PHYS 1444 – Section 002
Lecture #23

Monday, Nov. 25, 2019

Dr. Jaehoon Yu

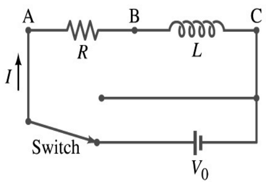
- Chapter 30: Inductance
  - LR Circuit
  - LC circuit and EM Oscillation
  - AC circuit with Inductor Only & Capacitor Only
- Chapter 31: Maxwell's equations
  - Expansion of Ampere's Law
  - Gauss' Law for Magnetism
  - Production of EM Waves
  - Light as EM Waves

Announcements

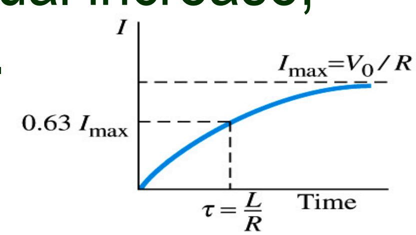
- Reading assignments: CH30.7 – 30.11
- Planetarium Extra Credit: bring to class Mon. Dec. 2
  - Be sure to tape one end of the ticket stub on a sheet of paper with your name on it
- Final comprehensive exam
  - In class 1:00 – 2:20pm, Wednesday, Dec. 4
  - Covers: CH21.1 – what we finish Monday, Dec. 2
  - Bring your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the quiz
  - No derivations, word definitions, set ups or solutions of any problems!
  - NO MAXWELL's Equations!!
  - No additional formulae or values of constants will be provided!
- No class this Wednesday, Nov. 27
  - Happy Thanksgiving!
- Course evaluation survey!!

LR Circuits

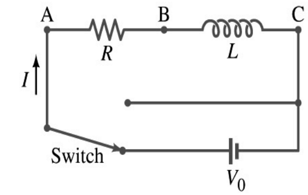
- What happens when an emf is applied to an inductor?
  - An inductor has some resistance, however negligible



    - So an inductor can be drawn as a circuit of separate resistance and coil. What is the name this kind of circuit? **LR Circuit**
  - What happens at the instance the switch is thrown to apply emf to the circuit?
    - The current starts to flow, gradually increasing from 0
    - This change is opposed by the induced emf in the inductor ➔ the emf at point B is higher than point C
    - However there is a voltage drop at the resistance which reduces the voltage across inductance
    - Thus the current increases less rapidly
    - The overall behavior of the current is a gradual increase, reaching to the maximum current $I_{max}=V_0/R$.

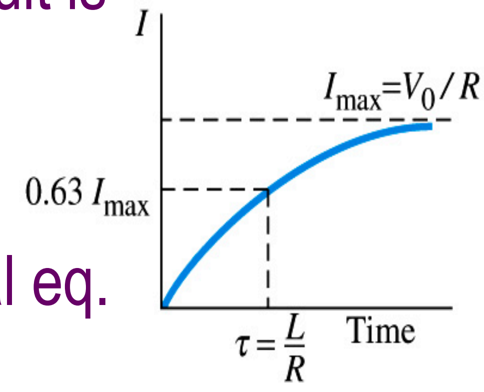


LR Circuits



- This can be shown w/ Kirchhoff loop rules
  - The emfs in the circuit are the battery voltage V_0 and the emf $\varepsilon=-\mathcal{L}(dI/dt)$ in the inductor opposing the current increase
  - The sum of the potential changes through the circuit is

$$V_0 + \varepsilon - IR = V_0 - L\,dI/dt - IR = 0$$

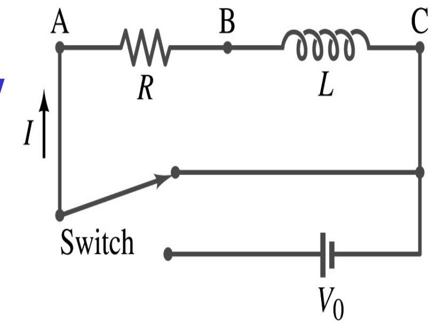


  - Where I is the current at any instance
  - By rearranging the terms, we obtain the differential eq.
  - $L\,dI/dt + IR = V_0$
  - We can integrate just as in RC circuit $\displaystyle\int_{I=0}^{I} \frac{dI}{V_0 - IR} = \int_{t=0}^{t} \frac{dt}{L}$
  - So the solution is $\displaystyle -\frac{1}{R}\ln\left(\frac{V_0 - IR}{V_0}\right) = \frac{t}{L}$

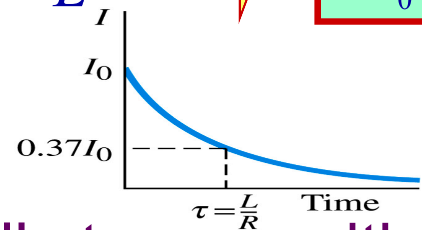
$$I = V_0\left(1 - e^{-t/\tau}\right)/R = I_{max}\left(1 - e^{-t/\tau}\right)$$

  - Where τ=L/R
    - This is the time constant τ of the LR circuit and is the time required for the current I to reach 0.63 of the maximum

Discharge of LR Circuits

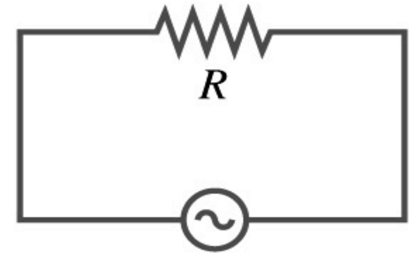


- If the switch is flipped away from the battery
  - The differential equation becomes
  - $L\, dI/dt + IR = 0$
  - So the integration is $\int_{I_0}^{I} \frac{dI}{IR} = -\int_{t=0}^{t} \frac{dt}{L}$ ⇒ $\ln \frac{I}{I_0} = -\frac{R}{L} t$
  - Which results in the solution
  - $I = I_0 e^{-\frac{R}{L}t} = I_0 e^{-t/\tau}$



  - The current decays exponentially to zero with the time constant τ=L/R
  - So there always is a reaction time when a system with a coil, such as an electromagnet, is turned on or off.
  - The current in LR circuit behaves almost the same as that in RC circuit but the time constant is inversely proportional to R in LR circuit unlike the RC circuit

AC Circuit w/ Resistance only



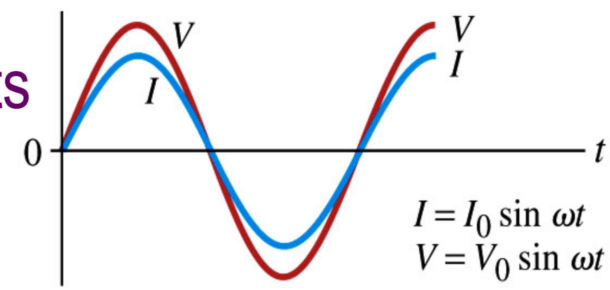
- What do you think will happen when an AC source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain

$$V - IR = 0$$

- Thus

$$V = I_0 R \sin \varpi t = V_0 \sin \varpi t$$

  - where $V_0 = I_0 R$
- What does this mean?
  - Current is 0 when voltage is 0 and current is in its peak when voltage is in its peak.
  - Current and voltage are "in phase"

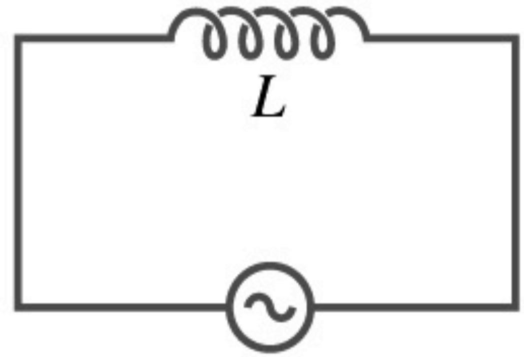


- Energy is lost via the transformation into heat at an average rate $\bar{P} = \bar{I}\,\bar{V} = I_{rms}^2 R = V_{rms}^2 / R$

AC Circuit w/ Inductance only

- From Kirchhoff's loop rule, we obtain

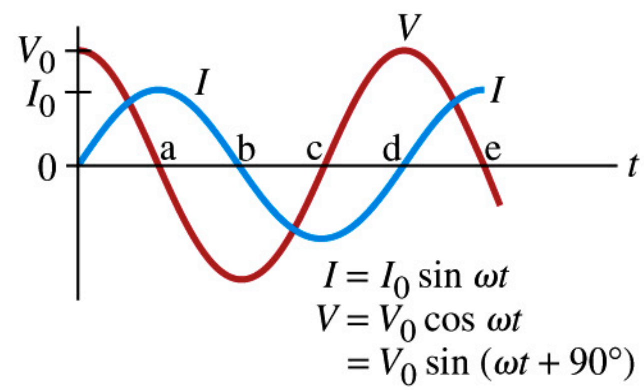
$$V - L\frac{dI}{dt} = 0$$



- Thus

$$V = L\frac{dI}{dt} = L\frac{d\left(I_0 \sin \varpi t\right)}{dt} = \varpi L I_0 \cos \varpi t$$

  – Using the identity $\cos\theta = \sin\left(\theta + 90^\circ\right)$
- $V = \varpi L I_0 \sin\left(\varpi t + 90^\circ\right) = V_0 \sin\left(\varpi t + 90^\circ\right)$
  – where $V_0 = \varpi L I_0$
- What does this mean?
  – Current and voltage are "out of phase by π/2 or 90°". In other words the current reaches its peak ¼ cycle after the voltage
- What happens to the energy?
  – No energy is dissipated
  – The average power is 0 at all times
  – The energy is stored temporarily in the magnetic field
  – Then released back to the source



AC Circuit w/ Inductance only

- How are the resistor and inductor different in terms of energy?
  - Inductor — Stores the energy temporarily in the magnetic field and then releases it back to the emf source
  - Resistor — Does not store energy but transforms it to thermal energy, losing it to the environment
- How are they similar?
  - They both impede the flow of charge
  - For a resistance R, the peak voltage and current are related to $V_0 = I_0 R$
  - Similarly, for an inductor we may write $V_0 = I_0 X_L$
    - Where X_L is the inductive reactance of the inductor $X_L = \varpi L$ — **0 when ω=0.**
    - What do you think is the unit of the reactance? Ω
    - The relationship $V_0 = I_0 X_L$ is not valid at a particular instance. Why not?
      - Since V_0 and I_0 do not occur at the same time

$V_{rms} = I_{rms} X_L$ is valid!

Example 30 – 9

Reactance of a coil. A coil has a resistance R=1.00Ω and an inductance of 0.300H. Determine the current in the coil if (a) 120 V DC is applied to it; (b) 120 V AC (rms) at 60.0Hz is applied.

Is there a reactance for DC? Nope. Why not? Since ω=0, $X_L = \varpi L = 0$

So for DC power, the current is from Kirchhoff's rule $V - IR = 0$

$$I_0 = \frac{V_0}{R} = \frac{120V}{1.00\Omega} = 120A$$

For an AC power with f=60Hz, the reactance is

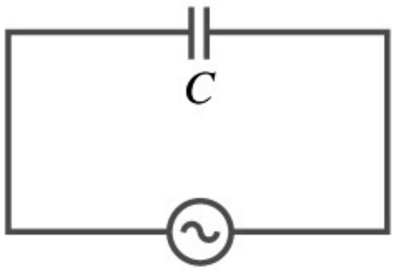
$$X_L = \varpi L = 2\pi fL = 2\pi \cdot \left(60.0s^{-1}\right) \cdot 0.300H = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is

$$I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120V}{113\Omega} = 1.06A$$

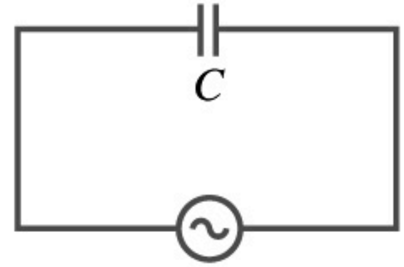
AC Circuit w/ Capacitance only

- What happens when a capacitor is connected to a DC power source?
  - The capacitor quickly charges up.
  - There is no steady current flow in the circuit
    - Since the capacitor prevents the flow of the DC current



- What do you think will happen if it is connected to an AC power source?
  - The current flows continuously. Why?
  - When the AC power turns on, charge begins to flow one direction, charging up the plates
  - When the direction of the power reverses, the charge flows in the opposite direction

AC Circuit w/ Capacitance only



- From Kirchhoff's loop rule, we obtain

$$V = \frac{Q}{C}$$

- The current at any instance is $I = \frac{dQ}{dt} = I_0 \sin \varpi t$

- The charge Q on the plate at any instance is

$$Q = \int_{Q=0}^{Q} dQ = \int_{t=0}^{t} I_0 \sin \varpi t dt = -\frac{I_0}{\varpi} \cos \varpi t$$

- Thus the voltage across the capacitor is

$$V = \frac{Q}{C} = -I_0 \frac{1}{\varpi C} \cos \varpi t$$

  – Using the identity $\cos\theta = -\sin\left(\theta - 90^\circ\right)$

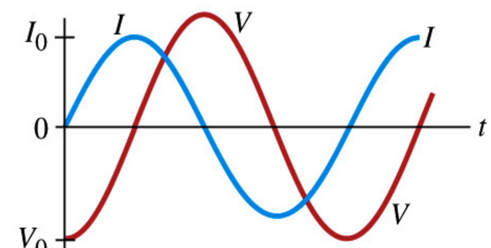
$$V = I_0 \frac{1}{\varpi C} \sin\left(\varpi t - 90^\circ\right) = V_0 \sin\left(\varpi t - 90^\circ\right)$$

  – Where

  – $V_0 = \frac{I_0}{\varpi C}$

AC Circuit w/ Capacitance only

- So the voltage is $V = V_0 \sin\left(\varpi t - 90^\circ\right)$
- What does this mean?
  - Current and voltage are "out of phase by π/2 or 90°" but in this case, the voltage reaches its peak ¼ cycle after the current
- What happens to the energy?
  - No energy is dissipated
  - The average power is 0 at all times
  - The energy is stored temporarily in the electric field
  - Then released back to the source



$I = I_0 \sin \omega t$
$V = -V_0 \cos \omega t$
$= V_0 \sin(\omega t - 90^\circ)$

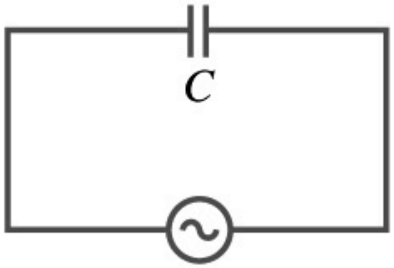
- Applied voltage and the current in the capacitor can be written as $V_0 = I_0 X_C$
  - Where the capacitive reactance X_C is defined as $X_C = \frac{1}{\varpi C}$
  - Again, this relationship is only valid for rms quantities

Infinite when ω=0.

$V_{rms} = I_{rms} X_C$

Example 30 – 10

Capacitor reactance. What are the peak and rms current in the circuit in the figure if C=1.0μF and V_{rms}=120V? Calculate for (a) f=60Hz, and then for (b) f=6.0x10^{5}Hz.



The peak voltage is $V_0 = \sqrt{2}V_{rms} = 120V \cdot \sqrt{2} = 170V$

The capacitance reactance is

$$X_C = \frac{1}{\varpi C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \cdot \left(60s^{-1}\right) \cdot 1.0 \times 10^{-6} F} = 2.7k\Omega$$

Thus the peak current is

$$I_0 = \frac{V_0}{X_C} = \frac{170V}{2.7k\Omega} = 63mA$$

The rms current is

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{120V}{2.7k\Omega} = 44mA$$