PHYS 1444 – Section 002
Lecture #18

Wednesday, Nov. 6, 2019

Dr. Jaehoon Yu

- Chapter 28:Sources of Magnetic Field
  - Sources of Magnetic Field
  - Magnetic Field Due to Straight Wire
  - Forces Between Two Parallel Wires
  - Ampére's Law and Its Verification
  - Solenoid and Toroidal Magnetic Field
  - Biot-Savart Law

Announcements

- Reading Assignments: CH27.6, 27.8, 27.9 and 28.5 – 10
- 2nd non-comprehensive term exam: Nov. 11
  - Monday, Nov. 11 in class ➔ Show up by 12:45.
  - Covers: CH25.5 through CH28.5
  - Bring your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the quiz
  - No derivations, word definitions, set ups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
- Remember the triple extra credit colloquia
  - At 4pm next Wednesday, Nov. 13
  - Professor Hitoshi Murayama of U.C. Berkeley
- Planetarium Extra Credit: bring to class Mon. Dec. 2
  - Be sure to tape one end of the ticket stub on a sheet of paper with your name on it

Reminder: Special Project #5

- Make a list of the power consumption and the resistance of all electric and electronic devices at your home and compile them in a table. (10 points total for the first 10 items and 0.5 points each additional item.)
- Estimate the cost of electricity for each of the items on the table using your own electric cost per kWh (if you don’t find your own, use $0.12/kWh) and put them in the relevant column. (5 points total for the first 10 items and 0.2 points each additional items)
- Estimate the the total amount of energy in Joules and the total electricity cost per day, per month and per year for your home. (8 points)
- Spreadsheet: http://www-hep.uta.edu/%7Eyu/teaching/fall19-1444-002/sp5-spreadsheet.xlsx
- Due: Beginning of the class Monday, Nov. 11

Sources of Magnetic Field

- We have learned so far about the effects of magnetic field on electric current and a moving charge
- We will now learn about the dynamics of magnetism
  – How do we determine magnetic field strengths in certain situations?
  – How do two wires with electric current interact?
  – What is the general approach to finding the connection between the electric current and the magnetic field?

Magnetic Field due to a Straight Wire

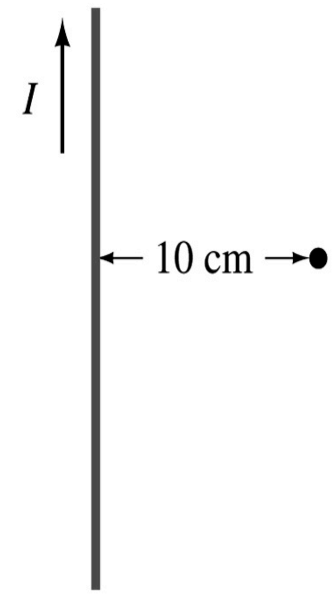
- The magnetic field due to the current flowing through a straight wire forms a circular pattern around the wire
  - What do you imagine the strength of the field is as a function of the distance from the wire?
    - It must be weaker as the distance increases
  - How about as a function of the electric current?
    - Directly proportional to the current
  - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
    - This is valid as long as r << the length of the wire
  - The proportionality constant is $\mu_0/2\pi$, thus the field strength becomes

$$B = \frac{\mu_0 I}{2\pi r}$$

  - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$

Merriam-Webster: Permeability is the property of a magnetizable substance that determines the degree in which it modifies the magnetic flux in the region occupied by it in a magnetic field

Example 28 – 1



Calculation of B near wire. A vertical electric wire in the wall of a building carries a DC current of 25A upward. What is the magnetic field at a point 10cm due East of this wire?

Using the formula for the magnetic field near a straight wire

$$B = \frac{\mu_0 I}{2\pi r}$$

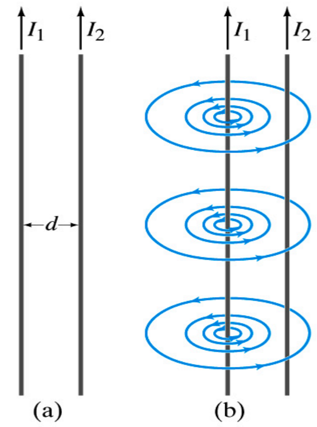
So we can obtain the magnitude of the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ T \cdot m/A\right) \cdot (25A)}{(2\pi) \cdot (0.01m)} = 5.0 \times 10^{-5}\ T$$

What direction? Into the page!

Force Between Two Parallel Wires

- We have learned that a wire carrying electric current produces magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
  - They will exert force onto each other. Repel or attract?
  - Depending on the direction of the currents

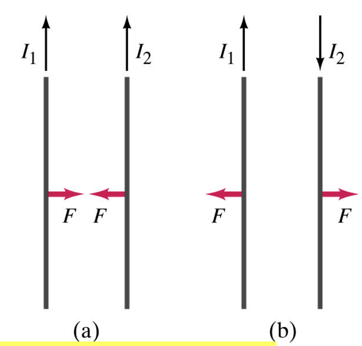


- This was first pointed out by Ampére.
- Let's consider two long parallel conductors separated by a distance **d**, carrying currents I_1 and I_2.
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force Between Two Parallel Wires

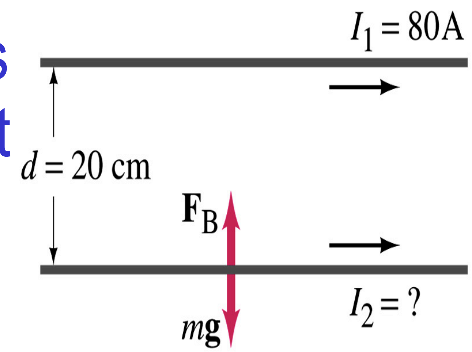
- The force **F** by a magnetic field **B_1** on a wire of length l, carrying the current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
  - So the force per unit length is $\frac{F}{l} = I_2 B_1 = I_2 \frac{\mu_0}{2\pi}\frac{I_1}{d}$
  - This force is only due to the magnetic field generated by the wire carrying the current I_1
    - There is the force exerted on the wire carrying the current I_1 by the wire carrying current I_2 of the same magnitude but in the opposite direction
- So the force per unit length is $$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$
- How about the direction of the force?



If the currents are in the same direction, the attractive force. If opposite, repulsive.

Example 28 – 5

Suspending a wire with current. A horizontal wire carries a current I_1=80A DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity and in which direction? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Down to the center of the Earth

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l}\frac{2\pi d}{\mu_0 I_1} =$$

$$\frac{2\pi\left(9.8\, m/s^2\right)\cdot\left(0.12\times10^{-3}\, kg\right)\cdot\left(0.20m\right)}{\left(4\pi\times10^{-7}\, T\cdot m/A\right)\cdot\left(80A\right)} = 15A$$

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\ T\cdot m/A$$

- The unit of current, ampere, is defined using the definition of the force between two wires each carrying **1A** of current and separated by **1m**

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{4\pi\times 10^{-7}\ T\cdot m/A}{2\pi}\frac{1A\cdot 1A}{1m} = 2\times 10^{-7}\ N/m$$

  - So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly $2\text{x}10^{-7}$N/m.

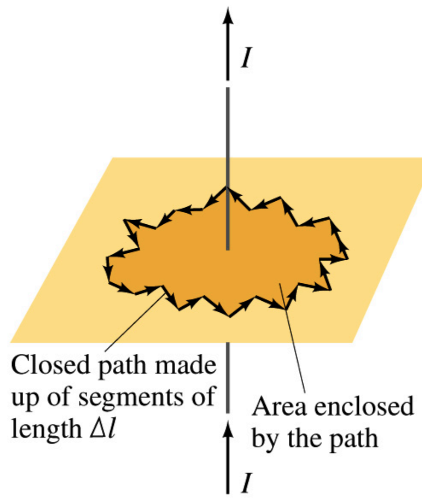
- Coulomb is then defined as exactly 1C=1A s.
- We do it this way since the electric current is measured more accurately and controlled more easily than the charge.

Ampére's Law

- What is the relationship between the magnetic field strength and the current?
$$B = \frac{\mu_0 I}{2\pi r}$$
  - Does this work in all cases?
    - Nope!
    - OK, then when?
    - Only valid for a long straight wire
- Then what would be the more generalized relationship between the current and the magnetic field for any shapes of the wire?
  - French scientist André Marie Ampére proposed such a relationship soon after Oersted's discovery

Ampére's Law

- Let's consider an arbitrary closed path around the current as shown in the figure.



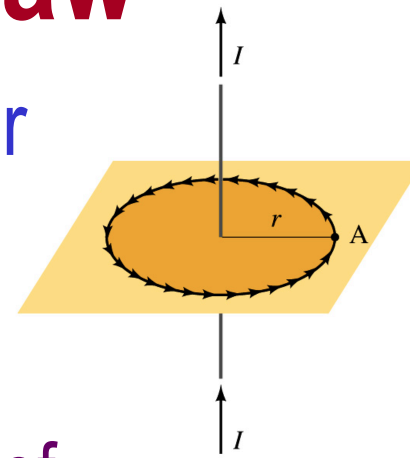
  - Let's cut this path in small segments, each Δl long.
  - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path
  - $$\sum B_{\parallel} \Delta l = \mu_0 I_{encl}$$
  - In the limit $\Delta l \rightarrow 0$, this relation becomes
  - $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

Ampére's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampére's Law



- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
  - This is a verification of Ampere's Law
  - We can apply Ampere's law to a circular path of radius r.

$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{l} = \oint B\,dl = B\oint dl = 2\pi r B$$

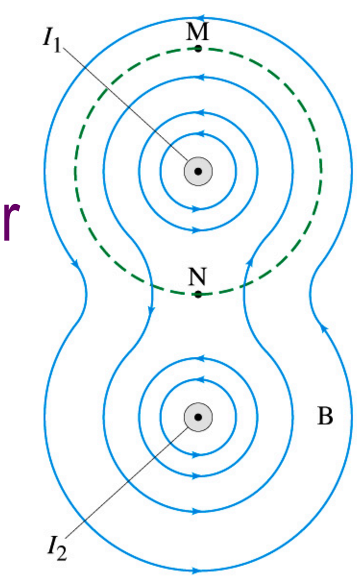
Solving for B

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi}\frac{I}{r}$$

  - We just verified that Ampere's law works in a simple case
  - Experiments verified that it works for other cases too
  - The importance of this formula, however, is that it provides means to relate magnetic field to an electric current

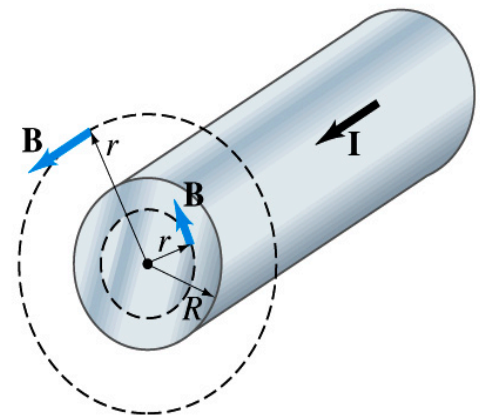
Verification of Ampére's Law

- Since Ampere's law is valid in general, B in Ampere's law is not just due to the current I_{encl}.
- B is the field at each point in space along the chosen path due to all sources
  - Including the current I enclosed by the path but also due to any other sources
  - How do you obtain B in the figure at any point?
    - Vector sum of the field by the two currents
  - The result of the closed path integral in Ampere's law for green dashed path is still $\mu_0 I_1$. Why?
  - While B in each point along the path varies, the integral over the closed path still comes out the same whether there is the second wire or not.



Example 28 – 6

Field inside and outside a wire. A long straight cylindrical wire conductor of radius R carries current I of uniform density in the conductor. Determine the magnetic field at (a) points outside the conductor (r>R) and (b) points inside the conductor (r<R). Assume that r, the radial distance from the axis, is much less than the length of the wire. (c) If R=2.0mm and I=60A, what is B at r=1.0mm, r=2.0mm and r=3.0mm?



Since the wire is long, straight and symmetric, the field should be the same at any point the same distance from the center of the wire.

Since B must be tangential to circles around the wire, let's choose a circular path of the closed-path integral outside the wire (r>R). What is I_{encl}? $I_{encl} = I$

So using Ampere's law

$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = 2\pi r B$$

Solving for B

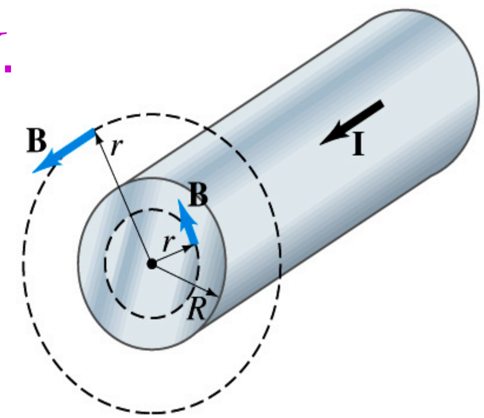
$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

Example 28 – 6 cont'd

For r<R, the current inside the closed path is less than I.

How much is it?

$$I_{encl} = I\frac{\pi r^2}{\pi R^2} = I\left(\frac{r}{R}\right)^2$$



So using Ampere's law

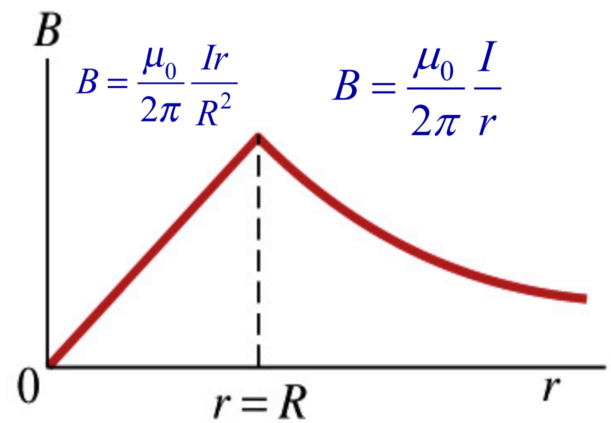
$$\mu_0 I\left(\frac{r}{R}\right)^2 = \oint \vec{B}\cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}\left(\frac{r}{R}\right)^2 = \frac{\mu_0}{2\pi}\frac{Ir}{R^2}$$

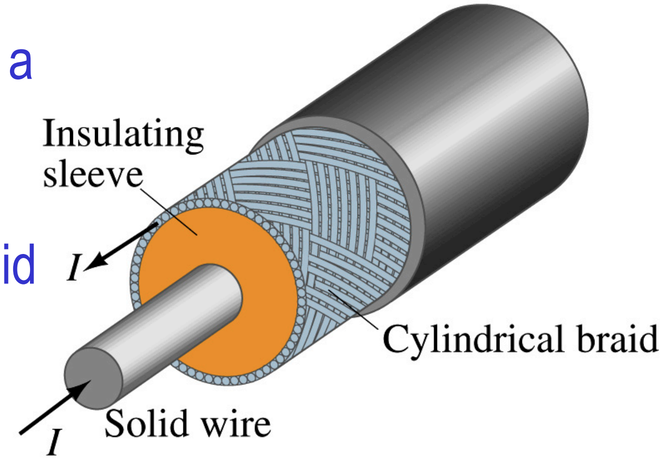
What does this mean?

The field is 0 at r=0 and increases linearly as a function of the distance from the center of the wire up to r=R then decreases as 1/r beyond the radius of the conductor.



Example 28 – 7

Coaxial cable. A coaxial cable is a single wire surrounded by a cylindrical metallic braid, as shown in the figure. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors and (b) outside the cable.



(a) The magnetic field between the conductors is the same as the long, straight wire case since the current in the outer conductor does not impact the enclosed current.

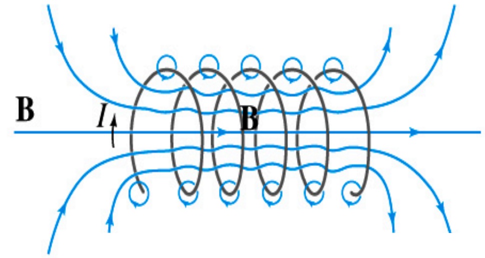
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

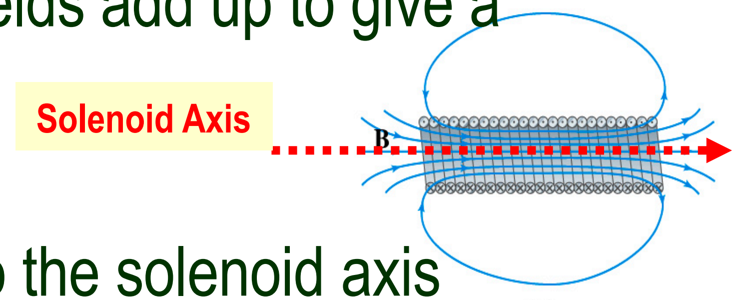
(b) Outside the cable, we can draw a similar circular path, since we expect the field to have a circular symmetry. What is the sum of the total current inside the closed path? $I_{encl} = I - I = 0.$

So there is no magnetic field outside a coaxial cable. In other words, the coaxial cable self-shields. The outer conductor also shields against an external electric field. Cleaner signal and less noise.

Solenoid and Its Magnetic Field

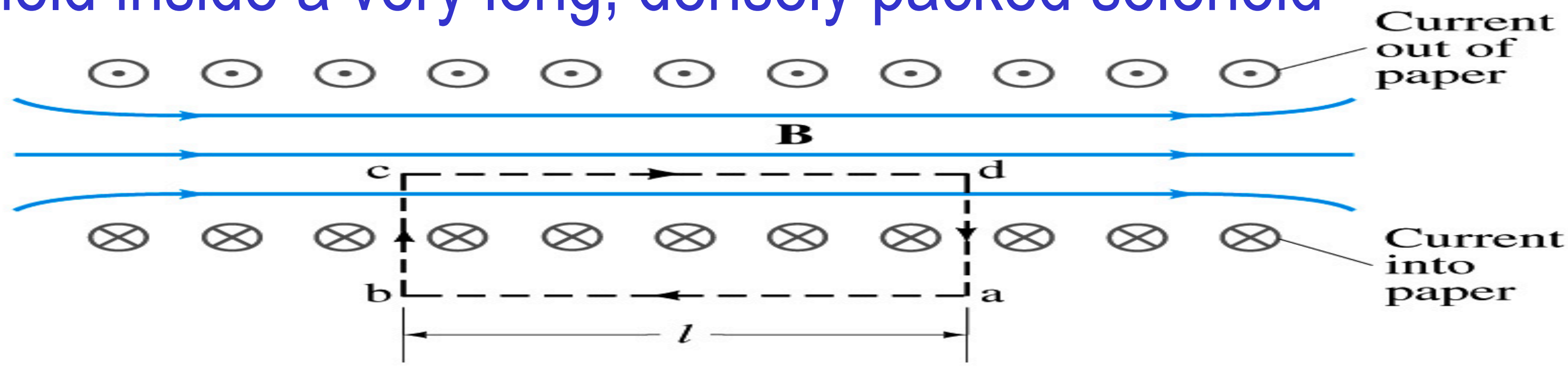
- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops are wide
    - The field near the wires are nearly circular
    - Between any two wires, the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform
  - For a long, densely packed loops
    - The field is nearly uniform and parallel to the solenoid axis within the entire cross section
    - The field outside the solenoid is very small compared to the field inside, except at the ends
      - The same number of field lines spread out to an open space





Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid



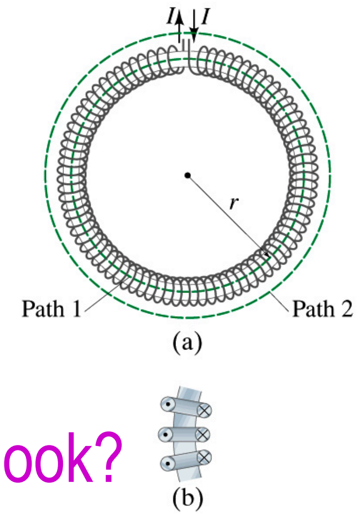
- Let's choose the path *abcd*, far away from the ends
  - We can consider four segments of the loop for integral
  - $\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$
  - Since the field outside the solenoid is negligible, the integral on $a \rightarrow b$ is 0.
  - Now the field B is perpendicular to the *bc* and *da* segments. So these integrals become 0, also.

Solenoid Magnetic Field

- Therefore, the sum becomes: $\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$
- If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$
  - Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed
- Thus Ampere's law gives us $Bl = \mu_0 NI$
- If we let $n=\mathcal{N}/\ell$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes
- $$B = \mu_0 nI$$
  - B depends on the number of loops per unit length, n, and the current I
    - B does not depend on the position within the solenoid but uniform inside it, like a bar magnet

Example 28 – 10

Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle.



(a) How do you think the magnetic field lines inside the toroid look?

Since it is a bent solenoid, it should be a circle concentric with the toroid. If we choose path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path, we obtain from Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 NI$$

Solving for B

$$B = \frac{\mu_0 NI}{2\pi r}$$

So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin. The field in this case is B = $\mu_0 n I$.

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.