PHYS 1441 – Section 002
Lecture #21

Monday, Nov. 23, 2020

Dr. Jaehoon Yu

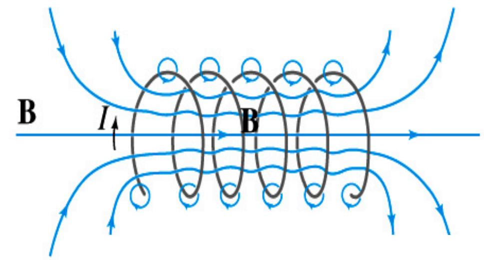
- CH28
  - Solenoid
  - Magnetic Materials and Their Behaviors
  - Hysteresis
- CH29
  - Induced EMF and EM Induction
  - Faraday's Law of Induction
  - Lenz's Law

Today's homework is homework #11, due 11pm, Friday, Dec. 11!!

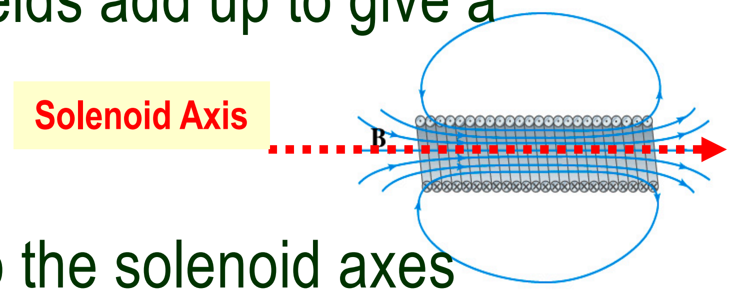
Announcements

- Reading assignments: CH28.6 – 10
- Quiz 4 on Quest
  - Beginning of the class Monday, Nov. 30
  - Covers: CH27.4 – CH29.2
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, setups or solutions of any problems, figures, pictures, diagrams or arrows, etc!
  - No additional formulae or values of constants will be provided!
  - Must send me the photos of front and back of the formula sheet, including the blank, no later than **11am the day of the test**
    - Once submitted, you cannot change, unless I ask you to delete some part of the sheet!
- The final comprehensive exam date and time
  - Last class of the course, Monday, Dec. 7
  - **11am – 12:30pm, 90min, Wed. Dec. 16**

Solenoid and Its Magnetic Field



- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops are wide
    - The field near the wires are nearly circular
    - Between any two wires, the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be large and uniform



  - For a long, densely packed loops
    - The field is nearly uniform and parallel to the solenoid axes within the entire cross section
    - The field outside the solenoid is very small compared to the field inside, except the ends
      - The same number of field lines spread out to an open space

Some Basic Formulae

- The force due to a uniform magnetic field ***B*** on the wire carrying current ***I*** in the wire w/ length ***l*** immersed in the field

$$\boxed{F = IlB\sin\theta} \qquad \boxed{\vec{F} = I\vec{l} \times \vec{B}}$$

- Magnetic field at distance ***r*** from the wire generated by the current ***I*** flowing in the wire

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

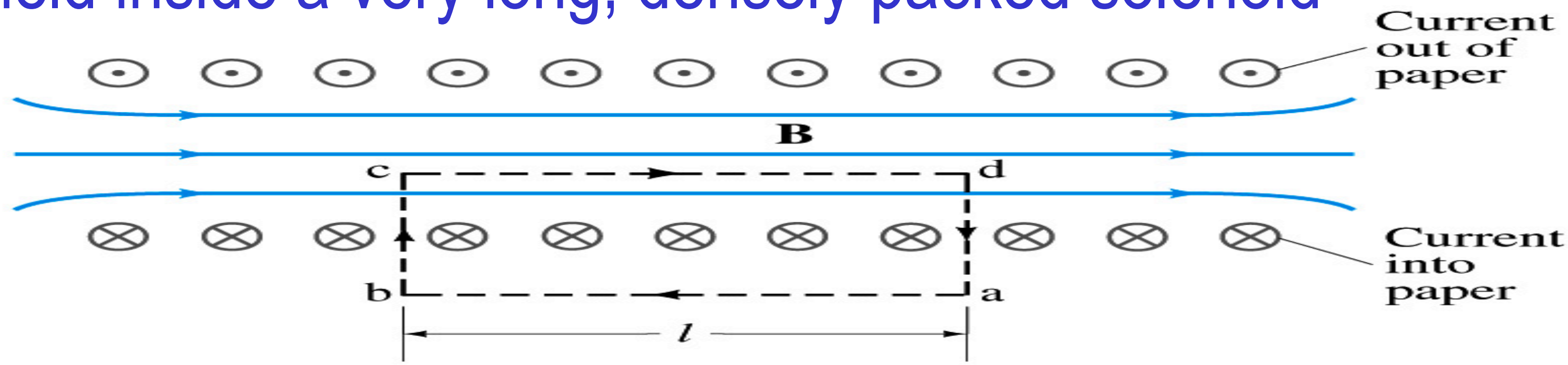
  - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$

- Ampére's law

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$$

Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid



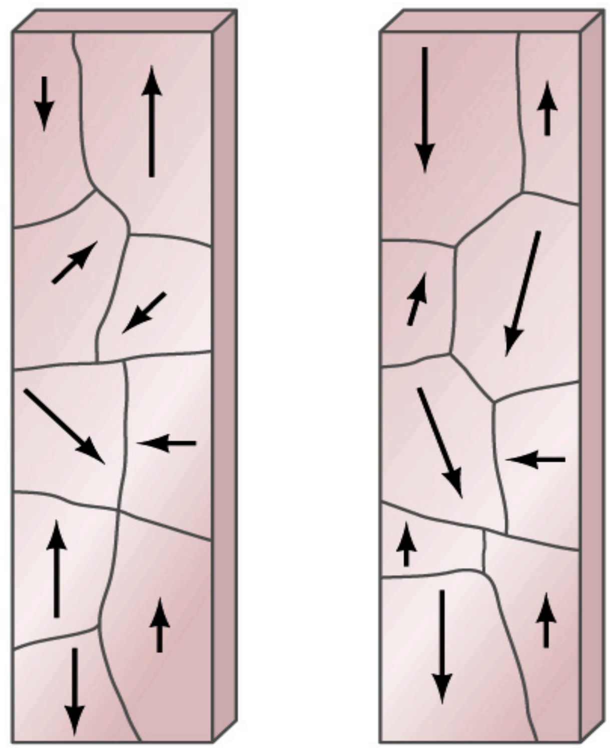
- Let's choose the path *abcd*, far away from the ends
  - We can consider four segments of the loop for integral
  - $\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$
  - The field outside the solenoid is negligible. So the integral on $a \rightarrow b$ is 0.
  - Now the field **B** is perpendicular to the bc and da segments. So these integrals become 0, also.

Solenoid Magnetic Field

- So the sum becomes: $\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$
- If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$
  - Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed
- Thus Ampere's law gives us $Bl = \mu_0 NI$
- If we let $n=\mathcal{N}/\ell$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes
- $$\boxed{B = \mu_0 nI}$$
  - B depends on the number of loops per unit length, n, and the current I
    - Does not depend on the position within the solenoid since the field is uniform inside it, like a bar magnet

Magnetic Materials - Ferromagnetism

- Iron is a material that can turn into a strong magnet
  - This kind of material is called the **ferromagnetic** material
- In a microscopic sense, ferromagnetic materials consist of many tiny magnetic regions called **domains**
  - Domains are like little magnets usually smaller than 1mm in length or width
- What do you think the alignment of domains are like when they are not magnetized? (poll 22)
  - Randomly arranged
- What if they are magnetized? (poll 22)
  - The number of the domains aligned with the external magnetic field direction grows while those of the domains not aligned reduce
  - This gives magnetization to the material
- How do we demagnetize a bar magnet?
  - Hit the magnet hard or heat it over the Curie temperature


(b)

B in Magnetic Materials

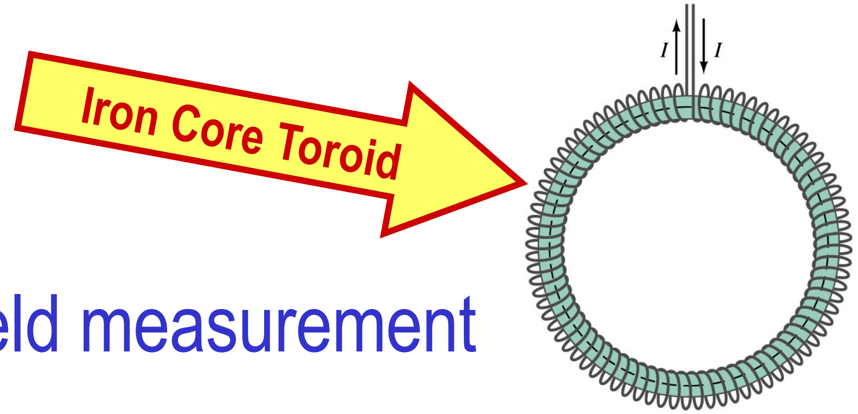
- What is the magnetic field inside a solenoid?
- $$B_0 = \mu_0 n I$$
  - Magnetic field in a long solenoid is directly proportional to the current.
  - This is valid only if air is inside the coil
- What do you think will happen to B if we have something other than the air inside the solenoid? (poll23)
  - It will be increased dramatically, when the current flows
    - Especially if a ferromagnetic material such as iron is put inside, the field could increase by several orders of magnitude ➔ Principle of the electromagnet
- Why?
  - Since the domains in the iron aligns permanently by the external field.
  - The resulting magnetic field is the sum of that due to current and due to the iron

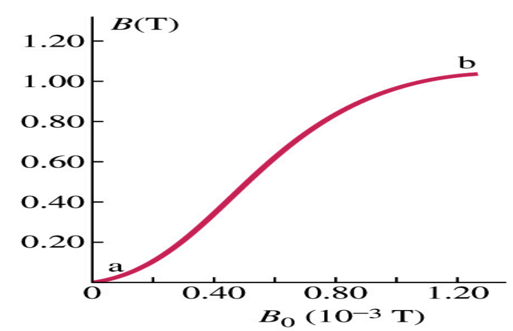
B in Magnetic Materials

- It is sometimes convenient to write the total field as the sum of two terms
- $$\vec{B} = \vec{B}_0 + \vec{B}_M$$
  - **B_0** is the field due only to the current in the wire, namely the external field
    - The field that would be present without a ferromagnetic material
  - **B_M** is the additional field due to the ferromagnetic material itself; often **$B_M \gg B_0$**
- The total field in this case can be written by replacing μ_0 with another proportionality constant μ, the magnetic permeability of the material $\boxed{B = \mu n I}$
  - μ is a property of a magnetic material
  - μ is not a constant but varies with the external field

Hysteresis

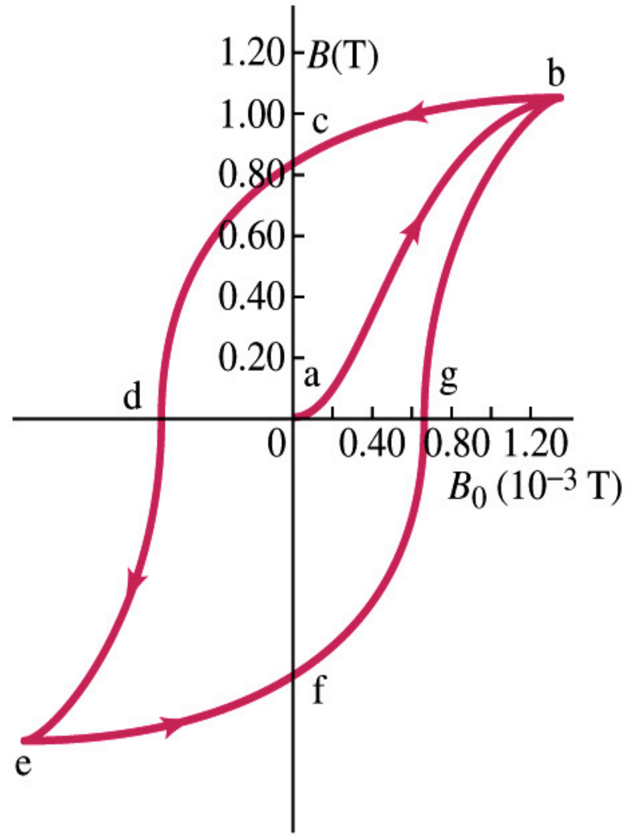
- What is a toroid?
  - A solenoid bent into a shape
- Toroid can be used for magnetic field measurement
  - Why?
  - Since it does not leak magnetic field outside of itself, it fully contains all the magnetic field created within it.
- Consider an un-magnetized iron core toroid, without any current flowing in the wire
  - What do you think will happen if the current slowly increases?
  - B_0 increases linearly with the current.
  - And **B** increases also but follows the curved line shown in the graph
  - As B_0 increases, the domains become more aligned until nearly all are aligned (point b on the graph)
    - The iron is said to be approaching saturation
    - Point b is typically at 70% of the max





Hysteresis

- What do you think will happen to **B** if the external field $\mathbf{B_0}$ is reduced to 0 by decreasing the current in the coil?
  - ~~Of course it goes to 0!!~~
  - Wrong! Wrong! Wrong! They do not go to 0. Why not?
  - The domains do not completely return to the random alignment state
- Now if the current direction is reversed, the external magnetic field direction is reversed, causing the total field **B** to pass **0**, and the direction reverses to the opposite side
  - If the current is reversed again, the total field **B** will increase but never goes through the origin
- This kind of curve whose path does not retrace themselves and does not go through the origin is called the **Hysteresis**.

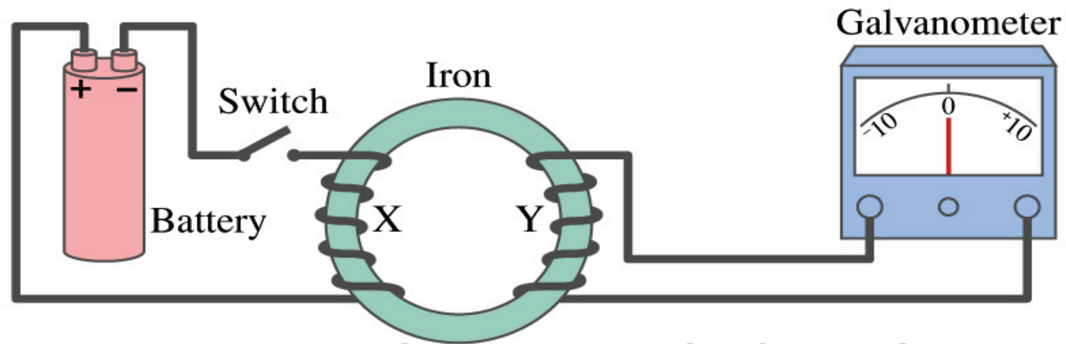


Induced EMF

- It has been discovered by Oersted and company in early 19th century that
  - Magnetic field can be produced by the electric current
  - Magnetic field can exert force on the electric charge
- So if you were scientists at that time, what would you wonder?
  - Yes, you are absolutely right! You would wonder if the magnetic field can create the electric current.
  - An American scientist Joseph Henry and an English scientist Michael Faraday independently found that this was possible
    - Faraday was given the credit since he published his work before Henry
      - He also did a lot of detailed studies on magnetic induction

Electromagnetic Induction

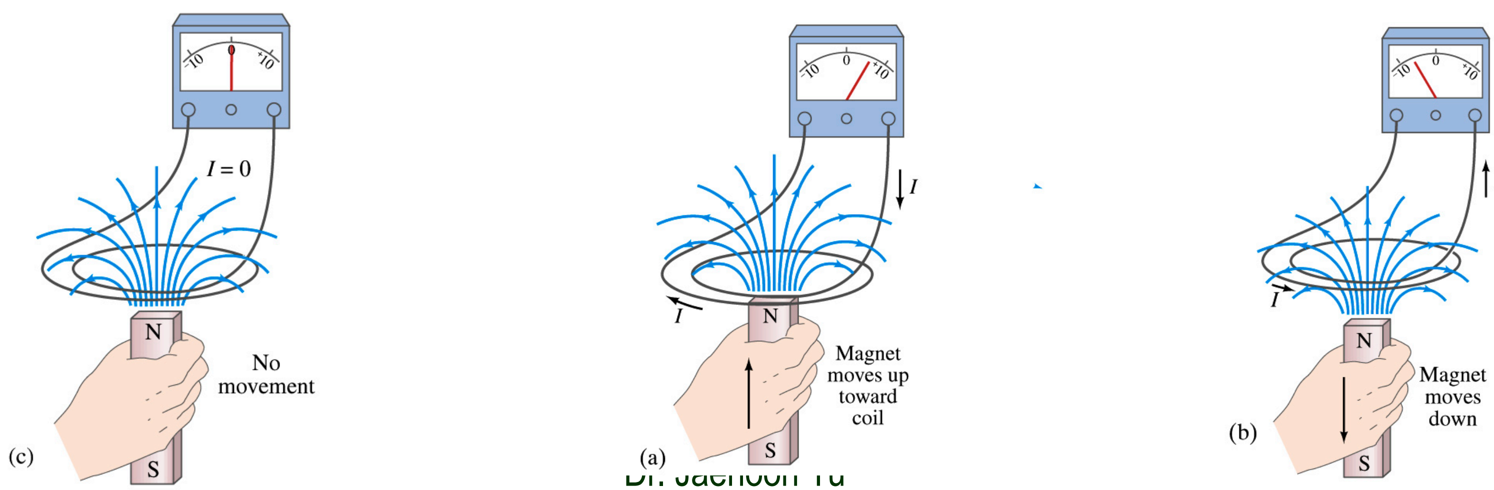
- Faraday used an apparatus below to show that magnetic field can induce current



- Despite his hope, he did not see steady current induced on the other side when the switch is thrown
- But he did see that the needle on the Galvanometer turns strongly when the switch is initially closed and is opened
  - When the magnetic field through coil Y changes, a current flows as if there were a source of emf
- Thus he concluded that **an induced emf is produced by a changing magnetic field** → **Electromagnetic Induction**

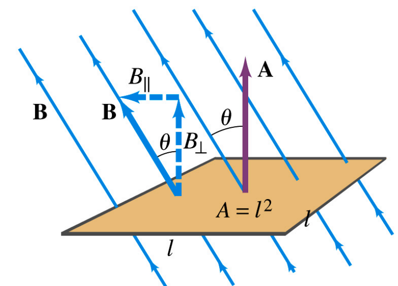
Electromagnetic Induction

- Further studies on electromagnetic induction taught
  - If a magnet is moved quickly into a coil of wire, a current is induced in the wire in one direction (poll 24)
  - If a magnet is removed from the coil, a current is induced in the wire in the opposite direction (poll24)
  - By the same token, the current can also be induced if the magnet stays put but the coil moves toward or away from the magnet (poll 24)
  - Current is also induced if the coil rotates.
- In other words, it does not matter whether the magnet or the coil moves. It is the relative motion that counts.



Magnetic Flux

- So what do you think is the induced emf proportional to?
  - The rate of changes of the magnetic field?
    - the higher the changes the higher the induction
  - Not really, it rather depends on the rate of change of the **magnetic flux**, Φ_B.
  - Magnetic flux is defined as (just like the electric flux)
  - $$\boxed{\Phi_B = B_\perp A = BA\cos\theta = \vec{B}\cdot\vec{A}}$$
    - θ is the angle between **B** and the area vector **A** whose direction is perpendicular to the face of the loop based on the right-hand rule



  - What kind of quantity is the magnetic flux? (å)
    - Scalar. Unit?
    - $T\cdot m^2$ or weber $\boxed{1Wb = 1T\cdot m^2}$
- If the area of the loop is not simple or B is not uniform, the magnetic flux can be written as $$\boxed{\Phi_B = \int \vec{B}\cdot d\vec{A}}$$

Faraday's Law of Induction

- In terms of magnetic flux, we can formulate Faraday's findings as follows:
  - The EMF induced in a circuit is equal to the rate of change of magnetic flux through the circuit

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}$$

Faraday's Law of Induction

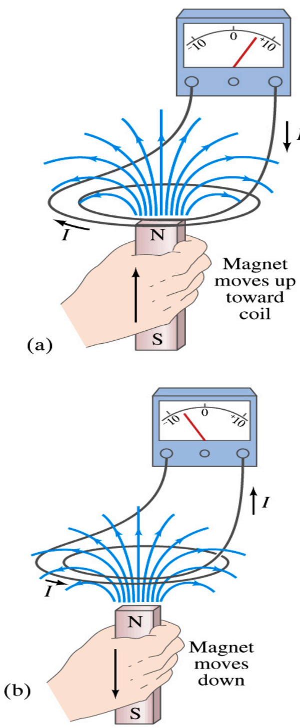
- If the circuit contains N closely wrapped loops, the total induced emf is the sum of EMF induced in each loop

$$\boxed{\varepsilon = -N\frac{d\Phi_B}{dt}}$$

  - Why negative?
    - Has got a lot to do with the direction of induced emf
    - The EMF is generated in the direction minimizing the change in flux

Lenz's Law

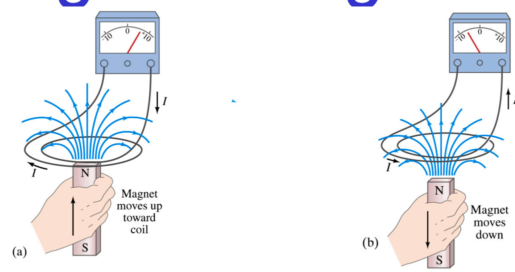
- It is experimentally found that
  - An induced EMF gives rise to the current whose magnetic field opposes the original change in flux ➔ This is known as **Lenz's Law**
  - In other words, an induced EMF is always in the direction that opposes the original change in flux that caused it. (poll 17)
  - We can use Lenz's law to explain the following cases in the figures
    - When the magnet is moving into the coil
      - Since the external flux increases, the field inside the coil takes the opposite direction to minimize the change and causes the current to flow clockwise
    - When the magnet is moving out
      - Since the external flux decreases, the field inside the coil takes the opposite direction to compensate the loss, causing the current to flow counter-clockwise



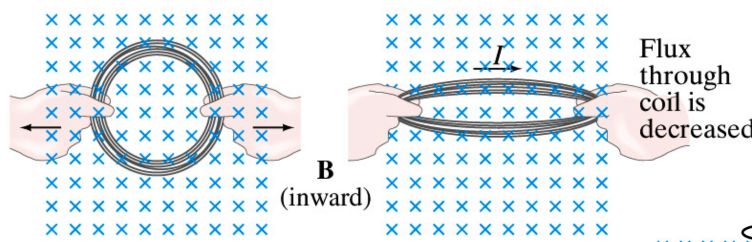
- Which law is Lenz's law result of? (poll 5)
  - Energy conservation. Why?

Induction of EMF

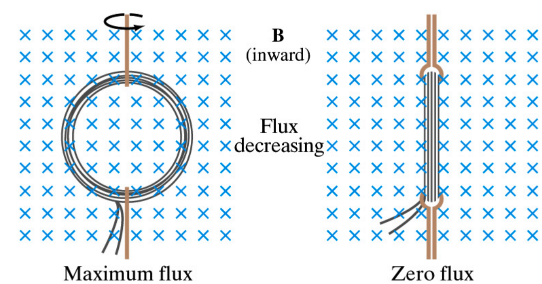
- How can we induce EMF?
- Let's look at the formula for the magnetic flux
- $$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos\theta \, dA$$
- What do you see? What are the things that can change with time to result in change of magnetic flux? (poll 8)
  - Magnetic field



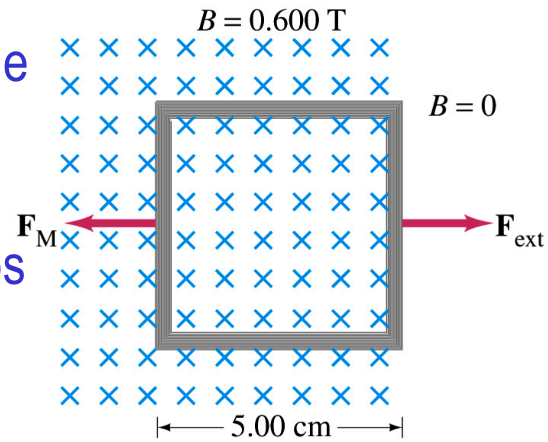
  - The area of the loop



  - The angle θ between the field and the area vector



Example 29 – 5



Pulling a coil from a magnetic field. A square coil of wire with side 5.00cm contains 100 loops and is positioned perpendicular to a uniform 0.600-T magnetic field. It is quickly and uniformly pulled from the field (moving perpendicular to B) to a region where B drops abruptly to zero. At t=0, the right edge of the coil is at the edge of the field. It takes 0.100s for the whole coil to reach the field-free region. Find (a) the rate of change in flux through the coil, (b) the emf and the current induced, and (c) how much energy is dissipated in the coil if its resistance is 100Ω. (d) what was the average force required?

What should be computed first? The initial flux at t=0.

The flux at t=0 is $\Phi_B = \vec{B}\cdot\vec{A} = BA = 0.600T\cdot\left(5\times10^{-2}m\right)^2 = 1.50\times10^{-3}Wb$

The change of flux is $\Delta\Phi_B = 0 - 1.50\times10^{-3}Wb = -1.50\times10^{-3}Wb$

Thus the rate of change of the flux is

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{-1.50\times10^{-3}Wb}{0.100s} = -1.50\times10^{-2}\,Wb/s$$

Example 29 – 5, cnt'd

Thus the total emf induced in this period is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -100\cdot\left(-1.50\times10^{-2}\,Wb/s\right) = 1.5V$$

The induced current in this period is

$$I = \frac{\varepsilon}{R} = \frac{1.5V}{100\Omega} = 1.50\times10^{-2}\,A = 15.0mA$$

Which direction would the induced current flow? Clockwise

The total energy dissipated is

$$E = Pt = I^2Rt = \left(1.50\times10^{-2}\,A\right)^2\cdot100\Omega\cdot0.100s = 2.25\times10^{-3}\,J$$

Force for each coil is $\vec{F} = I\vec{l}\times\vec{B}$ Force for N coil is $\vec{F} = NI\vec{l}\times\vec{B}$

$$\left|F\right| = NIlB = 100\cdot\left(1.50\times10^{-2}\,A\right)\cdot\left(4\times5\times10^{-2}\right)\cdot0.600T = 0.045N$$