PHYS 1444 – Section 002
Lecture #7

Monday, Sept. 16, 2019
Dr. Jaehoon Yu

CH 22

- Gauss' Law with Multiple Charges

CH 23

- Electric Potential Energy
- Electric Potential due to Point Charges
- Shape of the Electric Potential
- V due to Charge Distribution

Today's homework is homework #5, due 11pm, Wednesday, Sept. 25!!

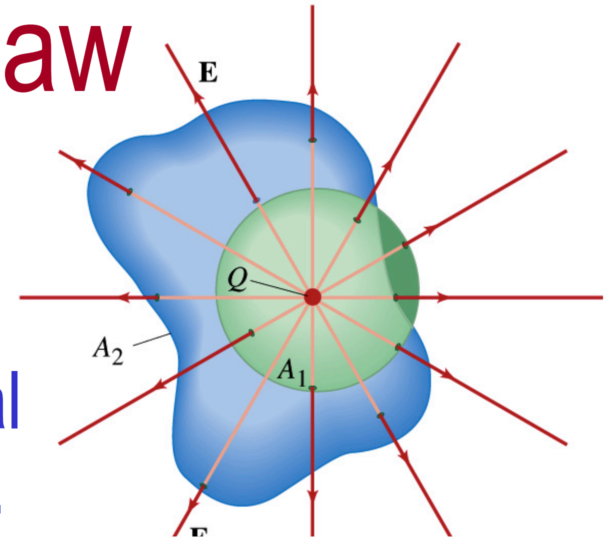
Announcements

- Reading assignment: CH23.9
- 1st Term Exam ➔ come ~10min earlier to the class
  - In class, this Wednesday, Sept. 18: DO NOT MISS THE EXAM!
  - CH21.1 to CH23.1+ Appendices A1 – A8
  - You can bring your calculator but it must not have any relevant formula pre-input
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the exam
  - No derivations, word definitions, or solutions of any problems !
  - No additional formulae or values of constants will be provided!
- Quiz 1 results
  - Class average: 28.2/50
    - Equivalent to 56.4/100
  - Top score: 49/50

Reminder: Special Project #2 – Angels & Demons

- Compute the total possible energy released from an annihilation of xx-grams of anti-matter and the same quantity of matter, where xx is the last two digits of your SS#. (20 points)
  - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in yy ns, where yy is the first two digits of your SS#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of world electricity usage (3.6GJ/mo) this energy corresponds to. (5 points)
- Due at the beginning of the class Monday, Sept. 23

Gauss' Law from Coulomb's Law Irregular Surface



- Let's consider the same single static point charge Q surrounded by a symmetric spherical surface A_1 and a randomly shaped surface A_2.
- What is the difference in the total number of field lines due to the charge Q, passing through the two surfaces?
  - None. What does this mean?
    - The total number of field lines passing through the surface is the same no matter what the shape the enclosing surface has.
  - So we can write: $\oint_{A_1} \vec{E} \cdot d\vec{A} = \oint_{A_2} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$
  - What does this mean?
    - The flux due to the given enclosed charge is the same no matter what the shape of the surface enclosing it is. ➔ Gauss' law, $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, is valid for any surface surrounding a single point charge Q.

Gauss' Law w/ more than one charge

- Let's consider several charges inside a closed surface.
- For each charge, Q_i inside the chosen closed surface,

$$\oint \vec{E}_i \cdot d\vec{A} = \frac{Q_i}{\varepsilon_0}$$

What is $\vec{E}_i$?

The electric field produced by Q_i alone!

- Since electric fields can be added vectorially, following the superposition principle, the total field **E** is equal to the sum of the fields due to each charge $\vec{E} = \sum \vec{E}_i$ plus any external fields. So

$$\oint \vec{E} \cdot d\vec{A} = \oint \left(\vec{E}_{ext} + \sum \vec{E}_i \right) \cdot d\vec{A} = \frac{\sum Q_i}{\varepsilon_0} = \frac{Q_{encl}}{\varepsilon_0}$$

What is Q_{encl}?

The total enclosed charge!

- The value of the flux depends only on the charge enclosed in the surface!! ➔ Gauss' law.

So what is Gauss' Law good for?

- Derivation of Gauss' law from Coulomb's law is only valid for **static electric charge**.
- Electric field can also be produced by changing magnetic fields.
  - Coulomb's law cannot describe this field while Gauss' law is still valid
- Gauss' law is more general than Coulomb's law.
  - Can be used to obtain electric field, forces or obtain charges

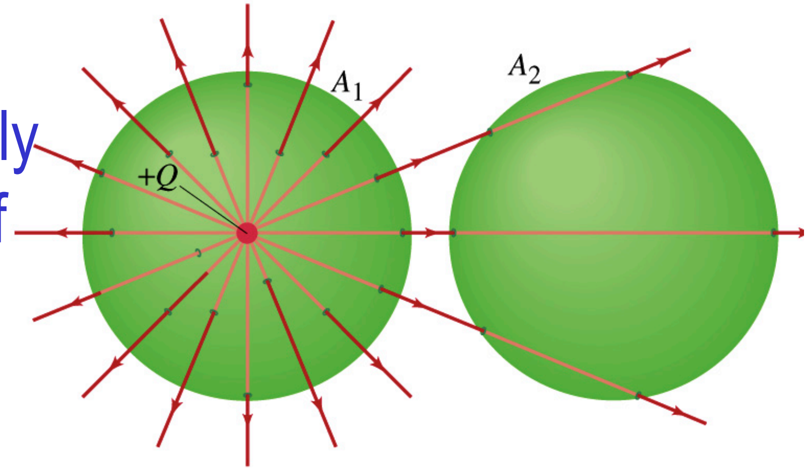
Gauss' Law: Any **differences** between the input and output flux of the electric field over any enclosed surface is due to the charge inside that surface!!!

Solving problems with Gauss' Law

- Identify the symmetry of the charge distributions
- Draw an appropriate Gaussian surface, making sure it pass through the point you want to know the electric field at
- Use the symmetry of charge distribution to determine the direction of E at the point of the Gaussian surface
- Evaluate the flux
- Calculate the enclosed charge by the Gaussian surface
  - Ignore all the charges outside the Gaussian surface
- Equate the flux to the enclosed charge and solve for E

Example 22 – 2

Flux from Gauss' Law: Consider two Gaussian surfaces, A_1 and A_2, shown in the figure. The only charge present is the charge +Q at the center of surface A_1. What is the net flux through each surface A_1 and A_2?



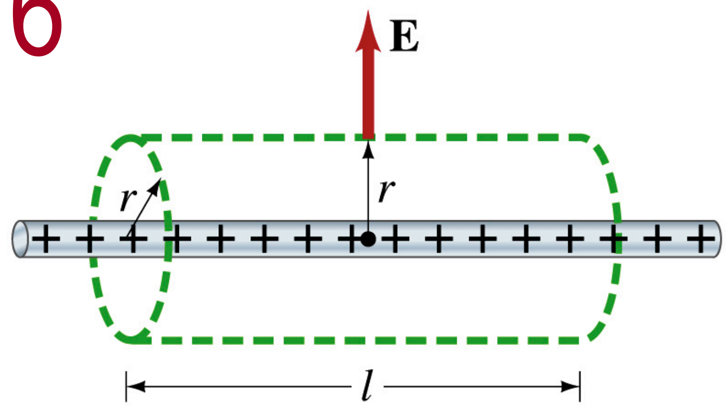
- The surface A_1 encloses the charge +Q, so from Gauss' law we obtain the total net flux

$$\oint \vec{E} \cdot d\vec{A} = \frac{+Q}{\varepsilon_0}$$

- For the surface A_2, the charge, +Q, is outside the surface, so the total net flux is 0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{0}{\varepsilon_0} = 0$$

Example 22 – 6



Long uniform line of charge: A very long straight wire possesses a uniform positive charge per unit length, λ. Calculate the electric field at points near but outside the wire, far from the ends.

- Which direction do you think the field due to the charge on the wire is?
  - Radially outward from the wire, the direction of the radial vector **r**.
- Due to the cylindrical symmetry, the field is the same on the Gaussian surface of the cylinder surrounding the wire.
  - The end surfaces do not contribute to the flux at all. Why?
    - Because the field vector **E** is perpendicular to the surface vector d**A**.
- From Gauss' law

$$\oint \vec{E}\cdot d\vec{A} = E\oint dA = E\left(2\pi rl\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

Solving for E

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Gauss' Law Summary

- The precise relationship between flux and the enclosed charge is given by Gauss' Law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$
    - ε_0 is the permittivity of free space in the Coulomb's law
- A few important points on Gauss' Law
    - Freedom to choose!!
        - The integral is performed over the value of **E** on a closed surface of our choice in any given situation.
    - Test of existence of an electric charge!!
        - The charge Q_{encl} is the net charge enclosed by the arbitrary closed surface of our choice.
    - Universality of the law!
        - It does NOT matter where or how much charge is distributed inside the surface or in which way they are distributed.
    - The charge outside the surface does not contribute to Q_{encl}. Why?
        - The charge outside the surface might impact field lines but not the total number of lines entering or leaving the surface

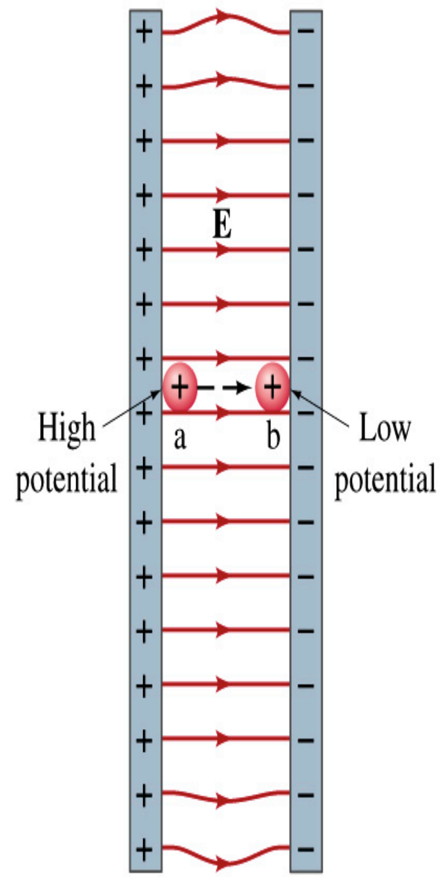
Electric Potential Energy

- Concept of energy is very useful solving mechanical problems
- Conservation of energy makes solving complex problems easier.
- When can the potential energy be defined?
  - Only for a conservative force.
  - The work done by a conservative force is independent of the path. What does it only depend on??
    - The difference between the initial and final positions
  - Can you give me an example of a conservative force?
    - Gravitational force
- Is the electrostatic force between two charges a conservative force?
  - Yes. Why?
  - The dependence of the force to the distance is identical to that of the gravitational force.
    - The only thing matters is the direct linear distance between the objects not the path.

Electric Potential Energy

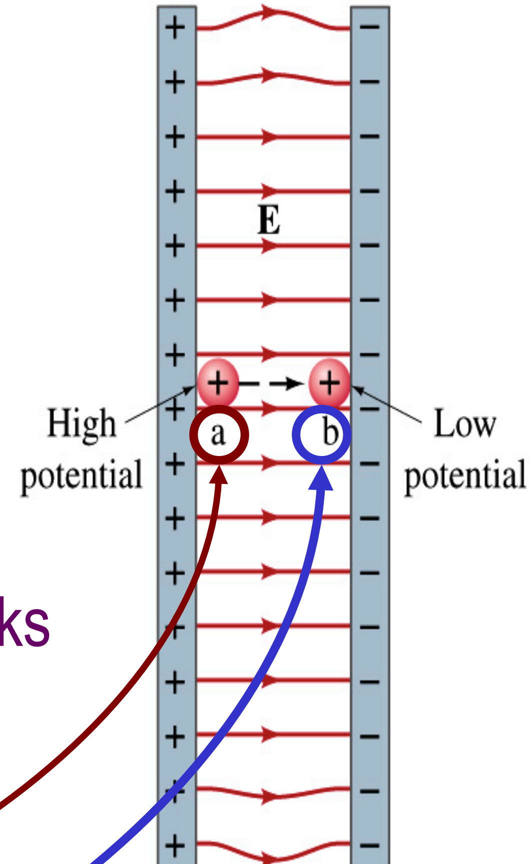
- How would you define the change in electric potential energy $U_b - U_a$?
  - The potential energy gained by the charge as it moves from point *a* to point *b*.
  - The negative of the work done on the charge by the electric force to move it from *a* to *b*.

- Let's consider an electric field between two parallel plates w/ equal but opposite charges
  - The field between the plates is uniform since the gap is small and the plates are infinitely long…
- What happens when we place a small charge, +q, on a point at the positive plate and let it go?
  - The electric force will accelerate the charge toward negative plate.
  - What kind of energy does the charged particle gain?
    - Kinetic energy



Electric Potential Energy

- What does this mean in terms of energies?
  - The electric force is a conservative force.
  - Thus, the mechanical energy (K+U) is conserved under this force.
  - The charged object has only the electric potential energy (no KE) at the positive plate.
  - The electric potential energy decreases and
  - Turns into kinetic energy as the electric force works on the charged object, and the charged object gains speed.



- Point of greatest potential energy for
  - Positively charged object
  - Negatively charged object

	a	b
PE=	U	0
KE=	0	K
ME=	U	K

U+K