantity Q*l* is called the electric dipole moment and is represented by the symbol *p*.

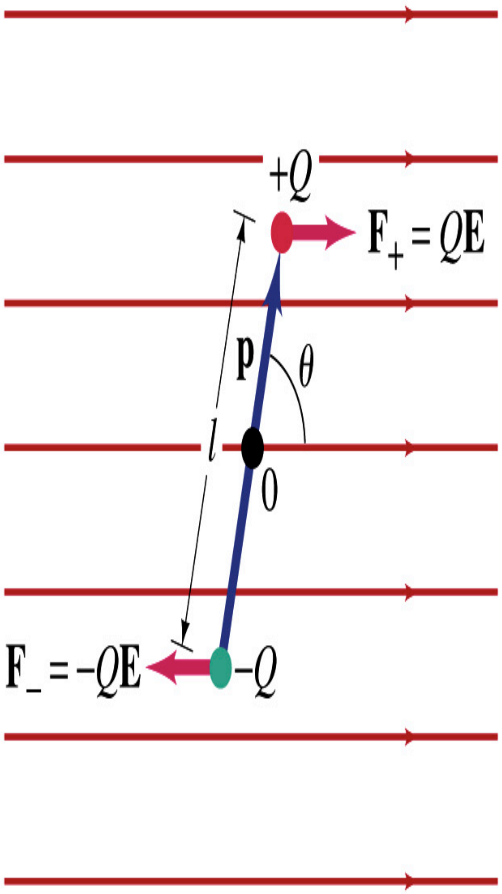


  - The dipole moment is a vector quantity, ***p***
  - The magnitude of the dipole moment is **Q*l*.** Unit? **C-m**
  - Its direction is from the negative to the positive charge.
  - Many of diatomic molecules like CO have a dipole moment. ➔ These are referred as polar molecules.
    - Even if the molecule is electrically neutral, their sharing of electron causes separation of charges
    - Symmetric diatomic molecules, such as O_2, do not have a dipole moment
  - The water molecule also has a dipole moment which is the vector sum of two dipole moments between Oxygen and each of Hydrogen atoms.

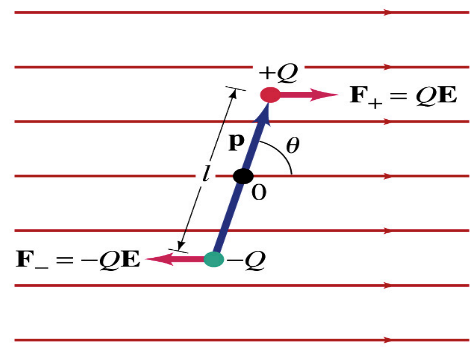


Dipoles in an External Field

- Let's consider a dipole placed in a uniform electric field **E**.
- What do you think will happen to the dipole in the figure?
  - Forces will be exerted on the charges.
    - The positive charge will get pushed toward right while the negative charge will get pulled toward left.
  - What is the net force acting on the dipole?
    - Zero
  - So will the dipole not move?
    - Yes, it will.
  - Why?
    - There is a torque applied on the dipole.



Dipoles in an External Field, cnt'd



- How much is the torque on the dipole?
  - Do you remember the formula for torque?
    - $\vec{\tau} = \vec{r} \times \vec{F}$
  - The magnitude of the torque exerting on each of the charges with respect to the rotational axis at the center is
    - $\tau_{+Q} = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta = \left|\left(\frac{l}{2}\right)(QE)\sin\theta\right| = \frac{l}{2}QE\sin\theta$
    - $\tau_{-Q} = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta = \left|\left(-\frac{l}{2}\right)(-QE)\sin\theta\right| = \frac{l}{2}QE\sin\theta$
  - Thus, the total torque is
    - $\tau_{Total} = \tau_{+Q} + \tau_{-Q} = \frac{l}{2}QE\sin\theta + \frac{l}{2}QE\sin\theta = lQE\sin\theta = pE\sin\theta$
  - So the torque on a dipole in vector notation is $\vec{\tau} = \vec{p} \times \vec{E}$
- The effect of the torque is to try to turn the dipole so that the dipole moment is parallel to **E**. Which direction?

Potential Energy of a Dipole in an External Field

- What is the work done on the dipole by the electric field to change the angle from θ_1 to θ_2?

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} \vec{\tau} \cdot d\vec{\theta} = \int_{\theta_1}^{\theta_2} -\tau d\theta$$

Why negative?

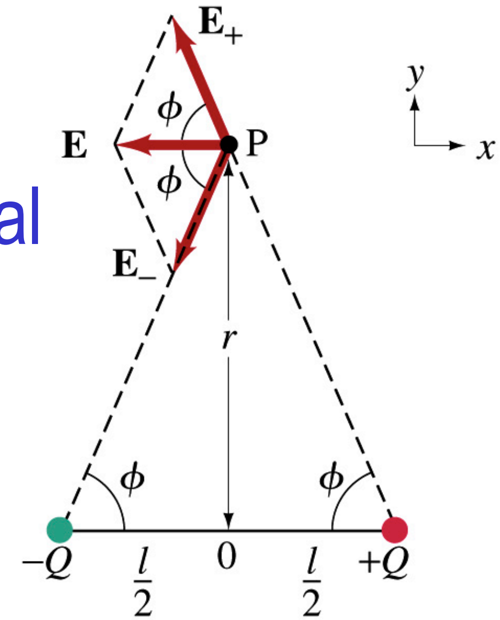
Because τ and θ are opposite directions to each other.

- The torque is $\tau = pE\sin\theta$.
- Thus the work done on the dipole by the field is

$$W = \int_{\theta_1}^{\theta_2} -pE\sin\theta d\theta = pE\left[\cos\theta\right]_{\theta_1}^{\theta_2} = pE\left(\cos\theta_2 - \cos\theta_1\right)$$

- What happens to the dipole's potential energy, U, when a positive work is done on it by the field?
  - It decreases.
- We choose U=0 when θ_1=90 degrees, then the potential energy at θ_2=θ becomes $U = -W = -pE\cos\theta = -\vec{p} \cdot \vec{E}$

Electric Field by a Dipole



- Let's consider the case in the picture.
- There are fields by both the charges. So the total electric field by the dipole is $\vec{E}_{Tot} = \vec{E}_{+Q} + \vec{E}_{-Q}$
- The magnitudes of the two fields are equal

$$E_{+Q} = E_{-Q} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\left(\sqrt{r^2 + (l/2)^2}\right)^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2 + (l/2)^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2 + l^2/4}$$

- Now we must work out the x and y components of the total field.
  - Sum of the two y components is
    - Zero since they are the same but in opposite direction
  - So the magnitude of the total field is the same as the sum of the two x-components:

$$E = 2E_+ \cos\phi = \frac{1}{2\pi\varepsilon_0} \frac{Q}{r^2 + l^2/4} \frac{l}{2\sqrt{r^2 + l^2/4}} = \frac{1}{4\pi\varepsilon_0} \frac{p}{\left(r^2 + l^2/4\right)^{3/2}}$$

Dipole Electric Field from Afar

- What happens when r>>l?.

$$E_D = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left(r^2 + l^2/4\right)^{3/2}} \approx \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \quad \left(\text{when } r \gg l\right)$$

- Why does this make sense?
  - Since from a long distance, the two charges are very close so that the overall charge gets close to 0!!
  - This dependence works for the point not on the bisecting line as well

Example 21 – 17

- **Dipole in a field**. The dipole moment of a water molecule is 6.1×10^{-30}C-m. A water molecule is placed in a uniform electric field with magnitude 2.0×10^{5}N/C. (a) What is the magnitude of the maximum torque that the field can exert on the molecule? (b) What is the potential energy when the torque is at its maximum? (c) In what position will the potential energy take on its greatest value? Why is this different than the position where the torque is maximized?

(a) The torque is maximized when θ=90 degrees. Thus the magnitude of the maximum torque is

$$\tau = pE\sin\theta = pE =$$

$$= \left(6.1 \times 10^{-30}\, C \cdot m\right)\left(2.5 \times 10^{5}\, N/C\right) = 1.2 \times 10^{-24}\, N \cdot m$$

What is the distance between a hydrogen atom and the oxygen atom?

Example 21 – 17

(b) What is the potential energy when the torque is at its maximum?

Since the dipole potential energy is $U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$

And τ is at its maximum at θ=90 degrees, the potential energy, U, is

$$U = -pE\cos\theta = -pE\cos\left(90^\circ\right) = 0$$

Is the potential energy at its minimum at θ=90 degrees? No

Why not? Because U will become negative as θ increases.

(c) In what position will the potential energy take on its greatest value?

The potential energy is maximum when cosθ= -1, θ=180 degrees.

Why is this different than the position where the torque is maximized?

The potential energy is maximized when the dipole is oriented so that it has to rotate through the largest angle against the direction of the field, to reach the equilibrium position at θ=0.

Torque is maximized when the field is perpendicular to the dipole, θ=90.