PHYS 1441 – Section 002
Lecture #11

Wednesday, Oct. 7, 2020

Dr. Jaehoon Yu

- CH24
  - Capacitors
  - Capacitors in Series and Parallel
  - Electric Energy Storage
  - Effect of Dielectric
  - Molecular description of Dielectric Material

Announcements

- Online Mid-term comprehensive exam on Quest
  - In class Monday, Oct. 19
  - Covers: CH21.1 through what we finish Wednesday, Oct. 12 + math refresher
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, setups or solutions of any problems, figures, pictures, diagrams or arrows, etc!
  - No additional formulae or values of constants will be provided!
  - Must send me the photos of front and back of the formula sheet, including the blank, no later than 11am the day of the exam
    - Once submitted, you cannot change, unless I ask you to delete some part of the sheet!
- Special seminar on COVID – 19: 3:30pm Sunday, Oct. 25
  - Dr. Linda Lee
  - Extra credit for participating in the seminar and
  - Asking a relevant question

Reminder: SP#3 – Civic Duty I: Voter Registration

- **Voter registration in Texas ends on Monday, Oct. 5, 2020**
  - Registration can be done: https://www.votetexas.gov/register/index.html
  - Check your registration: https://teamrv-mvp.sos.texas.gov/MVP/mvp.do
- For those who are legal to take part in the election
  - Your own registration to vote: 10 points
    - Include the screen shot your own voter registration check
  - You can have up to 3 more people who are not registered to register: 5 points each
    - Must include before and after the registration screen shots of the same person next to each other to show these are newly registered
- For those who are not legal to take part in the election
  - You can have up to 5 people who are not registered to register: 5 points each
    - Must include before and after the registration screen shots of the same person next to each other to show these are newly registered
- **Deadline: 1pm Wednesday, Oct. 14, 2020🡨 Note extended deadline**
- Put all screen shots in one pdf file following the naming convention – SP3-LastName-FirstName-Fall20.pdf and upload to the CANVAS assignment

SP#3 – Civic Duty I: Voter Registration – 2

TEXAS SECRETARY OF STATE

AM I REGISTERED?

TEXAS ELECTIONET ADMINISTRATION SYSTEM

Voter Information

Name: JAEHOON YU
[redacted]
Gender: MALE
Valid From: 01/01/2020
Effective Date of Registration: 05/20/2004
Voter Status: ACTIVE
County: TARRANT
Precinct: 2266
VUID: 1050748339

Change your Address

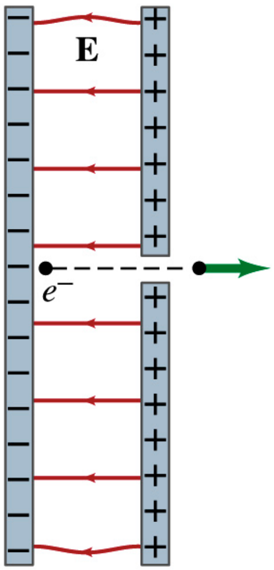
Upcoming Elections (Select Election for available polling information)

11/03/2020--2020 NOVEMBER 3RD GENERAL ELECTION

***Eligibility is determined by Effective Date of Registration (Must be on or before Election Day)

Reminder: Special Project #4

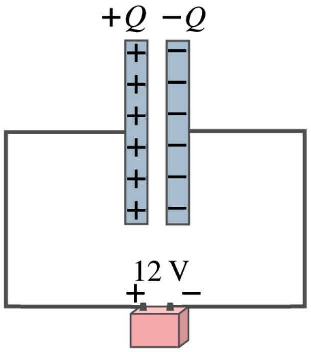
- **Particle Accelerator**. A charged particle of mass **M** with charge **-Q** is accelerated in the uniform field **E** between two parallel charged plates whose separation is **D** as shown in the figure on the right. The charged particle is accelerated from an initial speed $\mathbf{v_0}$ near the negative plate and passes through a tiny hole in the positive plate.



  – Derive the formula for the electric field E to accelerate the charged particle to a fraction *f* of the speed of light *c*. Express E in terms of **M, Q, D,** ***f*****, c** and $\mathbf{v_0}$.
  – (a) Using the Coulomb force and the kinematic equations. (8 points)
  – (b) Using the work-kinetic energy theorem. (8 points)
  – (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from the initial speed of 0.1% to 90% of the speed of light. You need to look up and write down the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)
- Must be handwritten and not copied from anyone else!
  – Follow the SP naming convention: SP4-first-last-fall20.pdf which includes all pages in one file ➔ Be sure to write your name onto all pages of the project report!
- Due beginning of the class **Monday, Oct. 12**, submitted on CANVAS!

Capacitors

- What do you think will happen if a battery is connected (or the voltage is applied) to a capacitor?
  - The capacitor gets charged quickly, one plate positive and the other negative in equal amount of charge.



- The battery terminals, the wires and the plates are conductors. What does this mean?
  - All conductors are at the same potential. And?
  - So the full battery voltage is applied across the capacitor plates.
- So for a given capacitor, the amount of charge stored on each capacitor plate is proportional to the potential difference V_{ba} between the plates. How would you write this formula?

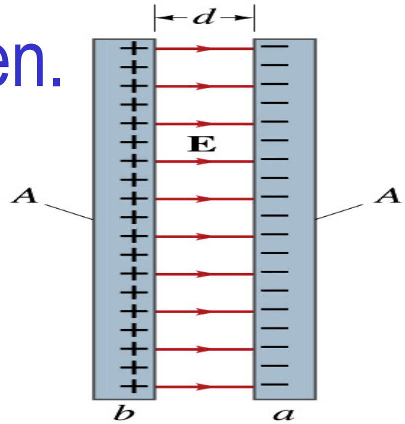
$$\boxed{Q = CV_{ba}}$$

C is the property of a capacitor so does not depend on Q or V.

  - C is a proportionality constant, called the capacitance of the device.
  - What is the unit (poll 3)? C/V or Farad (F) Normally use μF or pF.

Determination of Capacitance

- C, the capacitance can be determined analytically for a capacitor with a simple geometry and air in between.



- Let's consider a parallel plate capacitor.
  - Plates have area **A** each and separated by **d**.
    - d is smaller than the length, and so E is uniform.
  - E for parallel plates is $E=\sigma/\varepsilon_0$, $\sigma=Q/A$ is the surface charge density.
- E and V are related for uniform charge distribution $V_{ba}=-\int_a^b \vec{E}\cdot d\vec{l}$
- Since we take the integral from the lower potential (a) to the higher potential (b) along the field line, we obtain
- $V_{ba}=V_b-V_a=-\int_a^b E\,dl\cos 180^\circ=+\int_a^b E dl=\int_a^b \frac{\sigma}{\varepsilon_0}dl=\int_a^b \frac{Q}{\varepsilon_0 A}dl=\frac{Q}{\varepsilon_0 A}\int_a^b dl=\frac{Q}{\varepsilon_0 A}(b-a)=\frac{Qd}{\varepsilon_0 A}$
- So from the formula:
  - What do you notice?

$$\boxed{C=\frac{Q}{V_{ba}}=\frac{Q}{Qd/\varepsilon_0 A}=\frac{\varepsilon_0 A}{d}}$$

C only depends on the area and the distance of the plates and the permittivity of the medium between them.

Example 24 – 1

Capacitor calculations: (a) Calculate the capacitance of a capacitor whose plates are 20cmx3.0cm and are separated by a 1.0mm air gap. (b) What is the charge on each plate if the capacitor is connected to a 12-V battery? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1F, given the same air gap.

(a) Using the formula for a parallel plate capacitor, we obtain

$$C = \frac{\varepsilon_0 A}{d} =$$

$$= \left(8.85\times10^{-12}\, C^2/N\cdot m^2\right)\frac{0.2\times0.03m^2}{1\times10^{-3}\,m} = 53\times10^{-12}\, C^2/N\cdot m = 53\,pF$$

(b) From Q=CV, the charge on each plate is

$$Q = CV = \left(53\times10^{-12}\, C^2/N\cdot m\right)\left(12V\right) = 6.4\times10^{-10}\,C = 640\,pC$$

Example 24 – 1

(C) Using the formula for the electric field in two parallel plates

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{6.4\times10^{-10}C}{6.0\times10^{-3}m^2\times8.85\times10^{-12}C^2/N\cdot m^2} = 1.2\times10^4 N/C = 1.2\times10^4 V/m$$

Or, since $V = Ed$ we can obtain $E = \frac{V}{d} = \frac{12V}{1.0\times10^{-3}m} = 1.2\times10^4 V/m$

(d) Solving the capacitance formula for A, we obtain

$$C = \frac{\varepsilon_0 A}{d}$$

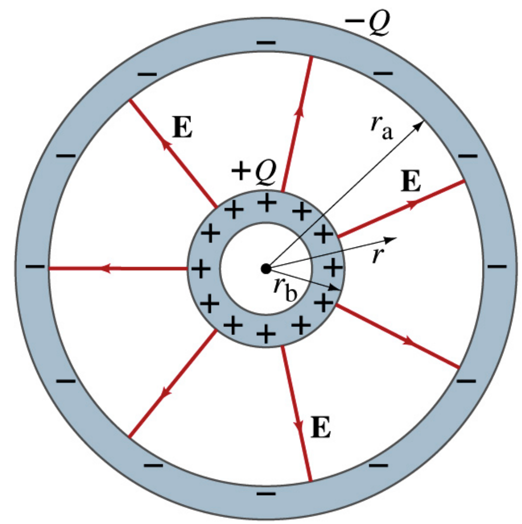
Solve for A

$$A = \frac{Cd}{\varepsilon_0} = \frac{1F\cdot1\times10^{-3}m}{\left(9\times10^{-12}C^2/N\cdot m^2\right)} \approx 10^8 m^2 \approx 100km^2$$

About 40% the area of Arlington ($256km^2$).

Example 24 – 3

Spherical capacitor: A spherical capacitor consists of two thin concentric spherical conducting shells, of radius r_a and r_b, as in the figure. The inner shell carries a uniformly distributed charge Q on its surface and the outer shell an equal but opposite charge –Q. Determine the capacitance of the two shells.



Using Gauss' law, the electric field outside a uniformly charged conducting sphere is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So the potential difference between a and b is

$$V_{ba} = -\int_a^b \vec{E}\cdot d\vec{l} =$$

$$= -\int_a^b E\cdot dr = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}dr = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)_{r_a}^{r_b} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a-r_b}{r_b r_a}\right)$$

Thus capacitance is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a-r_b}{r_b r_a}\right)} = \frac{4\pi\varepsilon_0 r_b r_a}{r_a - r_b}$$

Capacitor Cont'd

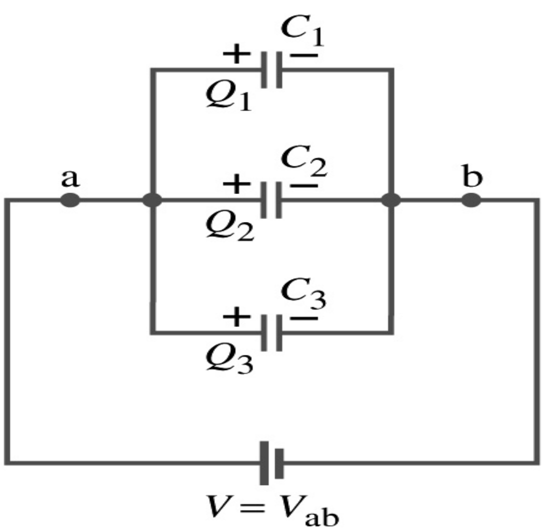
- A single isolated conductor can be said to have a capacitance, C.
- C can still be defined as the ratio of the charge to the absolute potential V on the conductor.
  - So Q=CV.
- The potential of a single conducting sphere of radius r_b can be obtained as

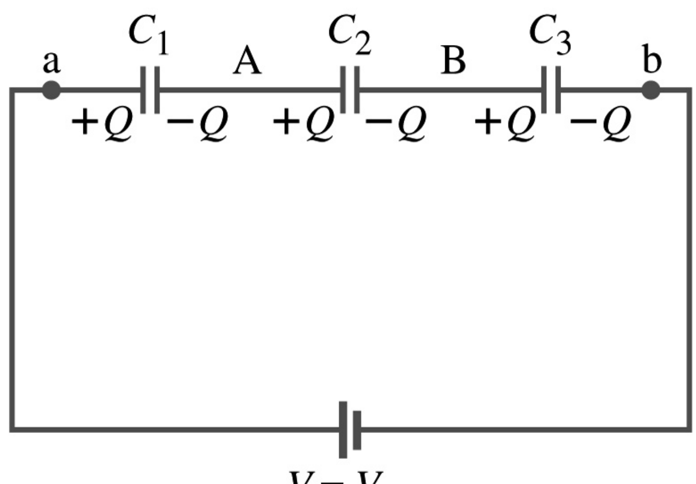
$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0 r_b} \qquad \text{where} \quad r_a \to \infty$$

- So its capacitance is $\boxed{C = \frac{Q}{V} = 4\pi\varepsilon_0 r_b}$

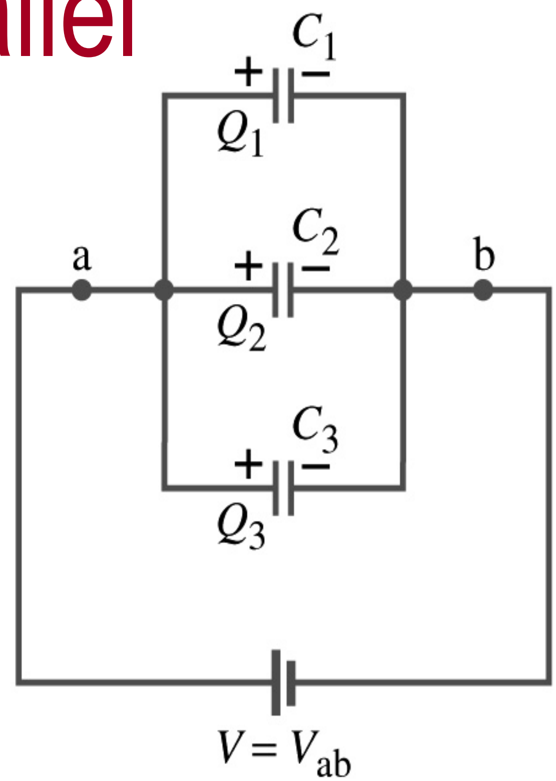
Capacitors in Series or Parallel

- Capacitors may be used in electric circuits
- What is an electric circuit?
  - A closed path of conductors, usually wires connecting capacitors and other electrical devices, in which
    - Charges can flow
    - And includes a voltage source such as a battery
- Capacitors can be connected in various ways.
  - In parallel, in series or in combination





Capacitors in Parallel

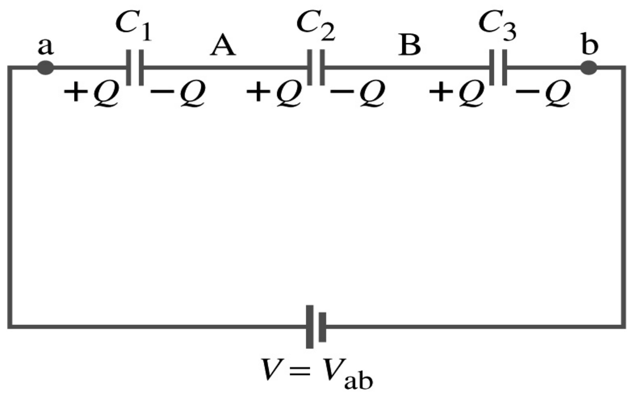


- Parallel arrangement provides the **<u>same voltage</u>** across all the capacitors.
  - Left hand plates are at V_a and right hand plates are at V_b
  - So each capacitor plate acquires the charge given by the formula
    - $Q_1=C_1V$, $Q_2=C_2V$, and $Q_3=C_3V$
- The total charge Q that must leave the battery is then
  - $Q=Q_1+Q_2+Q_3=V(C_1+C_2+C_3)$
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V= V(C_1+C_2+C_3)$
- Thus the equivalent capacitance in parallel is $\boxed{C_{eq} = C_1 + C_2 + C_3}$

What is the net effect?

The capacitance increases!!!

Capacitors in Series



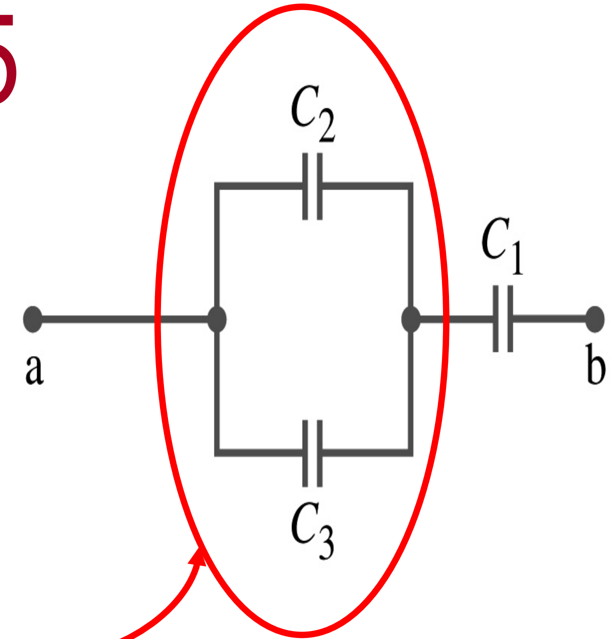
- Series arrangement is more interesting
  - When the battery is connected, +Q flows to the left plate of C_1 and –Q flows to the right plate of C_3.
  - Since capacitors in between were originally neutral, charges get induced to neutralize the ones in the middle.
  - So the charge on each capacitor plate is the same value, Q. (**Same charge**)
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V$
- The total voltage V across the three capacitors in series must be equal to the sum of the voltages across each capacitor.
  - $V=V_1+V_2+V_3=Q/C_1+Q/C_2+Q/C_3$
- Putting all these together, we obtain:
- $V=Q/C_{eq}=Q(1/C_1+1/C_2+1/C_3)$
- Thus the equivalent capacitance is $$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

What is the net effect?

The capacitance smaller than the smallest C!!!

Example 24 – 5

Equivalent Capacitor: Determine the capacitance of a single capacitor that will have the same effect as the combination shown in the figure. Take $C_1=C_2=C_3=C$.



We should do these first!!

How? These are in parallel so the equivalent capacitance is:

$$C_{eq1} = C_1 + C_2 = 2C$$

Now the equivalent capacitor is in series with C1.

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq1}} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

Solve for C_{eq}

$$C_{eq} = \frac{2C}{3}$$