PHYS 1444 – Section 002
Lecture #22

Wednesday, Nov. 20, 2019

Dr. Jaehoon Yu

- Chapter 29:EM Induction & Faraday's Law
  - Electric Field Due to Changing Magnetic Flux
- Chapter 30: Inductance
  - Inductance
  - Mutual and Self Inductance
  - Energy Stored in the Magnetic Field
  - LR Circuit
  - LC circuit and EM Oscillation

Announcements

- Reading Assignments: 28.6 – 10, CH29.5 and 29.8
- Final comprehensive: in class 1:00 – 2:20pm Wed. Dec. 4
- Planetarium Extra Credit: bring to class Mon. Dec. 2
  - Be sure to tape one end of the ticket stub on a sheet of paper with your name on it
- Quiz #4
  - Beginning of the class Monday, Nov. 25
  - Covers: CH28.6 – what we finish this today (CH30.4?)
  - Bring your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the quiz
  - No derivations, word definitions, set ups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
- No class Wednesday, Nov. 27

Electric Field due to Magnetic Flux Change

- When the electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
  - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This result applies not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- Recall the relationship between the electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field
- $\varepsilon = \int_a^b \vec{E} \cdot d\vec{l}$
- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

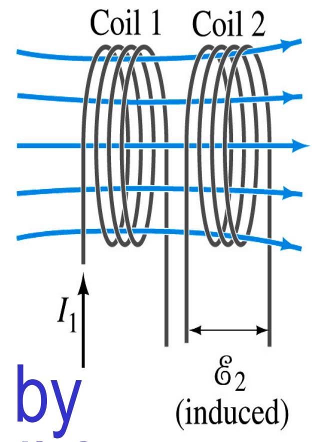
- The integral is taken around the path enclosing the area through which the magnetic flux Φ_B is changing.

Inductance

- A changing magnetic flux through a circuit induces an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - Or induce an emf in itself ➔ Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2, in coil 2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil 2 created by coil1 and N_2 is the number of closely packed loops in coil 2, then $N_2\Phi_{21}$ is the total flux passing through coil 2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21} I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined as $\boxed{M_{21} = N_2\Phi_{21}/I_1}$.
- The emf induced in coil 2 due to the changing current in coil 1 is



$$\boxed{\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d\left(N_2\Phi_{21}\right)}{dt} = -M_{21}\frac{dI_1}{dt}}$$

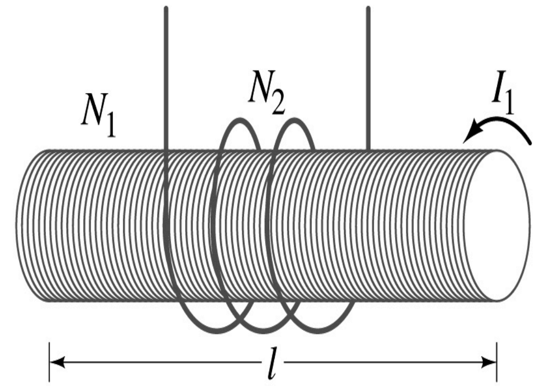
Mutual Inductance

- The mutual induction of coil 2 with respect to coil 1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

    What? Does this make sense?

    - The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
  - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil 2 will induce an emf in coil 1
- $$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain $$\varepsilon_1 = -M\frac{dI_2}{dt} \text{ and } \varepsilon_2 = -M\frac{dI_1}{dt}$$
  - SI unit for mutual inductance is Henry (H) $$1H = 1V\cdot s/A = 1\Omega\cdot s$$

Example 30 – 1



Solenoid and coil. A long thin solenoid of length l and cross-sectional area **A** contains $\mathbf{N_1}$ closely packed turns of wire. Wrapped around it is an insulated coil of $\mathbf{N_2}$ turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.

First, we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $B = \dfrac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil 2 is

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this?
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to **impedes its increase**, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = L\,I$
- We define self-inductance, $\mathcal{L}$: $\boxed{L = \dfrac{N\Phi_B}{I}}$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\dfrac{d\Phi_B}{dt} = -L\dfrac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V \cdot s/A = 1\Omega \cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

- It is an **impediment** onto the electrical current due to the existence of changing magnetic flux
- So what?
- In other words, it **behaves like a resistance** to the varying current, such as AC, that causes the constant change of magnetic flux
- But it also provides means to store energy, just like the capacitance

Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  - Precision resistors are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a "non-inductive winding"
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

Example 30 – 3

Solenoid inductance. (a) Determine the formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing **N** turns of wire in its length ℓ and whose cross-sectional area is **A**. (b) Calculate the value of $\mathcal{L}$ if **N=100**, **ℓ=5.0cm**, **A=0.30cm²** and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with **μ=4000μ_0**.

What is the magnetic field inside a solenoid? $B = \mu_0 nI = \mu_0 NI/l$

The flux is, therefore, $\Phi_B = BA = \mu_0 NIA/l$

Using the formula for self inductance: $L = \dfrac{N\Phi_B}{I} = \dfrac{N \cdot \mu_0 NIA/l}{I} = \dfrac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\left(4\pi \times 10^{-7}\, T\cdot m/A\right)100^2\left(0.30\times 10^{-4}\, m^2\right)}{5.0\times 10^{-2}\, m} = 7.5\mu H$$

(c) The magnetic field with an iron core solenoid is $B = \mu NI/l$

$$L = \frac{\mu N^2 A}{l} = \frac{4000\left(4\pi \times 10^{-7}\, T\cdot m/A\right)100^2\left(0.30\times 10^{-4}\, m^2\right)}{5.0\times 10^{-2}\, m} = 0.030H = 30mH$$

Energy Stored in the Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $$\boxed{U = \frac{1}{2} L I^2}$$

    Energy Stored in a magnetic field inside an inductor

  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V: $U = \frac{1}{2} C V^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without the fringe effect

$$L = \mu_0 N^2 A/l$$

  - The magnetic field in a solenoid is $B = \mu_0 NI/l$
  - Thus the energy stored in an inductor is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0}Al$$

What is this?

$$\boxed{U = \frac{1}{2}\frac{B^2}{\mu_0}Al}$$

E

  - Thus the energy density is

$$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2}\frac{B^2}{\mu_0}$$

Volume V

$$\boxed{u = \frac{1}{2}\frac{B^2}{\mu_0}}$$

E density

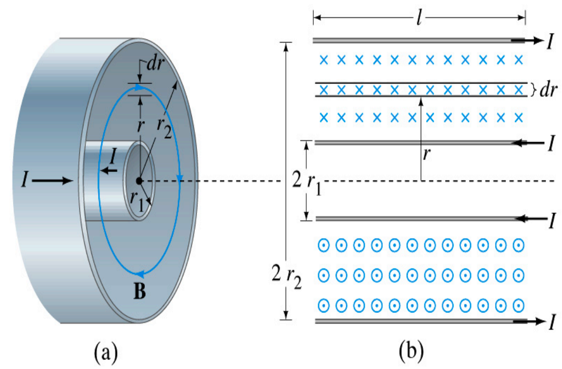
  - This formula is valid in any region of space
  - If a ferromagnetic material is present, μ_0 becomes μ.

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I? (b) Where is the energy density highest?



(a) The total flux through l of the cable is $\Phi_B = \int Bl\,dr = \frac{\mu_0 Il}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\mu_0 Il}{2\pi}\ln\frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}$

Thus the energy stored per unit length is $\frac{U}{l} = \frac{1}{2}\frac{LI^2}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{r_2}{r_1}$

(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2}\frac{B^2}{\mu_0}$

The energy density is highest where B is highest. Since B is highest close to $r=r_1$, near the surface of the inner conductor.