PHYS 1441 – Section 002
Lecture #24

Monday, Dec. 7, 2020

Dr. Jaehoon Yu

- CH30
  - Energy Stored in Magnetic Field
- CH31
  - Expansion of Ampere's Law
  - Gauss' Law for Magnetism
  - Production of EM Waves

The Pass/Fail Option

- You can choose pass or fail option for your course grade for this course.
  - The valid letter grades for P/F option choice are A, B, C or D
  - F, I or W cannot be turned to P/F
- You must be the one making the choice
  - Must speak to your department advisor to make sure the P/F choice does not impact fulfilling your major prerequisite requirements
- Some FAQ:
  - Is it allowed for a student to be assigned an incomplete, and then after they have completed the work receive a P in the class?
    - Answer: **No**. For a given class a student can either get an I (in consultation with professor), or a P/F, but not both.
  - What is the minimum grade needed to get a P?
    - Answer: Any non-failing grade (A,B,C,D) qualifies the student for a Pass. This is true across the university, although whether **a Pass meets a prerequisite requirement varies from department to department** (Some departments require a C or better to advance to the next class, so since a Pass only corresponds to D or better, the student will have to retake the class to meet the prereq) .

Announcements

- Reading assignments: CH30.7 – 30.11 and CH31.4
- Final comprehensive exam: 11am – 12:30pm, Wednesday, Dec. 16
    - Roll call begins at **10:45am, Wed. Dec. 16**
  - Do NOT miss the exam! **Must be in a quiet place to take the exam on Quest!**
    - You will get an F no matter how well you've been doing!!
  - Covers: CH21.1 – CH31.4 + math refresher
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, setups or solutions of any problems, figures, pictures, diagrams or arrows, etc!
  - No additional formulae or values of constants will be provided!
  - Must send me the photos of front and back of the formula sheet, including the blank, no later than **9:00am the day of the test**
    - Once submitted, you cannot change, unless I ask you to delete part of the sheet!
- Only 37 of you (<25%) filled the course feedback survey

Final Words

- I deeply appreciate your resilience and patience in the terrible circumstance
- Covid-19 was imposed on us what seems to be a natural mutation process
  - The whole humanity must work together to defeat the disease
    - One country alone CANNOT win over this kind of natural disaster
  - Human disregard to nature could have contributed to this disaster
  - We must realize that it is our responsibility to care for what is given to us
- Science must help and serve the entire humanity as a whole
- When you go out to the society, you must remember why you are do what you are do, for the whole humanity not for the selfish few
- Let's remember what happened in COVID-19 and think how we can prepare well and avoid this kind of disaster in the future
  - The climate change is something each and everyone of us must be responsible for our next generations
- Be a good person with true fundamental human decency!

Example 30 – 9

Reactance of a coil. A coil has a resistance R=1.00Ω and an inductance of 0.300H. Determine the current in the coil if (a) 120 V DC is applied to it; (b) 120 V AC (rms) at 60.0Hz is applied.

Is there a reactance for DC? Nope. Why not? Since ω=0, $X_L = \varpi L = 0$

So for DC power, the current is from Kirchhoff's rule $V - IR = 0$

$$I_0 = \frac{V_0}{R} = \frac{120V}{1.00\Omega} = 120A$$

For an AC power with f=60Hz, the reactance is

$$X_L = \varpi L = 2\pi f L = 2\pi \cdot \left(60.0 s^{-1}\right) \cdot 0.300H = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is

$$I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120V}{113\Omega} = 1.06A$$

Energy Stored in the Magnetic Field

- When an inductor of inductance $\mathcal{L}$ is carrying current I which is changing at a rate dI/dt, the energy is supplied to the inductor at a rate
  - $P = I\varepsilon = IL\frac{dI}{dt}$
- What is the work needed to increase the current in an inductor from 0 to I?
  - The work, dW, done in time dt is $dW = Pdt = LIdI$
  - Thus the total work needed to bring the current from 0 to I in an inductor is

$$W = \int dW = \int_0^I LIdI = L\left[\frac{1}{2}I^2\right]_0^I = \frac{1}{2}LI^2$$

Energy Stored in the Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $$U = \frac{1}{2}LI^2$$

    Energy Stored in the magnetic field inside an inductor
  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V: $U = \frac{1}{2}CV^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - The self inductance of an ideal solenoid without a fringe effect

$$L = \mu_0 N^2 A / l$$

  - The magnetic field in a solenoid is $B = \mu_0 NI/l$
  - Thus the energy stored in an inductor is

$$U = \frac{1}{2} L I^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \left(\frac{Bl}{\mu_0 N} \right)^2 = \frac{1}{2} \frac{B^2}{\mu_0} Al$$

What is this?

$$U = \frac{1}{2} \frac{B^2}{\mu_0} Al$$

E

  - Thus the energy density is

$$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2} \frac{B^2}{\mu_0}$$

Volume V

$$u = \frac{1}{2} \frac{B^2}{\mu_0}$$

E density

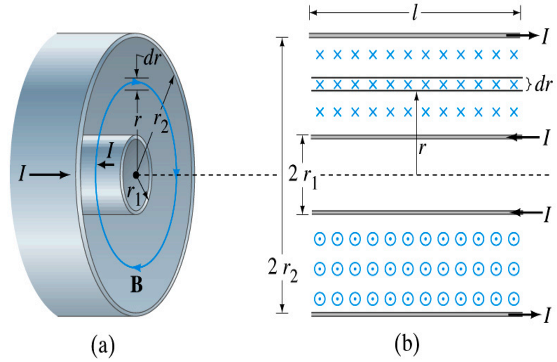
  - This formula is valid in any region of space
  - If a ferromagnetic material is present, μ_0 becomes μ.

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I? (b) Where is the energy density highest?



(a) The total flux through l of the cable is $\Phi_B = \int Bl\,dr = \frac{\mu_0 Il}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\mu_0 Il}{2\pi}\ln\frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}$

Thus the energy stored per unit length is $\frac{U}{l} = \frac{1}{2}\frac{LI^2}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{r_2}{r_1}$

(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2}\frac{B^2}{\mu_0}$

The energy density is highest where B is highest, close to $r=r_1$, near the surface of the inner conductor.

Maxwell's Equations

- The development of EM theory by Oersted, Ampere and others was not done in terms of EM fields
  - The idea of fields was introduced somewhat by Faraday
- Scottish physicist James C. Maxwell unified all the phenomena of electricity and magnetism in one theory with only four equations (Maxwell's Equations) using the concept of fields
  - This theory provided the prediction of EM waves
  - As important as Newton's law since it provides dynamics of electromagnetism
  - This theory is also in agreement with Einstein's special relativity
- The biggest achievement of 19th century electromagnetic theory is the prediction and experimental verifications that the electromagnetic waves can travel through the empty space
  - What do you think this accomplishment did?
    - Open a new world of communication
    - It also yielded the prediction that the light is an EM wave
- Since all of Electromagnetism is contained in the four Maxwell's equations, this is considered as one of the greatest achievements of human intellect

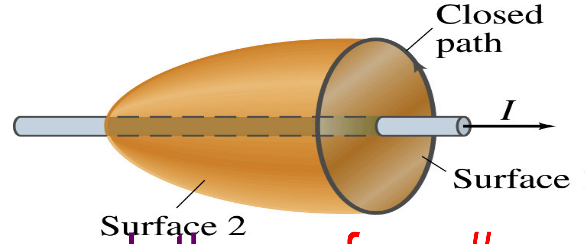
Ampere's Law

- Do you remember the mathematical expression of Oersted discovery of a magnetic field produced by an electric current, given by Ampere?

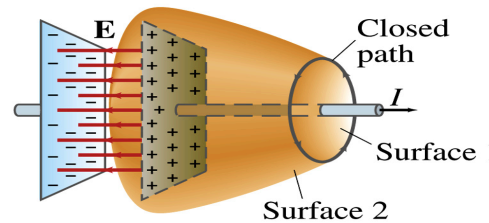
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- We've learned that a varying magnetic field produces an electric field
- Then, can the reverse phenomena, that a changing electric field producing a magnetic field, possible?
  - If this is the case, it would demonstrate a beautiful symmetry in nature between electricity and magnetism

Expanding Ampere's Law



- Let's consider a wire carrying current I
  - The current that is enclosed in the loop passes through the surface # 1 in the figure
  - We could imagine a different surface # 2 that shares the same enclosed path but cuts through the wire in a different location. What is the current that passes through the surface?
    - Still I.
  - So the Ampere's law still works



- We could then consider a capacitor being charged up or being discharged.
  - The current I enclosed in the loop passes through the surface #1
  - However the surface #2 that shares the same closed loop do not have any current passing through it.
    - There is magnetic field present since there is current ➔ In other words there is a changing electric field in between the plates
    - **Maxwell** resolved this by adding an additional term to Ampere's law involving the changing electric field

Modifying Ampere's Law

- To determine what the extra term should be, we have to first figure out what the electric field between the two plates is
  - The charge Q on the capacitor with capacitance C is Q=CV
    - Where V is the potential difference between the plates
  - Since V=Ed
    - Where E is the uniform field between the plates, and d is the separation of the plates
  - And for parallel plate capacitor $C=\varepsilon_0 A/d$
  - We obtain

$$Q = CV = \left(\varepsilon_0 \frac{A}{d}\right) Ed = \varepsilon_0 AE$$

Modifying Ampere's Law

– If the charge on the plate changes with time, we can write

$$\frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

– Using the relationship between the current and charge we obtain

$$I = \frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d(AE)}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Where $\Phi_E = EA$ is the electric flux through the surface between the plates

– In order to make Ampere's law work for the surface 2 in the figure, we must write it in the following form

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Extra term by Maxwell

– This equation represents the general form of Ampere's law

- This means that a magnetic field can be caused not only by an ordinary electric current but also by a changing electric flux

Example 31 – 1

Charging a capacitor. A 30-pF air-gap capacitor has circular plates of area $A=100cm^2$. It is charged by a 70-V battery through a 2.0-Ω resistor. At the instance the battery is connected, the electric field between the plates is changing most rapidly. At this instance, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Since this is an RC circuit, the charge on the plates is: $Q = CV_0\left(1 - e^{-t/RC}\right)$

For the initial current (t=0), we differentiate the charge with respect to time.

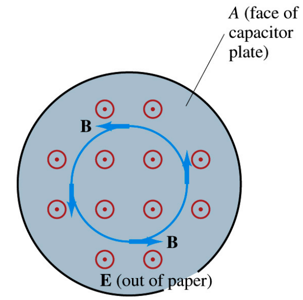
$$I_0 = \left.\frac{dQ}{dt}\right|_{t=0} = \left.\frac{CV_0}{RC}e^{-t/RC}\right|_{t=0} = \frac{V_0}{R} = \frac{70V}{2.0\Omega\cdot} = 35A$$

The electric field is $E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0}$

Change of the electric field is $\frac{dE}{dt} = \frac{dQ/dt}{A\varepsilon_0} = \frac{35A}{\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\cdot\left(1.0\times10^{-2}\,m^2\right)} = 4.0\times10^{14}\,V/m\cdot s$

Example 31 – 1, cnt'd

(c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.



The magnetic field lines generated by changing electric field is perpendicular to E and is circular due to symmetry

Whose law can we use to determine B?

Extended Ampere's Law w/ I_{encl}=0!

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

We choose a circular path of radius r, centered at the center of the plane, following the B.

For $r<r_{plate}$, the electric flux is $\Phi_E = EA = E\pi r^2$ since E is uniform throughout the plate

So from Ampere's law, we obtain $$B\cdot(2\pi r) = \mu_0\varepsilon_0 \frac{d\left(E\pi r^2\right)}{dt} = \mu_0\varepsilon_0 \pi r^2 \frac{dE}{dt}$$

Solving for B

$$B = \mu_0\varepsilon_0 \frac{r}{2}\frac{dE}{dt}$$

For $r<r_{plate}$

Since we assume E=0 for $r>r_{plate}$, the electric flux beyond the plate is fully contained inside the surface.

$$\Phi_E = EA = E\pi r_{plate}^2$$

So from Ampere's law, we obtain $$B\cdot(2\pi r) = \mu_0\varepsilon_0 \frac{d\left(E\pi r_{plate}^2\right)}{dt} = \mu_0\varepsilon_0 \pi r_{plate}^2 \frac{dE}{dt}$$

Solving for B

$$B = \frac{\mu_0\varepsilon_0 r_{plate}^2}{2r}\frac{dE}{dt}$$

For $r>r_{plate}$

Displacement Current

- Maxwell interpreted the second term in the generalized Ampere's law equivalent to an electric current
  - He called this term as the displacement current, I_D
  - While the other term is called as the conduction current, I
- Ampere's law then can be written as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{encl} + I_D \right)$$

  - Where

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

  - While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow itself

Gauss' Law for Magnetism

- If there is a symmetry between electricity and magnetism, there must be an equivalent law in magnetism as the Gauss' Law in electricity
- For a magnetic field B, the magnetic flux Φ_B through the surface is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

  - Where the integration is over the area of either an open or a closed surface
- The magnetic flux through a closed surface which completely encloses a volume is

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

- What was the Gauss' law in the electric case?
  - The electric flux through a closed surface is equal to the total net charge Q enclosed by the surface divided by ε_0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

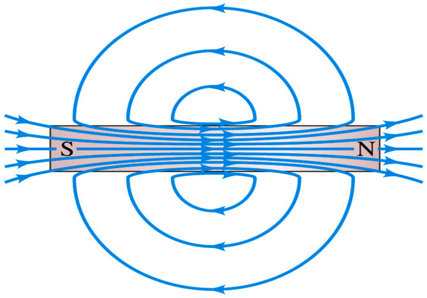
- Why is result of the integral zero?
  - There is no isolated magnetic poles, the magnetic equivalent of single electric charges

Gauss' Law for Magnetism

- What does the Gauss' law in magnetism mean physically?

$$\oint \vec{B} \cdot d\vec{A} = 0$$

  - There are as many magnetic flux lines that enter the enclosed volume as leave it
  - If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
    - Electricity do have the source and the sink
  - Magnetic field lines must be continuous
  - Even for bar magnets, the field lines exist both insides and outside of the magnet



Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampére's Law

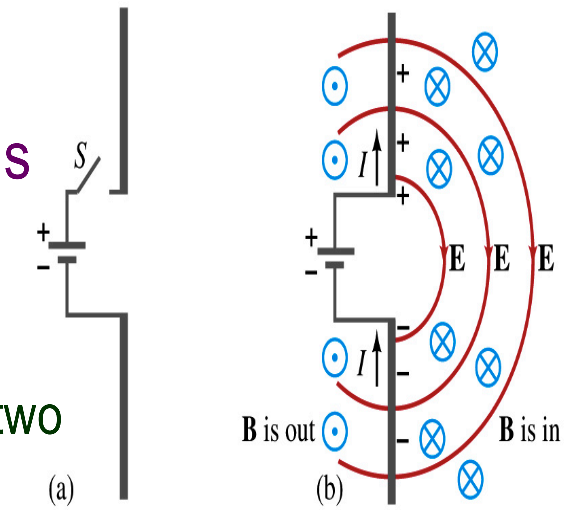
A magnetic field is produced by an electric current or by a changing electric field

Maxwell's Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
  - He then took this concept one step further and concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces the magnetic field that also changes.
    - This changing magnetic field then in turn produces the electric field that changes.
    - This process continues.
  - With the manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space

Production of EM Waves

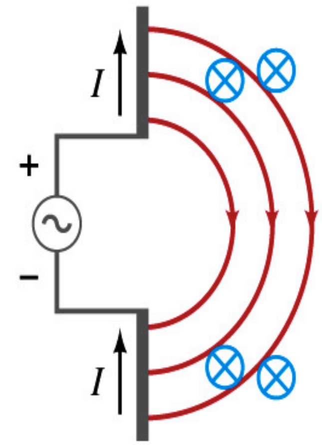
- Consider two conducting rods that will serve as an antenna are connected to a DC power source
  - What do you think will happen when the switch is closed?
    - The rod connected to the positive terminal is charged positively and the other negatively
    - Then the electric field will be generated between the two rods
    - Since there is the current that flows through, the rods generates a magnetic field around them

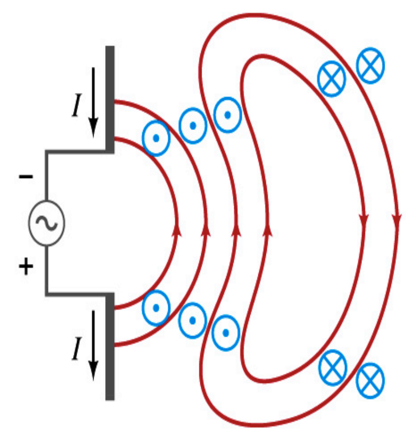


- How far would the electric and magnetic fields extend?
  - In static case, the field extends indefinitely
  - When the switch is closed, the fields are formed nearby the rods quickly but
  - The stored energy in the fields won't propagate w/ infinite speed

Production of EM Waves

- What happens if the antenna is connected to an AC power source?
  - When the connection was initially made, the rods are charging up quickly w/ the current flowing in one direction as shown in the figure
    - The field lines form as in the dc case
    - The field lines propagate away from the antenna
  - Then the direction of the voltage reverses
    - The new field lines with the opposite direction forms
    - While the original field lines still propagates away from the rod reaching out far
      - Since the original field propagates through an empty space, the field lines must form a closed loop (no charge exist)
    - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward is self supporting and do not need antenna with flowing charge
  - The fields far from the antenna is called the **radiation field**
  - Both electric and magnetic fields form closed loops perpendicular to each other





Properties of Radiation Fields – I

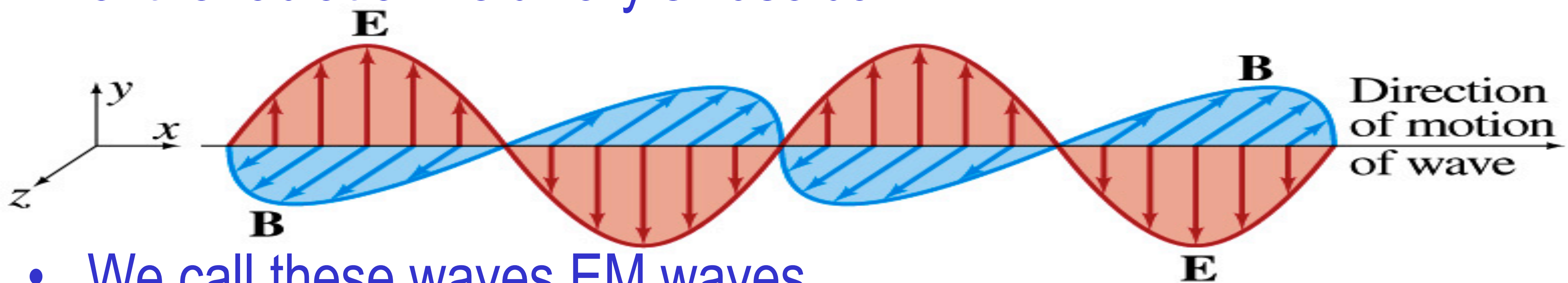
- The fields travel on the other side of the antenna as well
- The field strength is the greatest in the direction perpendicular to the oscillating charge while it is 0 along the direction of the current
- The magnitude of E and B in the radiation field decrease with distance as 1/r
- The energy carried by the EM wave is proportional to the square of the amplitude, E^2 or B^2
  - So the intensity of wave decreases as $1/r^2$

Properties of Radiation Fields – II

- The electric and magnetic fields at any point are perpendicular to each other and to the direction of the propagation (or motion)
- The fields alternate in direction
  - The field strengths vary from maximum in one direction, to 0 and to max in the opposite direction
- The electric and magnetic fields are in phase
  - B is maximum when E is maximum, vise versa
- Very far from the antenna, the field lines are quite flat over a reasonably large area
  - Called plane waves

EM Waves

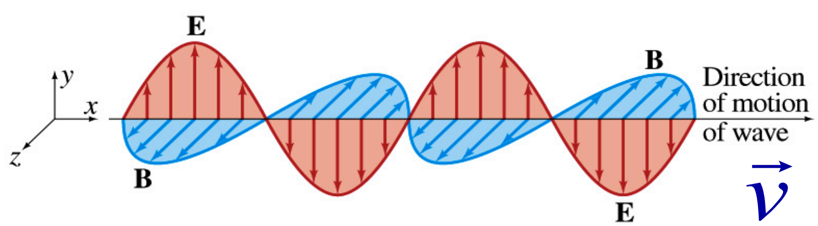
- If the voltage of the source varies sinusoidal, the field strengths of the radiation field vary sinusoidal



- We call these waves EM waves
- They are transverse waves
- EM waves are always those of fields
  - Since these are fields, they can propagate through an empty space
- In general **accelerating electric charges give rise to electromagnetic waves**
- This prediction from Maxwell's equations was experimentally proven by Heinrich Hertz through the **discovery of radio waves**

EM Waves and Their Speeds

- Let's consider a region of free space. What's a free space?
  - An area of space where there is no charges or conduction currents
  - In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
  - What are these flat waves called?
    - **Plane waves**
    - At any instance **E** and **B** are uniform over a large plane perpendicular to the direction of propagation
  - So we can also assume that the wave is traveling in the x-direction w/ velocity, **v**=v**i**, and that **E** is parallel to y axis and **B** is parallel to z axis



Maxwell's Equations w/ Q=I=0

- In this region of free space, Q=0 and I=0, thus the four Maxwell's equations become

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad \xrightarrow{Q_{encl}=0} \quad \oint \vec{E}\cdot d\vec{A} = 0$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \xrightarrow{\text{No Changes}} \quad \oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \xrightarrow{\text{No Changes}} \quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\varepsilon_0\frac{d\Phi_E}{dt} \quad \xrightarrow{I_{encl}=0} \quad \oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!