

PHYS 1444 – Section 002

Lecture 9

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Wednesday, Sept. 25, 2019

Dr. Jaehoon Yu

CH 23

- Electric Potential due to Point Charges
- Shape of the Electric Potential
- V due to Charge Distributions
- Equi-potential Lines and Surfaces
- Electric Potential Due to Electric Dipole
- Electrostatic Potential Energy



Announcements

- Quiz #2

- Next Wednesday, Oct. 2 at the beginning of the class
- Covers CH22.1 through what we cover in class today (CH23.8?)
- Bring your calculator but DO NOT input formula into it!
 - Cell phones or any types of computers cannot replace a calculator!
- BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the quiz
- No derivations, word definitions or solutions of any problems!
- No additional formulae or values of constants will be provided!

- Mid-term results

- Class average: 51.7/101
 - Equivalent to 51.2/100
- Top score: 94/101

- Colloquium today

- Dr. Maura Hagan of Utah State University and a member of US National Academy of Sciences





Distinguished Women in Science Speaker Series



MAURA HAGAN

- Dean, College of Science
Utah State University
- Member, National Academy
of Sciences

“Surfing Atmospheric Waves:
Their Impact on Space and the
Career of a Physicist”

Wednesday, September 25
4 p.m. SEIR Building Room 194

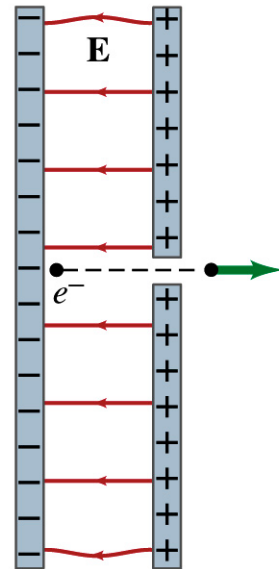
A reception will precede the talk at 3:30 p.m. outside SEIR 194

Sponsored by the UTA College of Science

This series is designed to highlight the achievements of leading women scientists who are making a difference in society through their research and leadership. It aims to inspire the next generation of leaders, especially female leaders, by putting a spotlight on the important contributions of women to research, industry, and academia.

Reminder: Special Project #3

- **Particle Accelerator.** A charged particle of mass M with charge $-Q$ is accelerated in the uniform field E between two parallel charged plates whose separation is D as shown in the figure on the right. The charged particle is accelerated from an initial speed v_0 near the negative plate and passes through a tiny hole in the positive plate.
 - Derive the formula for the electric field E to accelerate the charged particle to a fraction f of the speed of light c . Express E in terms of M , Q , D , f , c and v_0 .
 - (a) Using the Coulomb force and the kinematic equations. (8 points)
 - (b) Using the work-kinetic energy theorem. (8 points)
 - (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up and write down the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)
- Must be handwritten and not copied from anyone else!
- Due beginning of the class Monday, Sept. 30



Electric Potential due to Point Charges

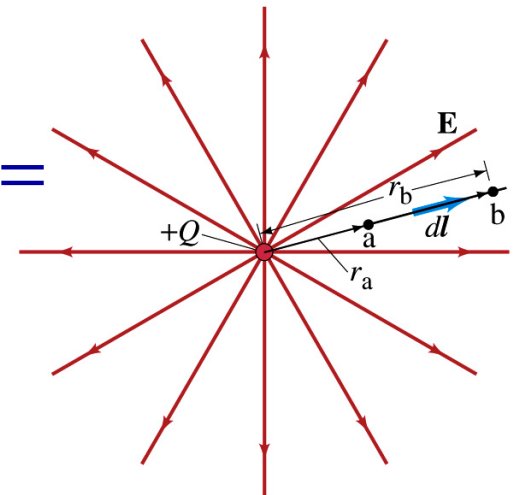
- What is the electric field by a single point charge Q at a distance r ?

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = k \frac{Q}{r^2}$$

- Electric potential due to the field E for moving from point r_a to r_b in the radial direction away from the charge Q is

$$V_b - V_a = - \int_{r_a}^{r_b} \vec{E} \cdot d\vec{l} = - \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{\hat{r}}{r^2} \cdot \hat{r} dr =$$

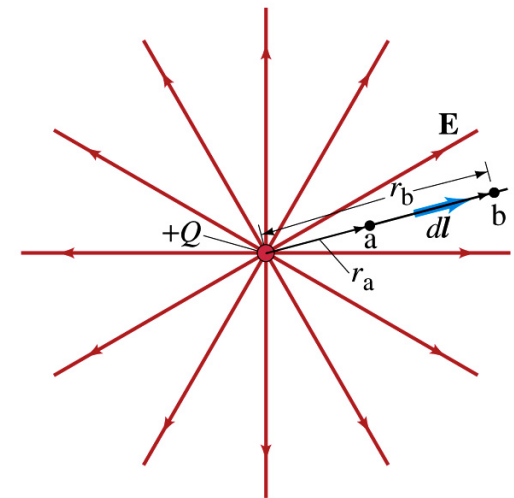
$$= - \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right)$$



Electric Potential due to Point Charges

- Since only the differences in potential have physical meaning, we can choose $V_b = 0$ at $r_b = \infty$.
- The electrical potential V at a distance r from a single point charge Q is

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$



- So the absolute potential by a single point charge can be thought of the potential difference by a single point charge between r and infinity

Properties of the Electric Potential

- What are the differences between the electric potential and the electric field?

- Electric potential

- Electric potential energy per unit charge

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

- Inversely proportional to the distance

- Simply add the potential by each of the source charges to obtain the total potential due to multiple charges, since potential is a scalar quantity

- Electric field

- Electric force per unit charge

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

- Inversely proportional to the **square** of the distance

- Need vector sums to obtain the total field due to multiple source charges

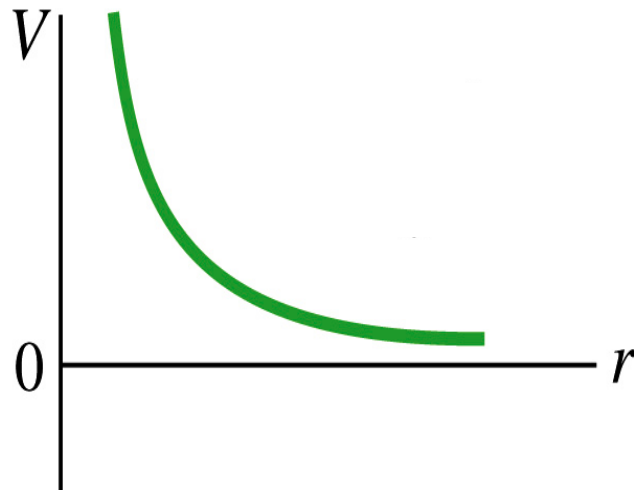
- Potential due to a positive charge is a large positive near the charge and decreases towards 0 at large distance.
- Potential due to a negative charge is a large negative near the charge and increases towards 0 at large distance.

Shape of the Electric Potential

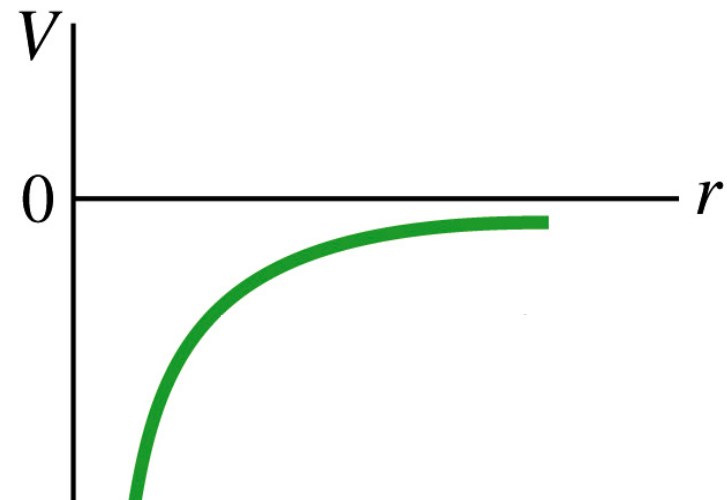
- So, what does the electric potential look like as a function of distance?
 - What is the formula for the potential by a single charge again?

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

Positive Charge



Negative Charge



Uniformly charged sphere would have the potential the same as a single point charge.

What does this mean?

Uniformly charged sphere behaves like all the charge is on the single point in the center.

Example 23 – 6

Work to bring two positive charges close together: What is minimum work required by an external force to bring the charge $q=3.00\mu\text{C}$ from a great distance away ($r=\infty$) to a point 0.500m from a charge $Q=20.0\mu\text{C}$?

What is the work done by the electric field in terms of potential energy and potential?

$$W = -qV_{ba} = -\frac{q}{4\pi\epsilon_0} \left(\frac{Q}{r_b} - \frac{Q}{r_a} \right)$$

Since $r_b = 0.500\text{m}$, $r_a = \infty$ we obtain

$$W = -\frac{q}{4\pi\epsilon_0} \left(\frac{Q}{r_b} - 0 \right) = -\frac{q}{4\pi\epsilon_0} \frac{Q}{r_b} = -\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \cdot (3.00 \times 10^{-6} \text{ C}) (20.00 \times 10^{-6} \text{ C})}{0.500\text{m}} = -1.08\text{J}$$

Electric force does negative work. In other words, an external force must do +1.08J of work to bring the $3.00\mu\text{C}$ charge from infinity to 0.500m to the charge $20.0\mu\text{C}$.



Electric Potential by Charge Distributions

- Let's consider a case of n individual point charges in a given space, and let $V=0$ at $r=\infty$.
- Then the potential V_{ia} due to the charge Q_i at point a , distance r_{ia} from Q_i is

$$V_{ia} = \frac{Q_i}{4\pi\epsilon_0} \frac{1}{r_{ia}}$$

- Thus the total potential V_a by all n point charges is

$$V_a = \sum_{i=1}^n V_{ia} = \sum_{i=1}^n \frac{Q_i}{4\pi\epsilon_0} \frac{1}{r_{ia}}$$

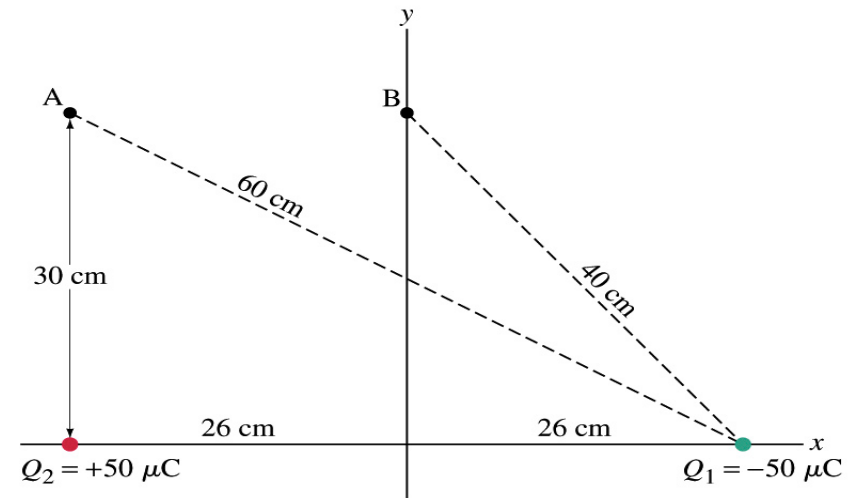
- For a continuous charge distribution, we obtain

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$



Example

- **Potential due to two charges:**
Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.
- Electric potential is a scalar quantity, so one adds the potential by each of the source charge, as if they are numbers.



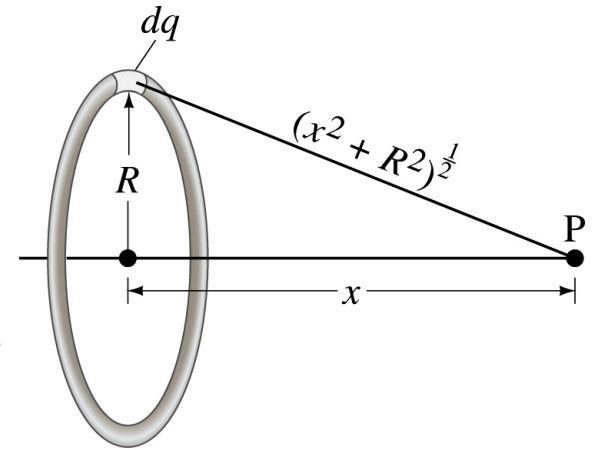
(a) potential at A is

$$V_A = V_{1A} + V_{2A} = \sum \frac{Q_i}{4\pi\epsilon_0 r_{iA}} =$$
$$= \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_{1A}} + \frac{1}{4\pi\epsilon_0} \frac{Q_2}{r_{2A}} = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{r_{1A}} + \frac{Q_2}{r_{2A}} \right)$$
$$= 9.0 \times 10^9 \left(\frac{-50 \times 10^{-6}}{0.60} + \frac{50 \times 10^{-6}}{0.30} \right) = 7.5 \times 10^5 \text{ V}$$

(b) How about potential at B?

Example 23 – 8

- **Potential due to a ring of charge:** A thin circular ring of radius R carries a uniformly distributed charge Q . Determine the electric potential at a point P on the axis of the ring at distance x from its center.



- Each point on the ring is at the same distance from the point P . What is the distance?

$$r = \sqrt{R^2 + x^2}$$

- So the potential at P is

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0 r} \int dq =$$

$$\frac{1}{4\pi\epsilon_0 \sqrt{x^2 + R^2}} \int dq = \frac{Q}{4\pi\epsilon_0 \sqrt{x^2 + R^2}}$$

What's this?

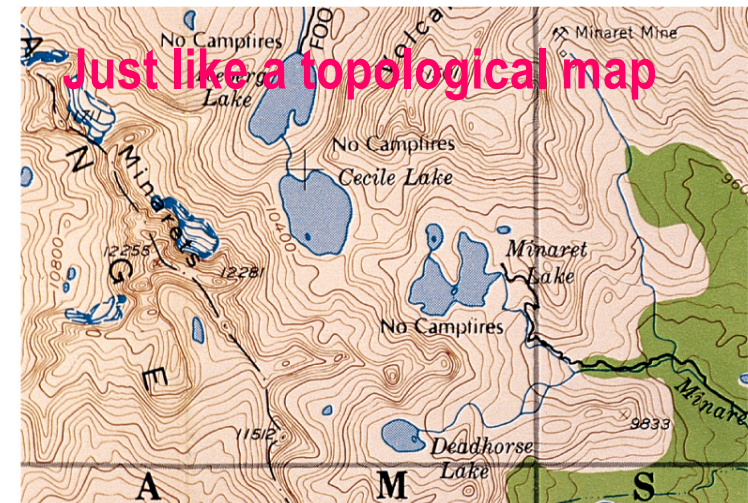
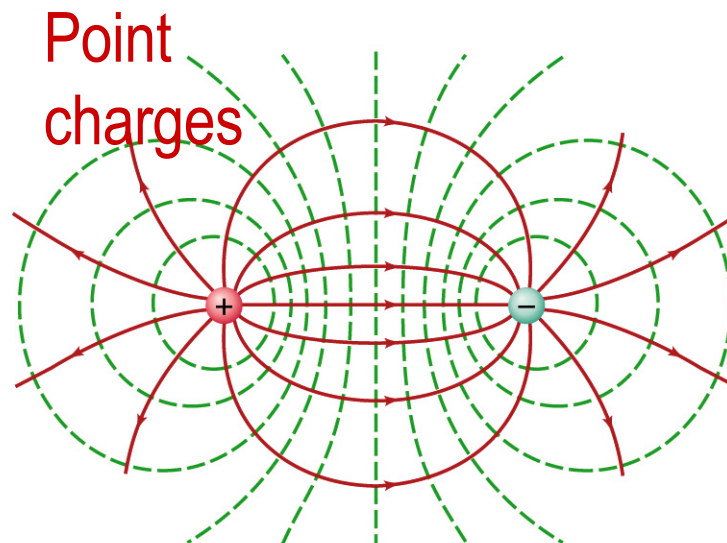
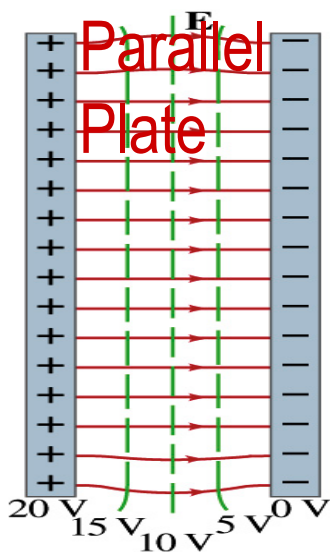
Equi-potential Surfaces

- Electric potential can be graphically shown using the equipotential lines in 2-D or the equipotential surfaces in 3-D
- Any two points on the equipotential surfaces (lines) are at the same potential
- What does this mean in terms of the potential difference?
 - The potential difference between any two points on an equipotential surface is 0.
- How about the potential energy difference?
 - Also 0.
- What does this mean in terms of the work to move a charge along the surface between these two points?
 - No work is necessary to move a charge between these two points.



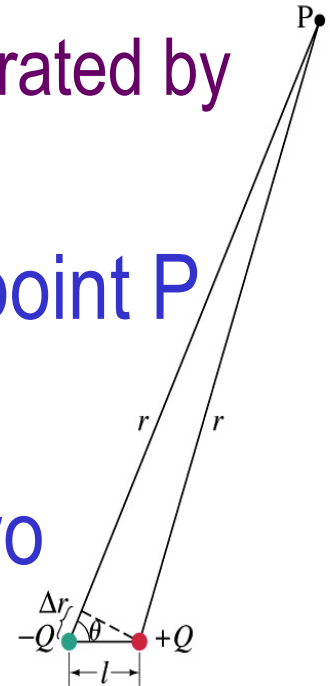
Equi-potential Surfaces

- An equipotential surface (line) must be perpendicular to the electric field. Why?
 - If there are any parallel components to the electric field, it would require work to move a charge along the surface.
- Since the equipotential surface (line) is perpendicular to the electric field, we can draw these surfaces or lines easily.
- Since there can be no electric field within a conductor in a static case, the entire volume of a solid conductor must be at the same potential.
- So the electric field must be perpendicular to the conductor surface.



Electric Potential due to Electric Dipoles

- What is an electric dipole?
 - Two equal point charge Q of opposite signs separated by a distance l and behaves like one entity: $p=Ql$
- For the electric potential due to a dipole at a point P
 - We take $V=0$ at $r=\infty$



- The simple sum of the potential at P by the two charges is

$$V = \sum \frac{Q_i}{4\pi\epsilon_0 r_{ia}} = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{r} + \frac{(-Q)}{r + \Delta r} \right) = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{r + \Delta r} \right) = \frac{Q}{4\pi\epsilon_0} \frac{\Delta r}{r(r + \Delta r)}$$

- Since $\Delta r = l \cos \theta$ and if $r \gg l$, $r \gg \Delta r$, thus $r \sim r + \Delta r$ and

$$V = \frac{Q}{4\pi\epsilon_0} \frac{l \cos \theta}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

V by a dipole at a distance r from the dipole

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

E Determined from V

- Potential difference between two points under an electric field is $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$

- So in a differential form, we can write

$$dV = -\vec{E} \cdot d\vec{l} = -E_l dl$$

– What are dV and E_l ?

- dV is the infinitesimal potential difference between the two points separated by a distance $d\ell$
 - E_l is the field component along the direction of $d\ell$
- Thus we can write the field component E_l as

$$E_l = -\frac{dV}{dl}$$

**Physical
Meaning?**

The component of the electric field in any direction is equal to the negative rate of change of the electric potential as a function of distance in that direction.!!



E Determined from V

- The quantity $dV/d\ell$ is called the **gradient of V** in a particular direction
 - If no direction is specified, the term gradient refers to the direction on which **V changes most rapidly** and this would be the direction of the field vector **E** at that point.
 - So if **E** and $d\ell$ are parallel to each other, $E = -\frac{dV}{d\ell}$
- If **E** is written as a function of x , y and z , the ℓ refers to x , y and z

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$
- $\frac{\partial V}{\partial x}$ is the “partial derivative” of V with respect to x , while y and z are held constant
- In vector form, $\vec{E} = -\text{grad}V = -\vec{\nabla}V = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)V$

$\vec{\nabla} = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)$ is called del or the gradient operator and is a vector operator.
 Wednesday, Sept. 25, 2019
 PHYS 1444-002, Fall 2019
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Electrostatic Potential Energy

- Consider a case in which a point charge q is moved between points a and b where the electrostatic potential due to other charges in the system is V_a and V_b
- The change in electrostatic potential energy of q in the field by other charges is

$$\Delta U = U_b - U_a = q(V_b - V_a) = qV_{ba}$$

- Now what is the electrostatic potential energy of a system of charges?
 - Let's choose $V=0$ at $r=\infty$
 - If there are no other charges around, single point charge Q_1 in isolation has no potential energy and is under no electric force



Electrostatic Potential Energy; Two charges

- If a second point charge Q_2 is brought close to Q_1 at a distance r_{12} , the potential due to Q_1 at the position of Q_2 is

$$V = \frac{Q_1}{4\pi\epsilon_0} \frac{1}{r_{12}}$$

- The potential energy of the two charges relative to $V=0$ at $r = \infty$ is

$$U = Q_2 V = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{12}}$$

- This is the work that needs to be done by an external force to bring Q_2 from infinity to the distance r_{12} from Q_1 .
- It is also a negative of the work needed to separate them to infinity.



Electrostatic Potential Energy; Three Charges

- So what do we do for three charges?
- Work is needed to bring all three charges together
 - Work needed to bring Q_1 to a certain location without the presence of any charge is 0.
 - Work needed to bring Q_2 to a distance to Q_1 is $U_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{12}}$
 - Work need to bring Q_3 to certain distances to Q_1 and Q_2 is

$$U_3 = U_{13} + U_{23} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0} \frac{Q_2 Q_3}{r_{23}}$$

- So the total electrostatic potential energy of the three charge system is

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}} \right) \left[V = 0 \text{ at } r = \infty \right]$$

- What about a four charge system or N charge system?



Electrostatic Potential Energy: electron Volt

- What is the unit of electrostatic potential energy?
 - Joules
- Joules is a very large unit in dealing with electrons, atoms or molecules atomic scale problems
- For convenience a new unit, electron volt (eV), is defined
 - 1 eV is defined as the energy acquired by a particle carrying the magnitude of the charge equal to that of an electron ($q=e$) when it moves across a potential difference of 1V.
 - How many Joules is 1 eV then? $1eV = 1.6 \times 10^{-19} C \cdot 1V = 1.6 \times 10^{-19} J$
- eV however is **NOT a standard SI unit**. You must convert the energy to Joules for computations.
- What is the speed of an electron with kinetic energy 5000eV?

