

# PHYS 1444 – Section 501

## Lecture #6

*Monday, Feb. 6, 2006*

*Dr. Jaehoon Yu*

- Electric Potential
- Electric Potential and Electric Field
- Electric Potential due to Point Charges
- Shape of the Electric Potential
- $V$  due to Charge Distributions
- Equi-potential Lines and Surfaces
- Electric Potential Due to Electric Dipole



# Announcements

- Distribution list
  - All but 6 of you have responded back.
  - If you did not receive the message, please check your trash can or the spam filter to see if the message is junked
  - Otherwise, please contact me again from your favorite e-mail address so that I can add you back on.
- Quiz next Monday, Feb. 13
  - Covers CH21 – CH 23
- 1<sup>st</sup> term exam Wednesday, Feb. 22
  - Covers CH21 – CH25
- Reading assignments
  - CH23–9



# Electric Potential and Potential Energy

- What is the definition of the electric potential?
  - The potential energy difference per unit charge
- OK, then, how would you express the potential energy that a charge  $q$  would obtain when it is moved between point  $a$  and  $b$  with the potential difference  $V_{ba}$ ?

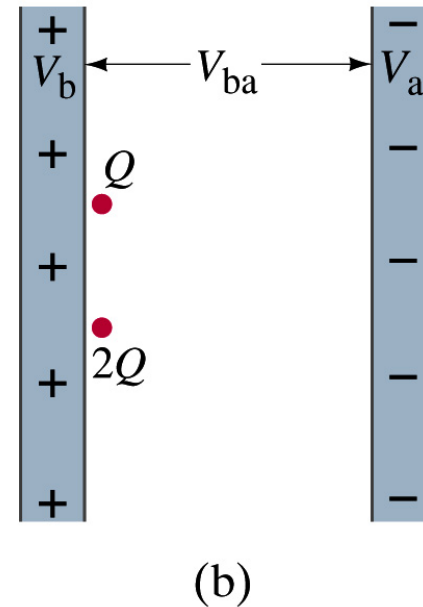
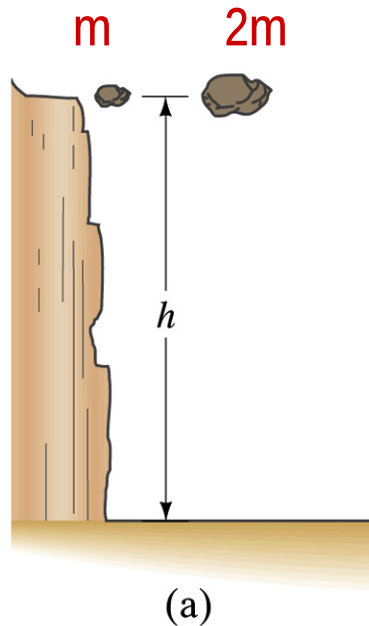
$$U_b - U_a = q(V_b - V_a) = qV_{ba}$$

- In other words, if an object with charge  $q$  moves through a potential difference  $V_{ba}$ , its potential energy changes by  $qV_{ba}$ .
- So based on this, how differently would you describe the electric potential in words?
  - A measure of how much energy an electric charge can acquire in a given situation
  - A measure of how much work a given charge can do.



# Comparisons of Potential Energies

- Let's compare gravitational and electric potential energies



- What are the potential energies of the rocks?
  - $mgh$  and  $2mgh$
- Which rock has a bigger potential energy?
  - The rock with a larger mass
- Why?
  - It's got a bigger mass.
- What are the potential energies of the charges?
  - $QV_{ba}$  and  $2QV_{ba}$
- Which object has a bigger potential energy?
  - The object with a larger charge.
- Why?
  - It's got a bigger charge.

The potential is the same but the heavier rock or larger charge can do a greater work.

# Electric Potential and Potential Energy

- The electric potential difference gives potential energy or possibility to do work based on the charge of the object.
- So what is happening in batteries or generators?
  - They maintain a potential difference.
  - The actual amount of energy used or transformed depends on how much charge flows.
  - How much is the potential difference maintained by a car's battery?
    - 12Volts
  - If for a given period, 5C charge flows through the headlight lamp, what is the total energy transformed?
    - $E_{\text{tot}} = 5\text{C} \cdot 12\text{V} = 60$  Umm... What is the unit? **Joules**
  - If it is left on twice as long?  $E_{\text{tot}} = 10\text{C} \cdot 12\text{V} = 120\text{J}$ .



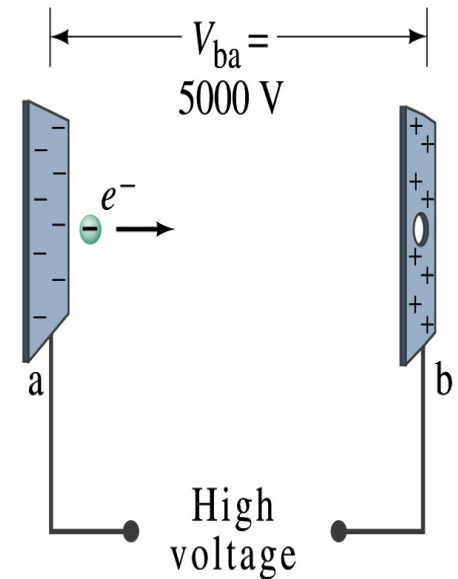
# Some Typical Voltages

| Sources                                 | Approximate Voltage |
|-----------------------------------------|---------------------|
| Thundercloud to ground                  | $10^8 \text{ V}$    |
| High-Voltage Power Lines                | $10^6 \text{ V}$    |
| Power supply for TV tube                | $10^4 \text{ V}$    |
| Automobile ignition                     | $10^4 \text{ V}$    |
| Household outlet                        | $10^2 \text{ V}$    |
| Automobile battery                      | $12 \text{ V}$      |
| Flashlight battery                      | $1.5 \text{ V}$     |
| Resting potential across nerve membrane | $10^{-1} \text{ V}$ |
| Potential changes on skin (EKG and EEG) | $10^{-4} \text{ V}$ |



## Example 23 – 2

**Electrons in TV tube:** Suppose an electron in the picture tube of a television set is accelerated from rest through a potential difference  $V_{ba}=+5000\text{V}$ . (a) What is the change in potential energy of the electron? (b) What is the speed of the electron ( $m=9.1\times 10^{-31}\text{kg}$ ) as a result of this acceleration? (c) Repeat for a proton ( $m=1.67\times 10^{-27}\text{kg}$ ) that accelerates through a potential difference of  $V_{ba}=-5000\text{V}$ .



- (a) What is the charge of an electron?
  - $e = -1.6 \times 10^{-19}\text{ C}$
- So what is the change of its potential energy?

$$\Delta U = qV_{ba} = eV_{ba} = (-1.6 \times 10^{-19}\text{ C})(+5000\text{ V}) = -8.0 \times 10^{-16}\text{ J}$$

## Example 23 – 2

- (b) Speed of the electron?
  - The entire potential energy of the electron turns to its kinetic energy. Thus the equation is

$$\Delta K = \frac{1}{2} m_e v_e^2 - 0 = W = -\Delta U = -eV_{ba} = \\ = -(-1.6 \times 10^{-19} \text{ C}) 5000 \text{ V} = 8.0 \times 10^{-16} \text{ J}$$

$$v_e = \sqrt{\frac{2 \times eV_{ba}}{m_e}} = \sqrt{\frac{2 \times 8.0 \times 10^{-16}}{9.1 \times 10^{-31}}} = 4.2 \times 10^7 \text{ m/s}$$

- (C) Speed of a proton?

$$\Delta K = \frac{1}{2} m_p v_p^2 - 0 = W = -\Delta U = -\{(-e)(-V_{ba})\} = -eV_{ba} = 8.0 \times 10^{-16} \text{ J}$$

$$v_p = \sqrt{\frac{2 \times eV_{ba}}{m_p}} = \sqrt{\frac{2 \times 8.0 \times 10^{-16}}{1.67 \times 10^{-27}}} = 9.8 \times 10^5 \text{ m/s}$$





# Electric Potential and Electric Field

- The effect of a charge distribution can be described in terms of electric field or electric potential.
  - What kind of quantities are the electric field and the electric potential?
    - Electric Field: Vector
    - Electric Potential: Scalar
  - Since electric potential is a scalar quantity, it is often easier to handle.
- Well other than the above, what are the connections between these two quantities?



# Electric Potential and Electric Field

- The potential energy is expressed in terms of a conservative force

$$U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{l}$$

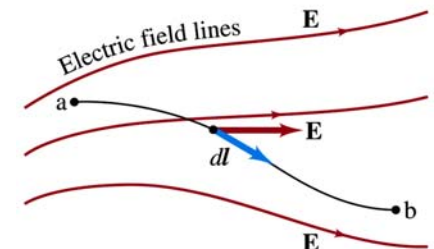
- For the electrical case, we are more interested in the potential difference:

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = -\int_a^b \frac{\vec{F}}{q} \cdot d\vec{l} = -\int_a^b \vec{E} \cdot d\vec{l}$$

- This formula can be used to determine  $V_{ba}$  when the electric field is given.

- When the field is uniform

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = -E \int_a^b dl = -Ed \quad \text{or} \quad V_{ba} = -Ed$$

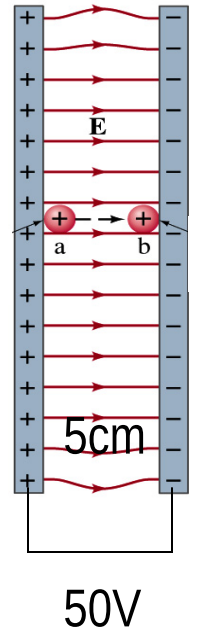


Unit of the electric field in terms of potential?  $\text{V/m}$

Can you derive this from N/C?

## Example 23 – 3

Uniform electric field obtained from voltage:  
Two parallel plates are charged to a voltage of 50V. If the separation between the plates is 5.0cm, calculate the magnitude of the electric field between them, ignoring any fringe effect.



What is the relationship between electric field and the potential for a uniform field?  $V = -Ed$

Solving for E  $E = \frac{V}{d} = \frac{50V}{5.0cm} = \frac{50V}{5 \times 10^{-2}m} = 1000V / m$

# Electric Potential due to Point Charges

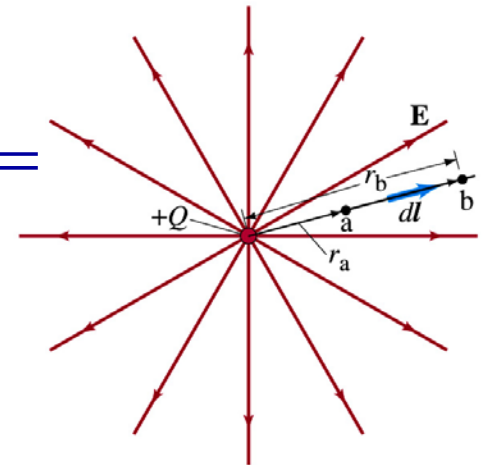
- What is the electric field by a single point charge  $Q$  at a distance  $r$ ?

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = k \frac{Q}{r^2}$$

- Electric potential due to the field  $E$  for moving from point  $r_a$  to  $r_b$  in radial direction away from the charge  $Q$  is

$$V_b - V_a = - \int_{r_a}^{r_b} \vec{E} \cdot d\vec{l} = - \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{\hat{r}}{r^2} \cdot \hat{r} dr =$$

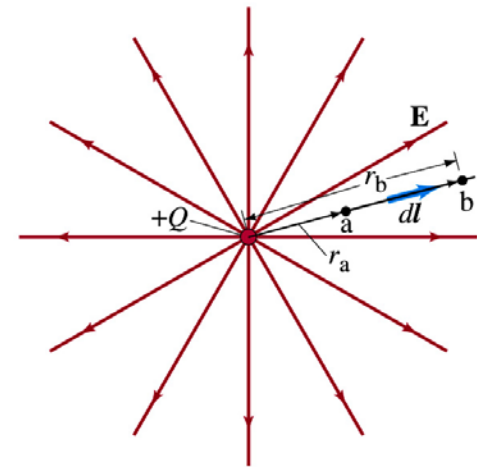
$$= - \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left( \frac{1}{r_b} - \frac{1}{r_a} \right)$$



# Electric Potential due to Point Charges

- Since only the differences in potential have physical meaning, we can choose  $V_b = 0$  at  $r_b = \infty$ .
- The electrical potential  $V$  at a distance  $r$  from a single point charge is

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$



- So the absolute potential by a single point charge can be thought of as the potential difference by a single point charge between  $r$  and infinity

# Properties of the Electric Potential

- What are the differences between the electric potential and the electric field?

- Electric potential

- Electric potential energy per unit charge
    - Inversely proportional to the distance
    - Simply add the potential by each of the charges to obtain the total potential from multiple charges, since potential is a scalar quantity

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

- Electric field

- Electric force per unit charge
    - Inversely proportional to the square of the distance
    - Need vector sums to obtain the total field from multiple charges

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

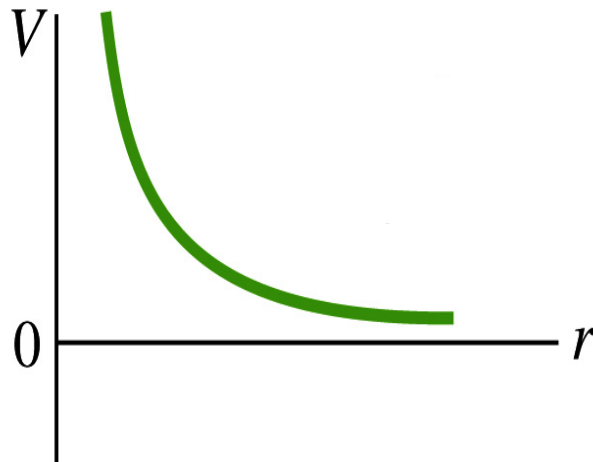
- Potential for the positive charge is large near the charge and decreases towards 0 at a large distance.
- Potential for the negative charge is large negative near the charge and increases towards 0 at a large distance.

# Shape of the Electric Potential

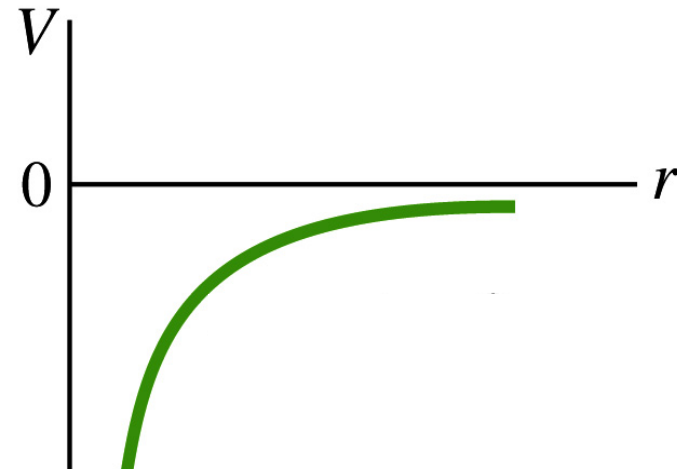
- So, how does the electric potential look like as a function of distance?
  - What is the formula for the potential by a single charge?

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

Positive Charge



Negative Charge



Uniformly charged sphere would have the potential the same as a single point charge.

What does this mean?

Uniformly charged sphere behaves like all the charge is on the single point in the center.

## Example 23 – 6

Work to bring two positive charges close together: What minimum work is required by an external force to bring a charge  $q=3.00\mu\text{C}$  from a great distance away ( $r=\text{infinity}$ ) to a point  $0.500\text{m}$  from a charge  $Q=20.0\mu\text{C}$ ?

What is the work done by the electric field in terms of potential energy and potential?

$$W = -qV_{ba} = -\frac{q}{4\pi\epsilon_0} \left( \frac{Q}{r_b} - \frac{Q}{r_a} \right)$$

Since  $r_b = 0.500\text{m}$ ,  $r_a = \infty$  we obtain

$$W = -\frac{q}{4\pi\epsilon_0} \left( \frac{Q}{r_b} - 0 \right) = -\frac{q}{4\pi\epsilon_0} \frac{Q}{r_b} = -\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \cdot (3.00 \times 10^{-6} \text{ C}) (20.00 \times 10^{-6} \text{ C})}{0.500\text{m}} = -1.08\text{J}$$

Electric force does negative work. In other words, the external force must work +1.08J to bring the charge  $3.00\mu\text{C}$  from infinity to  $0.500\text{m}$  to the charge  $20.0\mu\text{C}$ .





# Electric Potential by Charge Distributions

- Let's consider that there are  $n$  individual point charges in a given space and  $V=0$  at  $r=\text{infinity}$ .
- Then the potential due to the charge  $Q_i$  at a point  $a$ , distance  $r_{ia}$  from  $Q_i$  is

$$V_{ia} = \frac{Q_i}{4\pi\epsilon_0} \frac{1}{r_{ia}}$$

- Thus the total potential  $V_a$  by all  $n$  point charges is

$$V_a = \sum_{i=1}^n V_{ia} = \sum_{i=1}^n \frac{Q_i}{4\pi\epsilon_0} \frac{1}{r_{ia}}$$

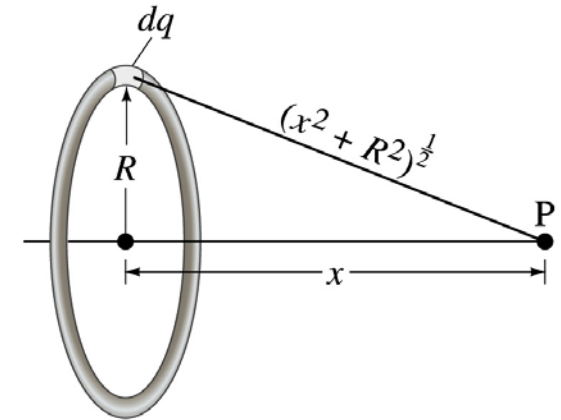
- For a continuous charge distribution, we obtain

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$



## Example 23 – 8

- Potential due to a ring of charge: A thin circular ring of radius  $R$  carries a uniformly distributed charge  $Q$ . Determine the electric potential at a point  $P$  on the axis of the ring a distance  $x$  from its center.



- Each point on the ring is at the same distance from the point  $P$ . What is the distance?

$$r = \sqrt{R^2 + x^2}$$

- So the potential at  $P$  is

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0 r} \int dq =$$
$$\frac{1}{4\pi\epsilon_0 \sqrt{x^2 + R^2}} \int dq = \frac{Q}{4\pi\epsilon_0 \sqrt{x^2 + R^2}}$$

What's this?