PHYS 1441 – Section 002
Lecture #19

Monday, Nov. 16, 2020

Dr. Jaehoon Yu

- CH27
  - Cyclotron Frequency
  - Torque on a Current Loop
  - Magnetic Dipole Moment
- CH28
  - Source of Magnetic Field
  - Magnetic Field Due to a Straight Wire
  - Ampere's Law

Today's homework is homework #10, due 11pm,Tuesday, Dec. 1!!

Announcements

- Reading assignments: CH28.6 – 10
- The final exam date and time
  - 11am – 12:20pm, 80min, Wed. Dec. 16
  - No class Monday, Dec. 14
- Term 2 results
  - Class average: 62.2/104 equivalent to 58.5/100
    - Previous quizzes: 65.7/100 and 58.6/100
  - Top score: 104/104

Example 27 – 7

Electron's path in a uniform magnetic field. An electron travels at the speed of 2.0×10^{7}m/s in a plane perpendicular to a 0.010-T magnetic field. What is the radius of the electron's path?

What is formula for the centripetal force? $F = ma = m\dfrac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is $F = evB$

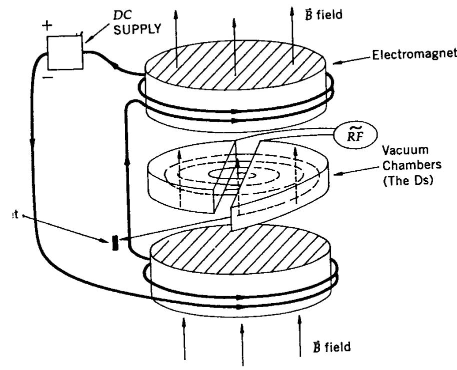
Since the magnetic force provides the centripetal force, we can establish an equation with the two forces $F = evB = m\dfrac{v^2}{r}$

Solving for r

$$r = \frac{mv}{eB} = \frac{\left(9.1\times10^{-31}\,kg\right)\cdot\left(2.0\times10^{7}\,m/s\right)}{\left(1.6\times10^{-19}\,C\right)\cdot\left(0.010T\right)} = 1.1\times10^{-2}\,m$$

Cyclotron Frequency

- The time required for a particle of charge **q** moving w/ a constant speed **v** to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



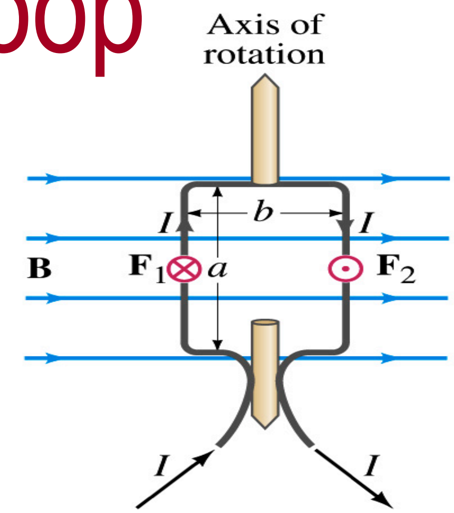
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$\boxed{f = \frac{1}{T} = \frac{qB}{2\pi m}}$$

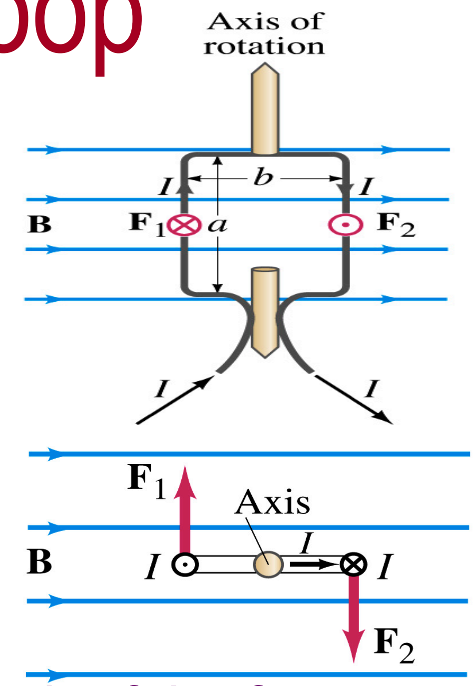
- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.

Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with an electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts force on both vertical sections of wire.
  - Where is this principle used in?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque in the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire? (Poll 20)
  - It will not turn beyond 90 degrees without changing current direction

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length a?
    - $F_a = IaB$
    - The moment arm of the coil is $b/2$
  - So the total torque is the sum of the torques by each of the forces

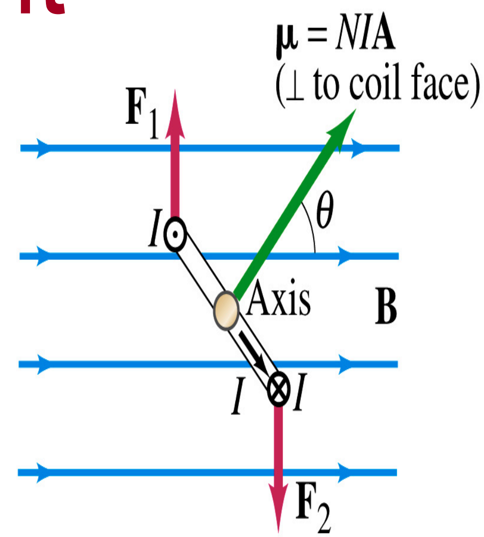
$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

    - Where $A = ab$ is the area of the coil loop
  - What is the total net torque if the coil consists of N loops of wire?

$$\boxed{\tau = NIAB}$$

  - If the coil makes an angle θ w/ the field $\boxed{\tau = NIAB\sin\theta}$

Magnetic Dipole Moment



- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity N$I\mathcal{A}$ is called the **magnetic dipole moment of the coil** (Poll 2)
  - It is a vector quantity $\boxed{\vec{\mu} = NI\vec{A}}$
  - Its direction is the same as that of the area vector **A** and is perpendicular to the plane of the coil consistent with the right-hand rule
    - When your fingers cup around the loop in the same direction as the current, your thumb points to the direction of the magnetic moment
  - The tendency of an object to interact with an external magnetic field
  - Using the definition of magnetic moment, the torque can be rewritten in vector form

$$\boxed{\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}}$$

Magnetic Dipole Potential Energy

- Where else did you see the same form of the torque?
  - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
  - The potential energy of the electric dipole is
  - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
  - The work done by the torque is
  - $U = \int \tau d\theta = \int NIAB \sin\theta \, d\theta = -\mu B \cos\theta + C$
  - If we chose U=0 at θ=π/2, then C=0
  - Thus the potential energy is $\boxed{U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}}$
    - Very similar to the electric dipole

Example 27 – 12

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius 0.529×10^{-10}m.

What provides the centripetal force? The Coulomb force

So we can obtain the speed of the electron from $F = \dfrac{e^2}{4\pi\varepsilon_0 r^2} = \dfrac{m_e v^2}{r}$

Solving for v

$$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}} = \sqrt{\frac{\left(8.99\times10^9\,N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(9.1\times10^{-31}\,kg\right)\cdot\left(0.529\times10^{-10}\,m\right)}} = 2.19\times10^6\,m/s$$

Since the electric current is the charge that passes through the given point per unit time, we can obtain the current $I = \dfrac{e}{T} = \dfrac{ev}{2\pi r}$

Since the area of the orbit is $A=\pi r^2$, we obtain the hydrogen magnetic moment

$$\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2} = \frac{er}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}} = \frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m_e}}$$

Sources of Magnetic Field

- We have learned so far about the effects of magnetic field on the electric current and the moving charge
- We will now learn about the dynamics of magnetism
  - How do we determine magnetic field strengths in certain situations?
  - How do two wires with electric current interact?
  - What is the general approach to finding the connection between current and magnetic field?

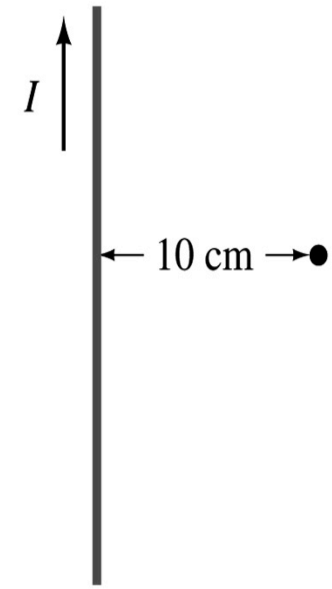
Magnetic Field due to a Straight Wire

- The magnetic field due to the current flowing through a straight wire forms a circular pattern around the wire
  - What do you imagine the strength of the field is as a function of the distance from the wire?
    - It must be weaker as the distance increases
  - How about as a function of current?
    - Directly proportional to the current
  - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
    - This is valid as long as r << the length of the wire
  - The proportionality constant is $\mu_0/2\pi$, thus the field strength becomes

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

  - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$

Example 28 – 1



Calculation of B near a wire. A vertical electric wire in the wall of a building carries a DC current of 25A upward. What is the magnetic field at a point 10cm due East of this wire?

Using the formula for the magnetic field near a straight wire

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

So we can obtain the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\, T \cdot m/A\right) \cdot \left(25A\right)}{\left(2\pi\right) \cdot \left(0.01m\right)} = 5.0 \times 10^{-5}\, T$$