PHYS 1441 – Section 002
Lecture #23

Wednesday, Dec. 2, 2020

Dr. Jaehoon Yu

- CH29
  - Electric Field due to Changing Magnetic Flux
  - Generalized Faraday's Law
- CH30
  - Inductance
  - Mutual and Self Inductance
  - Inductor
  - Energy Stored in Magnetic Field

Announcements

- Reading assignments: CH30.7 – 30.11 and CH31.4
- Final comprehensive exam on Quest
  - 11am – 12:30pm, Wednesday, Dec. 16
    - Roll call begins at **10:45am, Wed. Dec. 16**
  - Covers: CH21.1 – what we finish next Monday, Dec. 7 + math refresher
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, setups or solutions of any problems, figures, pictures, diagrams or arrows, etc!
  - No additional formulae or values of constants will be provided!
  - Must send me the photos of front and back of the formula sheet, including the blank, no later than **9:00am the day of the test**
    - Once submitted, you cannot change, unless I ask you to delete part of the sheet!
- Quiz 4 results
  - Class average: 30.7/50 equivalent to 61.4/100
    - Previous quizzes: 80.3/100, 58.8/100, 68.3/100
  - Top score: 50/50
- Course feedback survey should be done ASAP! (only 29 of you submitted!)

Ex. 29 – 13 Power Transmission – Why HV?

Transmission lines. An average of 120kW of electric power is sent to a small town from a power plant 10km away. The transmission lines have a total resistance of 0.4Ω. Calculate the power loss if the power is transmitted at (a) 240V and (b) 24,000V.

We cannot use $P=V^2/R$ for the power loss since we do not know the voltage along the transmission line. We, however, can use $P=I^2R$ since the current is the same along the entire transmission line.

(a) If 120kW is sent at 240V, the total current is $I=\frac{P}{V}=\frac{120\times10^3}{240}=500A.$

Thus the power loss due to transmission line is

$$P=I^2R=(500A)^2\cdot(0.4\Omega)=100kW$$

(b) If 120kW is sent at 24,000V, the total current is $I=\frac{P}{V}=\frac{120\times10^3}{24\times10^3}=5.0A.$

Thus the power loss due to transmission line is

$$P=I^2R=(5A)^2\cdot(0.4\Omega)=10W$$

The higher the transmission voltage, the smaller the current, causing less loss of energy. This is why power is transmitted w/ HV, as high as 170kV.

Electric Field due to Magnetic Flux Change

- When the electric current flows through a wire, there is an electric field in the wire that moves electrons
- We also learned that changing magnetic flux induces a current in the wire. (poll 17) What does this mean?
  - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- Recall the relationship between the electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field
- $\varepsilon = \int_a^b \vec{E} \cdot d\vec{l}$
- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

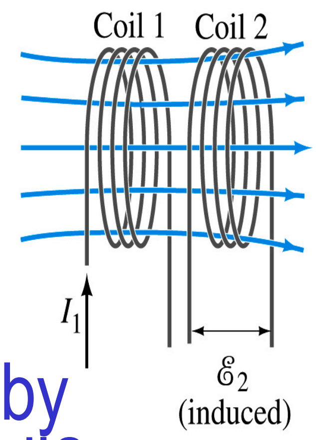
- The integral is taken around a path enclosing the area through which the magnetic flux Φ_B is changing.

Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit (Poll 17)
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - and induce an emf in itself ➔ Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.



- What is the induced emf, ε_2, in coil2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21} I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined as $\boxed{M_{21} = N_2\Phi_{21}/I_1}$.
- The emf induced in coil2 due to the changing current in coil1 is

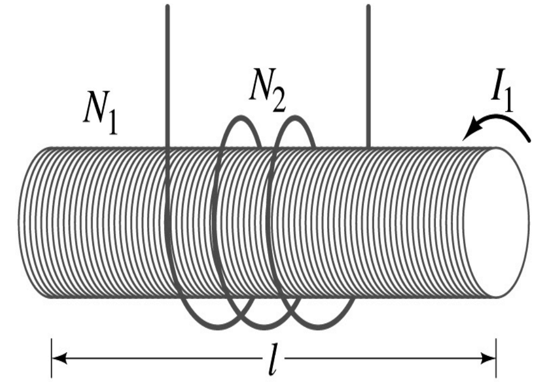
$$\boxed{\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d\left(N_2\Phi_{21}\right)}{dt} = -M_{21}\frac{dI_1}{dt}}$$

Mutual Inductance

- The mutual induction of coil2 with respect to coil1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present
    - The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be smaller.
  - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain $$\varepsilon_1 = -M\frac{dI_2}{dt} \text{ and } \varepsilon_2 = -M\frac{dI_1}{dt}$$
  - SI unit for mutual inductance is Henry (H) $$1H = 1V\cdot s/A = 1\Omega\cdot s$$

Example 30 – 1

Solenoid and coil. A long thin solenoid of length ℓ and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $$B=\frac{\mu_0 N_1 I_1}{l}$$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21}=BA=\frac{\mu_0 N_1 I_1}{l}A$$

Thus the mutual inductance of coil 2 is $$M_{21}=\frac{N_2\Phi_{21}}{I_1}=\frac{N_2}{I_1}\frac{\mu_0 N_1 I_1}{l}A=\frac{\mu_0 N_1 N_2}{l}A$$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this? (Poll 17)
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impedes its increase, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through an N turn coil is proportional to current I in the coil, $N\Phi_B = L\,I$
- We define self-inductance, $\mathcal{L}$: $\boxed{L = \frac{N\Phi_B}{I}}$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V\cdot s/A = 1\Omega\cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

- It is an impediment on the electrical current due to the existence of changing flux (poll 25)
- So what?
- In other words, it behaves like a resistance to the varying current, such as AC, that causes the constant change of magnetic flux
- But it also provides means to store energy, just like the capacitance

Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  - Precision resisters are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in the opposite direction to cancel the magnetic flux
    - This is called a “non-inductive winding”
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of **alternating current** (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

$$X_L = \varpi L$$

Example 30 – 3

Solenoid inductance. (a) Determine the formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing N turns of wire in its length ℓ and whose cross-sectional area is A. (b) Calculate the value of $\mathcal{L}$ if N=100, ℓ=5.0cm, A=0.30cm^2 and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with $\mu=4000\mu_0$.

What is the magnetic field inside a solenoid? $B=\mu_0 nI=\mu_0 NI/l$

The flux is, therefore, $\Phi_B=BA=\mu_0 NIA/l$

Using the formula for self inductance: $L=\dfrac{N\Phi_B}{I}=\dfrac{N\cdot\mu_0 NIA/l}{I}=\dfrac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L=\frac{\mu_0 N^2 A}{l}=\frac{\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}\,m^2\right)}{5.0\times10^{-2}\,m}=7.5\mu H$$

(c) The magnetic field with an iron core solenoid is $B=\mu NI/l$

$$L=\frac{\mu N^2 A}{l}=\frac{4000\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}\,m^2\right)}{5.0\times10^{-2}\,m}=0.030H=30mH$$