PHYS 1444 – Section 002
Lecture #6

Wednesday, Sept. 11, 2019

Dr. Jaehoon Yu

CH 22

– Gauss' Law

– Electric Flux

– Gauss' Law with Multiple Charges

CH 23

– Electric Potential Energy

– Electric Potential

Announcements

- Reading assignments
  - CH21.11, CH21.12, CH21.13 and CH22.4
- 1st Term Exam
  - In class, Wednesday, Sept. 18: DO NOT MISS THE EXAM!
  - CH1.1 to what we learn on Monday, Sept. 16 + Appendices A1 – A8
  - You can bring your calculator but it must not have any relevant formula pre-input
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations, word definitions, or solutions of any problems !
  - No additional formulae or values of constants will be provided!
- Colloquium at 4pm today in SH100
  - Dr. Isaac Rutel of U. of Oklahoma

PHYSICS DEPARTMENT

UNIVERSITY OF TEXAS AT ARLINGTON

Colloquium:

Career and Research Opportunities in Medical Physics

The field of medical physics provides technical services to the medical field. While generally performing this work "behind-the-scenes", the knowledge, experience and critical thinking abilities of the medical physicist can be invaluable when lives are on the line. An overview of the various career pathways in medical physics is presented, highlighting diagnostic, therapeutic, nuclear medicine and health physics tracks, specifically. Beyond clinical duties, many medical physicists perform clinical and quasi-fundamental research to provide benefits, enhancements, and new modalities to the medical field. Some current topics of research are presented with their applications to medicine.

Dr. Isaac Rutel
University of Oklahoma Health Sciences
Department of Radiological Sciences

WEDNESDAY, SEPTEMBER 11
4PM ROOM 100 SCIENCE HALL
REFRESHMENTS AT 3:30PM IN 108 SCIENCE HALL

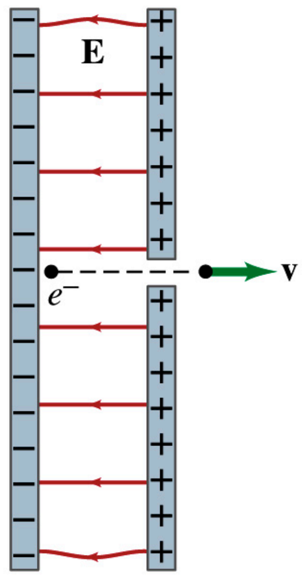
UNIVERSITY OF TEXAS ARLINGTON

Reminder: Special Project #2 – Angels & Demons

- Compute the total possible energy released from an annihilation of xx-grams of anti-matter and the same quantity of matter, where xx is the last two digits of your SS#. (20 points)
  - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in yy ns, where yy is the first two digits of your SS#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of world electricity usage (3.6GJ/mo) this energy corresponds to. (5 points)
- Due at the beginning of the class Monday, Sept. 23

Example 21 – 14

- **Electron accelerated by electric field.** An electron (mass m = 9.1×10^{-31}kg) is accelerated in a uniform field E (E=2.0×10^4N/C) between two parallel charged plates. The separation of the plates is 1.5cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate. (a) With what speed does it leave the hole? (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.



The magnitude of the force on the electron is F=qE and is directed to the right. The equation to solve this problem is

$$F = qE = ma$$

The magnitude of the electron's acceleration is $a = \frac{F}{m} = \frac{qE}{m}$

Between the plates the field **E** is uniform, thus the electron undergoes a uniform acceleration

$$a = \frac{eE}{m_e} = \frac{\left(1.6 \times 10^{-19} C\right)\left(2.0 \times 10^4 N/C\right)}{\left(9.1 \times 10^{-31} kg\right)} = 3.5 \times 10^{15}\, m/s^2$$

Example 21 – 14

Since the travel distance is 1.5x10^{-2}m, using one of the kinetic eq. of motions,

$$v^2 = v_0^2 + 2ax \quad \therefore v = \sqrt{2ax} = \sqrt{2 \cdot 3.5 \times 10^{15} \cdot 1.5 \times 10^{-2}} = 1.0 \times 10^7 \, m/s$$

Since there is no electric field outside the conductor, the electron continues moving with this speed after passing through the hole.

- (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.

The magnitude of the electric force on the electron is

$$F_e = qE = eE = \left(1.6 \times 10^{-19} C\right)\left(2.0 \times 10^4 N/C\right) = 3.2 \times 10^{-15} N$$

The magnitude of the gravitational force on the electron is

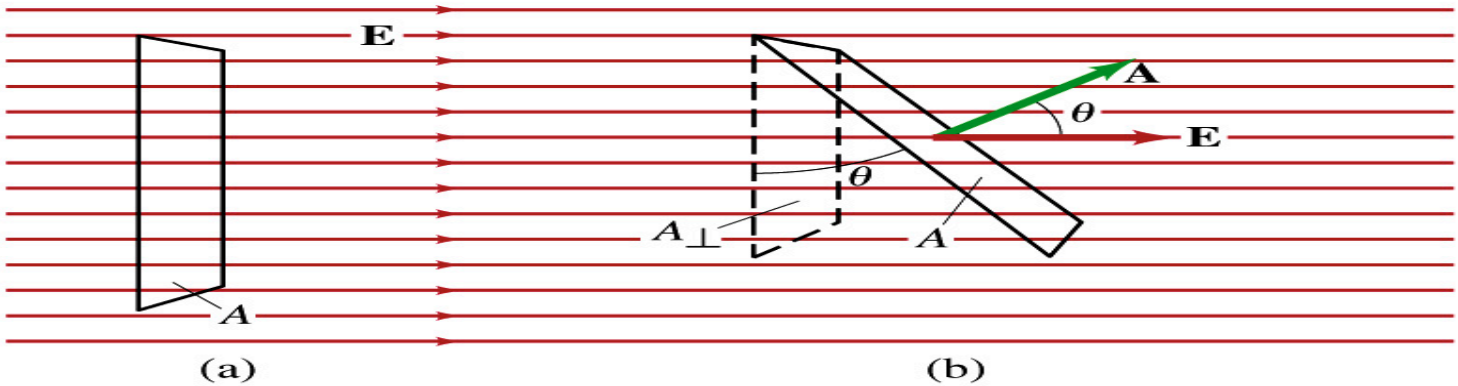
$$F_G = mg = 9.8 m/s^2 \times \left(9.1 \times 10^{-31} kg\right) = 8.9 \times 10^{-30} N$$

Thus the gravitational force on the electron is negligible compared to the electromagnetic force.

Gauss' Law

- Gauss' law states the relationship between electric charge and the electric field.
  - More generalized and elegant form of Coulomb's law.
- The electric field by a distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions vectorially.
- Gauss' law, however, gives an additional insight into the nature of the electrostatic field and a more general relationship between the charge and the field

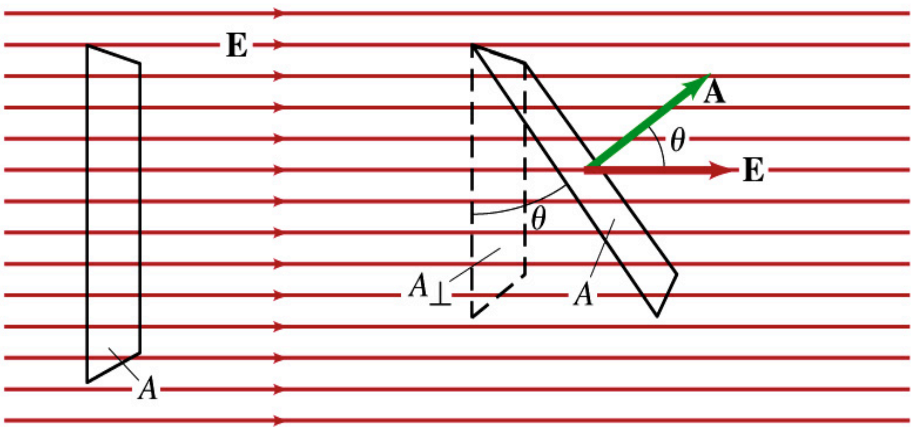
Electric Flux



- Let's imagine a surface of area A through which a uniform electric field E passes
- The electric flux Φ_E is defined as
  - Φ_E=EA, if the field is perpendicular to the surface
  - Φ_E=EAcosθ, if the field makes an angle θ to the surface
- So the electric flux is defined as $\Phi_E = \vec{E}\cdot\vec{A}$. Unit? $\mathrm{N}\cdot\mathrm{m}^2/C$
- How would you define the electric flux in words?
  - The total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_\perp = \Phi_E$

Example 22 – 1

- **Electric flux**. (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux in figure if the angle is 30 degrees?



(a) (b)

The electric flux is defined as

$$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$$

So when (a) θ=0, we obtain

$$\Phi_E = EA\cos\theta = EA = \left(200N/C\right)\cdot\left(0.1\times0.2m^2\right) = 4.0\,\mathrm{N}\cdot\mathrm{m}^2/C$$

And when (b) θ=30 degrees, we obtain

$$\Phi_E = EA\cos30^\circ = \left(200N/C\right)\cdot\left(0.1\times0.2m^2\right)\cos30^\circ = 3.5\,\mathrm{N}\cdot\mathrm{m}^2/C$$

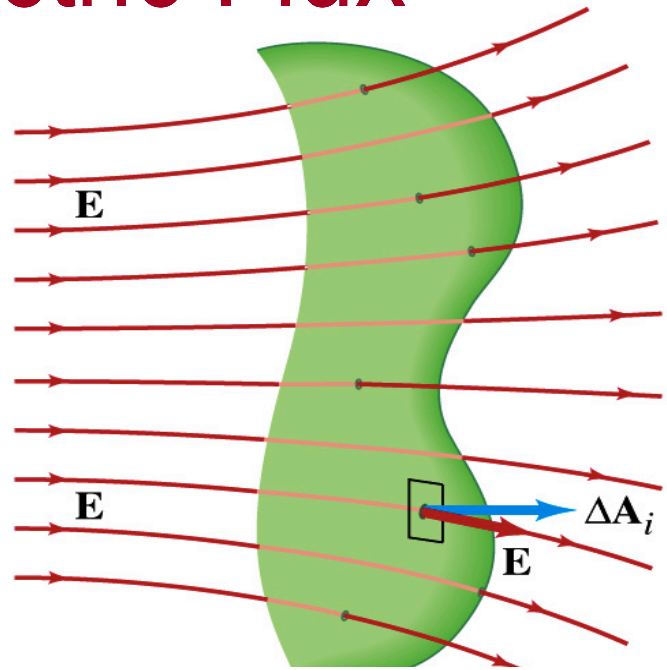
A Brain Teaser of Electric Flux

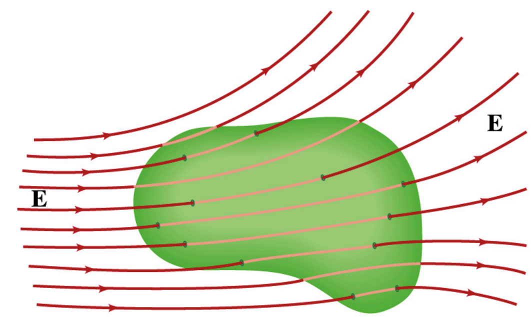
- What would change the electric flux through a circle lying in the xz plane where the electric field is (10N/C)**j**?
  1. Changing the magnitude of the electric field
  2. Changing the surface area of the circle
  3. Tipping the circle so that it is lying in the xy plane
  4. All of the above
  5. None of the above

Generalization of the Electric Flux

- Let's consider a surface of area A that is not a square or flat but in some random shape, and that the field is not uniform.
- The surface can be divided up into infinitesimally small areas of $\Delta\mathbf{A}_i$ that can be considered flat.
- And the electric field through this area can be considered uniform since the area is very small.
- Then the electric flux through the entire surface is approximately

$$\Phi_E \approx \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i$$



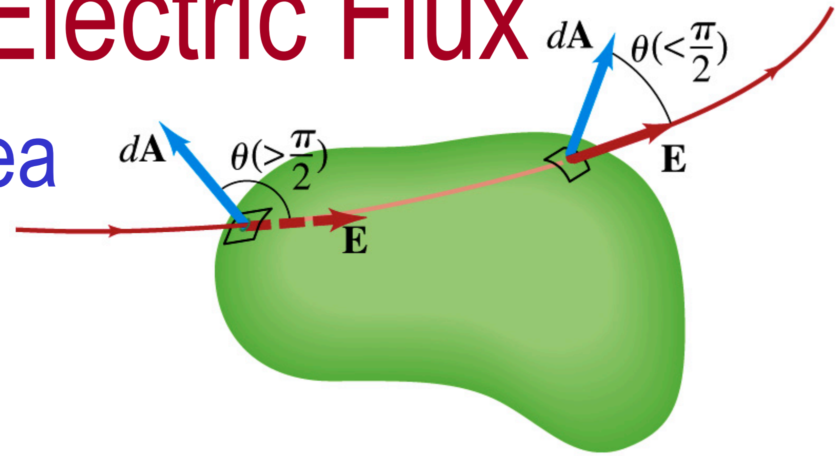


- In the limit where $\Delta\mathbf{A}_i \rightarrow 0$, the discrete summation becomes an integral.

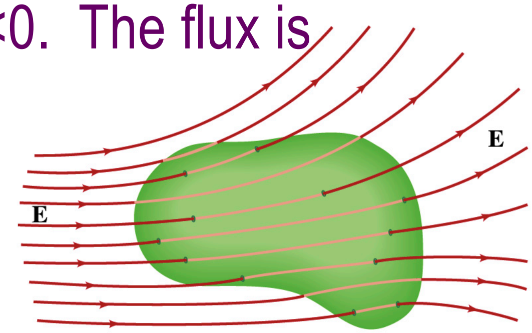
$$\Phi_E = \int \vec{E}_i \cdot d\vec{A}$$ open surface

$$\Phi_E = \oint \vec{E}_i \cdot d\vec{A}$$ enclosed surface

Generalization of the Electric Flux

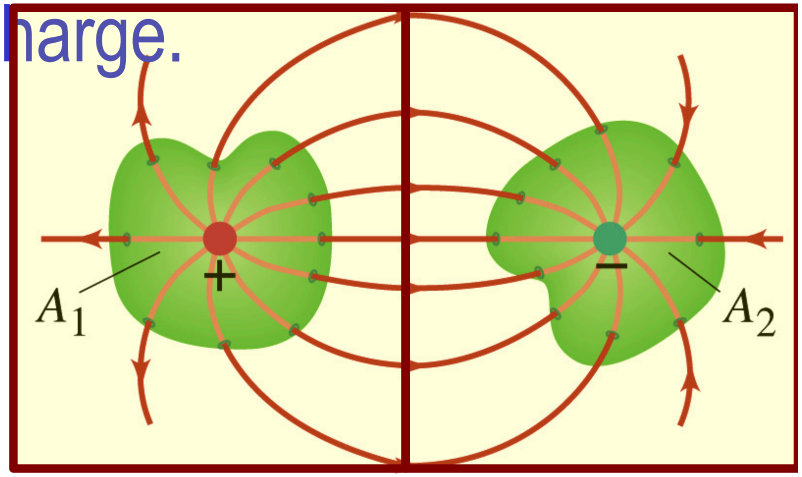


- We arbitrarily define that the area vector points outward from the enclosed volume.
  - For the line leaving the volume, $\theta<\pi/2$ and $\cos\theta>0$. The flux is positive.
  - For the line coming into the volume, $\theta>\pi/2$ and $\cos\theta<0$. The flux is negative.
  - If $\Phi_E>0$, there is net flux out of the volume.
  - If $\Phi_E<0$, there is flux into the volume.

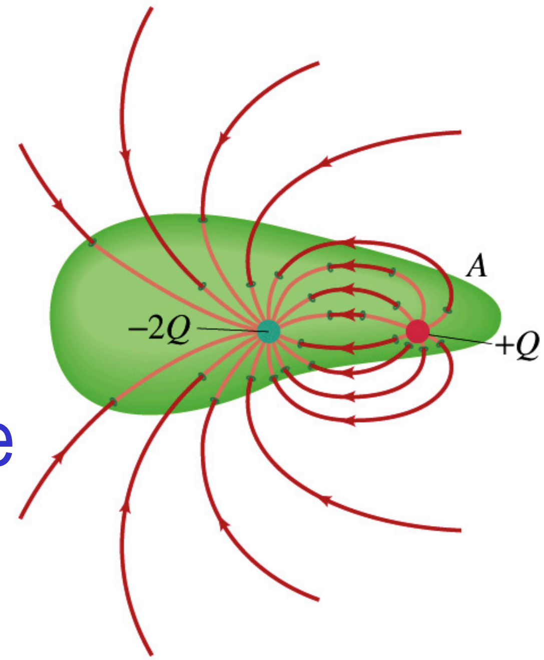


- In the above figures, each field that enters the volume also leaves the volume, so $\Phi_E = \oint \vec{E}\cdot d\vec{A} = 0.$
- The flux is non-zero only if one or more lines start or end inside the surface.

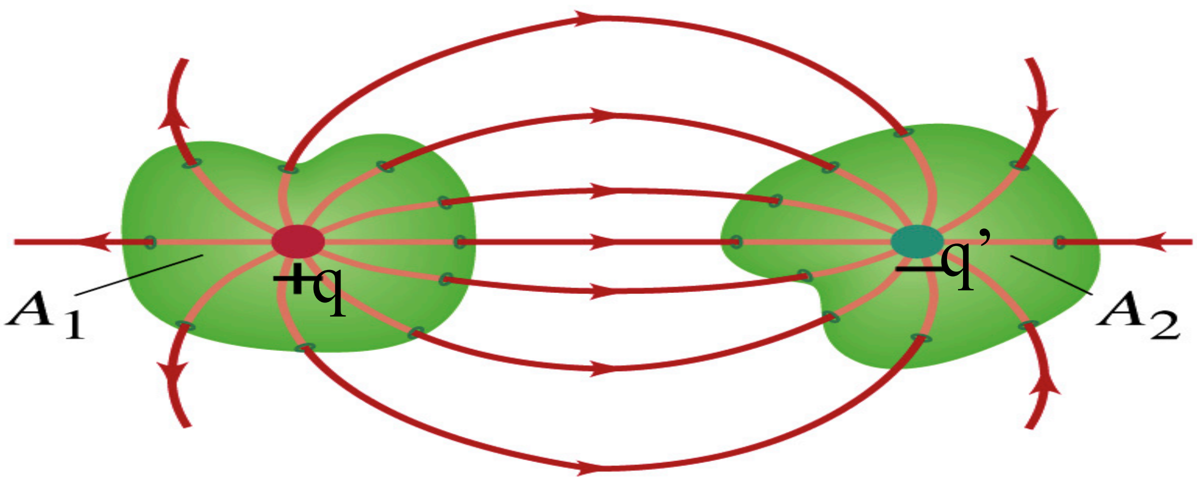
Generalization of the Electric Flux



- The field line starts or ends only on a charge.
- Sign of the net flux on the surface A_1?
  - The net outward flux (positive flux)
- How about A_2?
  - Net inward flux (negative flux)
- What is the flux in the bottom figure?
  - There should be a net inward flux (negative flux) since the total charge inside the volume is negative.
- The net flux that crosses an enclosed surface is proportional to the total charge inside the surface. ➔ This is the crux of Gauss' law.

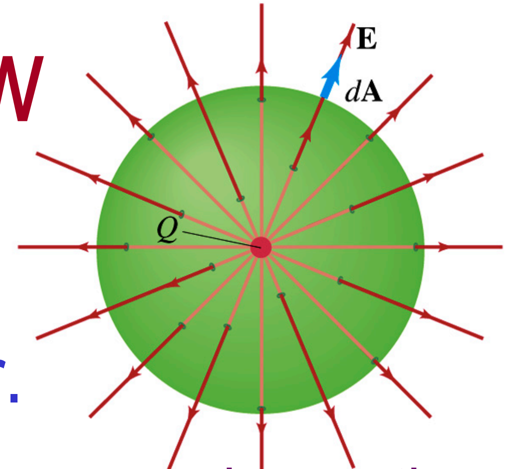


Gauss' Law



- Let's consider the case in the above figure.
- What are the results of the closed integral of the Gaussian surfaces A_1 and A_2?
  - For A_1 $\oint \vec{E} \cdot d\vec{A} = \frac{+q}{\varepsilon_0}$
  - For A_2 $\oint \vec{E} \cdot d\vec{A} = \frac{-q'}{\varepsilon_0}$

Coulomb's Law from Gauss' Law



- Let's consider a charge Q enclosed inside our imaginary Gaussian surface of sphere of radius r.
  - Since we can choose any surface enclosing the charge, we choose the simplest possible one! ☺
- The surface is symmetric about the charge.
  - What does this tell us about the field E?
    - Must have the same magnitude (uniform) at any point on the surface
    - Points radially outward parallel to the surface vector d**A**.
- The Gaussian integral can be written as

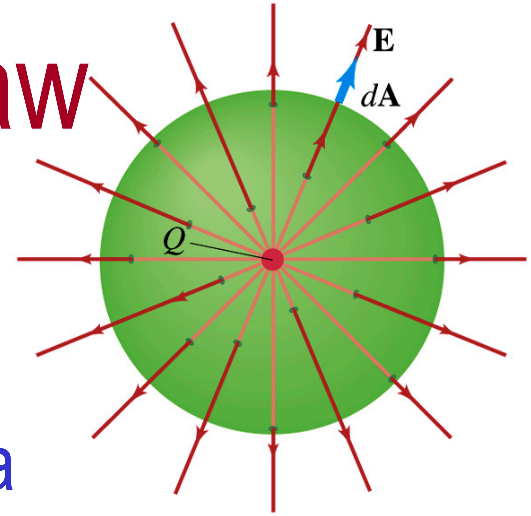
$$\oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA = E\left(4\pi r^2\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Solve for E

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Electric Field of Coulomb's Law

Gauss' Law from Coulomb's Law



- Let's consider a single static point charge Q surrounded by an imaginary spherical surface.
- Coulomb's law tells us that the electric field at a spherical surface of radius r is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

- Performing a closed integral over the surface, we obtain

$$\oint \vec{E}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dA$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\oint dA = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

Gauss' Law