PHYS 1444 – Section 501
Lecture #7

Wednesday, Feb. 8, 2006
Dr. Jaehoon Yu

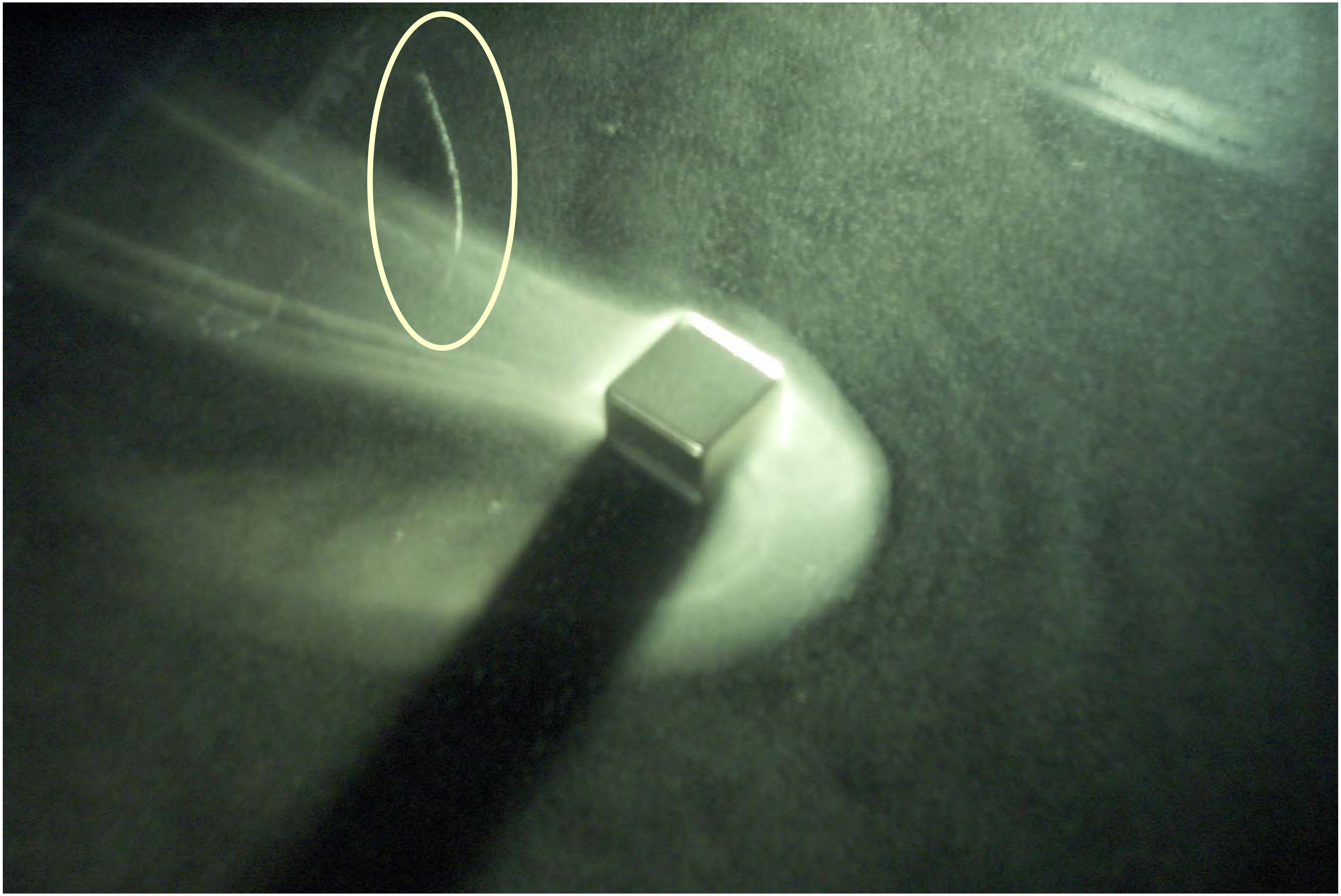
- Equi-potential Lines and Surfaces
- Electric Potential Due to Electric Dipole
- E determined from V
- Electrostatic Potential Energy of a System of Charges
- Capacitors and Capacitance

Today's homework is #4, due 7pm, Thursday, Feb. 16!!

Announcements

- Distribution list
  - I still have 5 of you incommucado…
- A Large Cloud Chamber workshop
  - Date and time: 10am, this Saturday, Feb. 11
  - Will build and operate a prototype chamber with all the new gadgets
- Quiz next Monday, Feb. 13
  - Covers CH21 – CH 23
- 1st term exam Wednesday, Feb. 22
  - Covers CH21 – CH25
- Reading assignments
  - CH23–9

A Cloud Chamber Image of A Cosmic Track

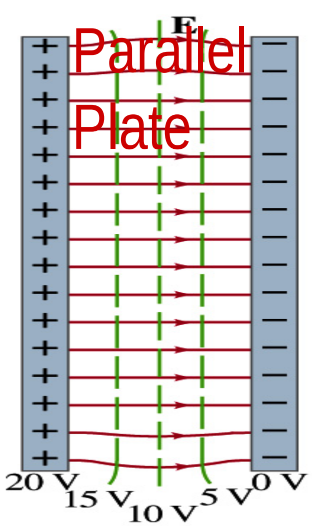


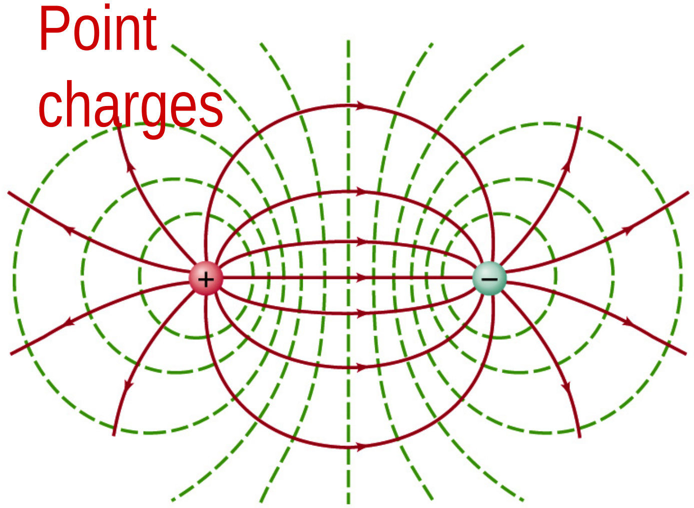
Equi-potential Surfaces

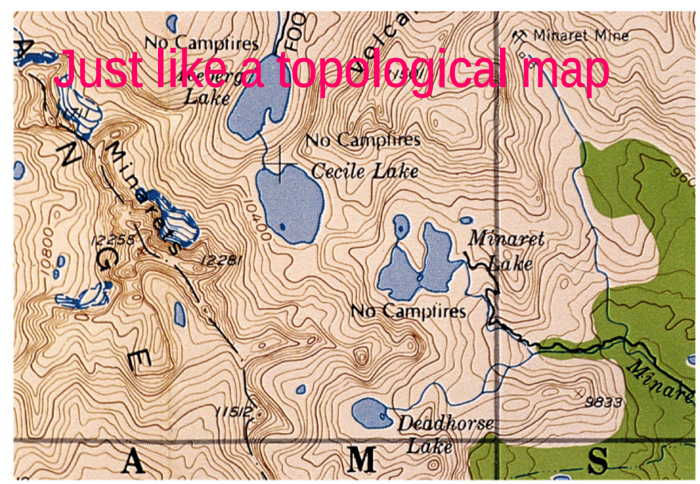
- Electric potential can be visualized using equipotential lines in 2-D or equipotential surfaces in 3-D
- Any two points on equipotential surfaces (lines) are on the same potential
- What does this mean in terms of the potential difference?
  – The potential difference between the two points on an equipotential surface is 0.
- How about the potential energy difference?
  – Also 0.
- What does this mean in terms of the work to move a charge along the surface between these two points?
  – No work is necessary to move a charge between these two points.

Equi-potential Surfaces

- An equipotential surface (line) must be perpendicular to the electric field. Why?
  - If there are any parallel components to the electric field, it would require work to move a charge along the surface.
- Since the equipotential surface (line) is perpendicular to the electric field, we can draw these surfaces or lines easily.
- There can be no electric field inside a conductor in static case, thus the entire volume of a conductor must be at the same potential.
- So the electric field must be perpendicular to the conductor surface.

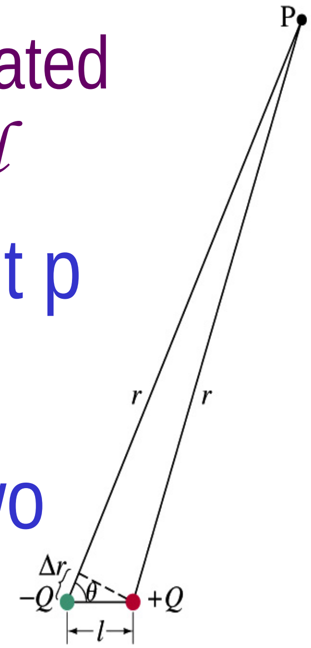






Electric Potential due to Electric Dipoles

- What is an electric dipole?
  - Two equal point charge Q of opposite sign separated by a distance ℓ and behaves like one entity: P=Qℓ
- The electric potential due to a dipole at a point p
  - We take V=0 at r=infinity
- The simple sum of the potential at P by the two charges is



$$V = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} + \frac{(-Q)}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{\Delta r}{r+\Delta r}$$

- Since $\Delta r = \ell\cos\theta$ and if $r \gg \ell$, $r \gg \Delta r$, thus $r \sim r+\Delta r$ and

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{l\cos\theta}{r} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r}$$

V by a dipole at a distance r from the dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r}$$

E Determined from V

- Potential difference between two points under the electric field is $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$
- So in a differential form, we can write

$$dV = -\vec{E} \cdot d\vec{l} = -E_l dl$$

  – What are dV and E_ℓ?
    - dV is the infinitesimal potential difference between two points separated by the distance $d\ell$
    - E_ℓ is the field component along the direction of $d\ell$.
- Thus we can write the field component E_ℓ as

$$E_l = -\frac{dV}{dl}$$

Physical Meaning?

The component of the electric field in any direction is equal to the negative rate of change of the electric potential as a function of distance in that direction.!!

E Determined from V

- The quantity $dV/d\ell$ is called the **gradient of V** in a particular direction
  – If no direction is specified, the term gradient refers to the direction V changes most rapidly, and this would be the direction of the field vector **E** at that point.
  – So if **E** and $d\ell$ are parallel to each other, $E = -\frac{dV}{dl}$
- If E is written as a function of x, y and z, ℓ refers to x, y and z $E_x = -\frac{\partial V}{\partial x}$ $E_y = -\frac{\partial V}{\partial y}$ $E_z = -\frac{\partial V}{\partial z}$
- $\frac{\partial V}{\partial x}$ is the "partial derivative" of V with respect to x, with y and z held constant
- In vector form, $\vec{E} = -gradV = -\vec{\nabla}V = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)V$

$\vec{\nabla} = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)$ is called the *del* or the *gradient operator* and is a *vector operator*.

Electrostatic Potential Energy

- Consider a point charge q is moved between points a and b where the electrostatic potentials due to other charges are V_a and V_b
- The change in electrostatic potential energy of q in the field by other charges is

$$\Delta U = U_b - U_a = q\left(V_b - V_a\right) = qV_{ba}$$

- Now what is the electrostatic potential energy of a system of charges?
  - Let's choose V=0 at r=infinity
  - If there are no other charges around, single point charge Q_1 in isolation has no potential energy and is exerted on with no electric force

Electrostatic Potential Energy; Two charges

- If a second point charge Q_2 is brought close to Q_1 at the distance r_{12}, the potential due to Q_1 at the position of Q_2 is

$$V = \frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r_{12}}$$

- The potential energy of the two charges relative to V=0 at r=infinity is $U = Q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$
  - This is the work that needs to be done by an external force to bring Q_2 from infinity to a distance r_{12} from Q_1.
  - It is also a negative of the work needed to separate them to infinity.

Electrostatic Potential Energy; Three Charges

- So what do we do for three charges?
- Work is needed to bring all three charges together
  - Work needed to bring Q_1 to a certain place without the presence of any charge is 0.
  - Work needed to bring Q_2 to a distance to Q_1 is $U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$
  - Work need to bring Q_3 to a distance to Q_1 and Q_2 is

$$U_3 = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2 Q_3}{r_{23}}$$

- So the total electrostatic potential of the three charge system is

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}}\right) \quad [V = 0 \text{ at } r = \infty]$$

  - What about a four charge system?

Electrostatic Potential Energy: electron Volt

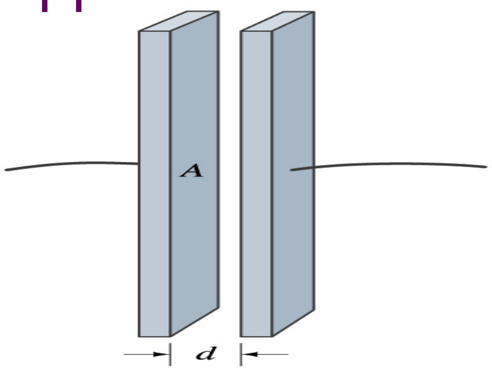
- What is the unit of electrostatic potential energy?
  - Joules
- Joules is a very large unit in dealing with electrons, atoms or molecules in atomic scale problems
- For convenience a new unit, electron volt (eV), is defined
  - 1 eV is defined as the energy acquired by a particle carrying the charge equal to that of an electron (q=e) when it moves across a potential difference of 1V.
  - How many Joules is 1 eV then? $1eV = 1.6\times10^{-19}C\cdot1V = 1.6\times10^{-19}J$
- eV however is <u>not a standard SI unit</u>. You must convert the energy to Joules for computations.
- What is the speed of an electron with kinetic energy 5000eV?

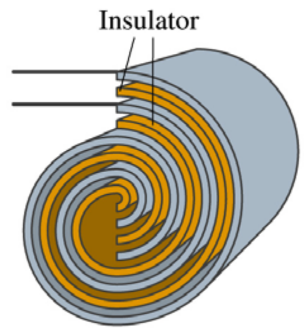
Capacitors (or Condensers)

- What is a capacitor?
  - A device that can store electric charge
  - But does not let them flow through
- What does it consist of?
  - Usually consists of two conducting objects (plates or sheets) placed near each other without touching
  - Why can't they touch each other?
    - The charge will neutralize…
- Can you give some examples?
  - Camera flash, UPS, Surge protectors, binary circuits, etc…
- How is a capacitor different than a battery?
  - Battery provides potential difference by storing energy (usually chemical energy) while the capacitor stores charges but very little energy.

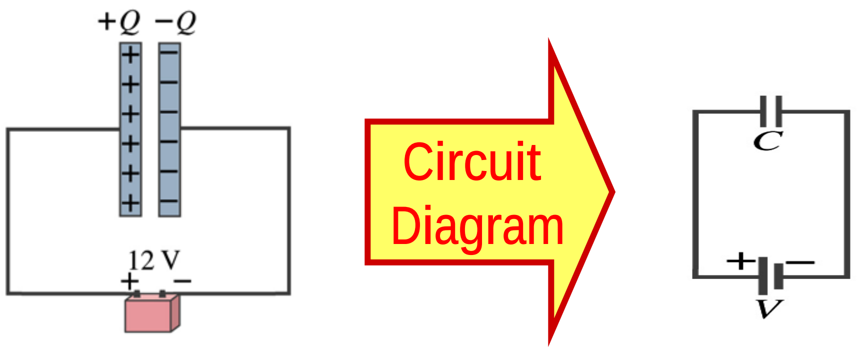
Capacitors

- A simple capacitor consists of a pair of parallel plates of area $\mathcal{A}$ separated by a distance d.
  - A cylindrical capacitors are essentially parallel plates wrapped around as a cylinder.



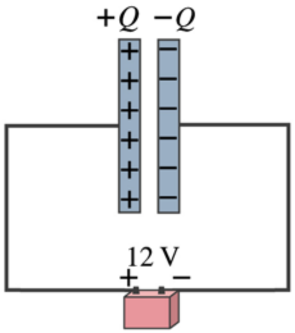


- How would you draw symbols for a capacitor and a battery?
  - Capacitor -||-
  - Battery (+) -|i- (-)



Capacitors

- What do you think will happen if a battery is connected (or the voltage is applied) to a capacitor?
  - The capacitor gets charged quickly, one plate positive and the other negative in equal amount.



- Each battery terminal, the wires and the plates are conductors. What does this mean?
  - All conductors are at the same potential. And?
  - So the full battery voltage is applied across the capacitor plates.
- So for a given capacitor, the amount of charge stored in the capacitor is proportional to the potential difference V_{ba} between the plates. How would you write this formula?

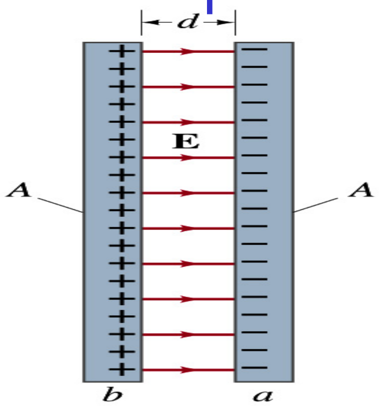
$$Q = CV_{ba}$$

C is a property of a capacitor so does not depend on Q or V.

  - C is a proportionality constant, called capacitance of the device.
  - What is the unit? C/V or Farad (F) Normally use μF or pF.

Determination of Capacitance

- C can be determined analytically for capacitors w/ simple geometry and air in between.
- Let's consider a parallel plate capacitor.
  - Plates have area A each and separated by d.
    - d is smaller than the length, and so E is uniform.
  - E for parallel plates is $E=\sigma/\varepsilon_0$, σ is the surface charge density.



- E and V are related $V_{ba} = -\int_a^b \vec{E} \cdot d\vec{l}$
- Since we take the integral from lower potential point *a* to higher potential point *b* along the field line, we obtain
- $V_{ba} = V_b - V_a = -\int_a^b Edl \cos 180^\circ = +\int_a^b Edl = \int_a^b \frac{\sigma}{\varepsilon_0} dl = \int_a^b \frac{Q}{\varepsilon_0 A} dl = \frac{Q}{\varepsilon_0 A}\int_a^b dl = \frac{Q}{\varepsilon_0 A}(b-a) = \frac{Qd}{\varepsilon_0 A}$
- So from the formula: $C = \frac{Q}{V_{ba}} = \frac{Q}{Qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}$
  - What do you notice?

C only depends on the area and the distance of the plates and the permittivity of the medium between them.

Example 24 – 1

Capacitor calculations: (a) Calculate the capacitance of a capacitor whose plates are 20cmx3.0cm and are separated by a 1.0mm air gap. (b) What is the charge on each plate if the capacitor is connected to a 12-V battery? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1F, given the same air gap.

(a) Using the formula for a parallel plate capacitor, we obtain

$$C = \frac{\varepsilon_0 A}{d} =$$

$$= \left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\frac{0.2\times0.03m^2}{1\times10^{-3}m} = 53\times10^{-12}\,C^2/N\cdot m = 53pF$$

(b) From Q=CV, the charge on each plate is

$$Q = CV = \left(53\times10^{-12}\,C^2/N\cdot m\right)\left(12V\right) = 6.4\times10^{-10}C = 640pC$$

Example 24 – 1

(C) Using the formula for the electric field in two parallel plates

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{6.4\times10^{-10}C}{6.0\times10^{-3}m^2\times8.85\times10^{-12}C^2/N\cdot m^2} = 1.2\times10^4 N/C = 1.2\times10^4 V/m$$

Or, since $V = Ed$ we can obtain $E = \frac{V}{d} = \frac{12V}{1.0\times10^{-3}m} = 1.2\times10^4 V/m$

(d) Solving the capacitance formula for A, we obtain

$$C = \frac{\varepsilon_0 A}{d}$$

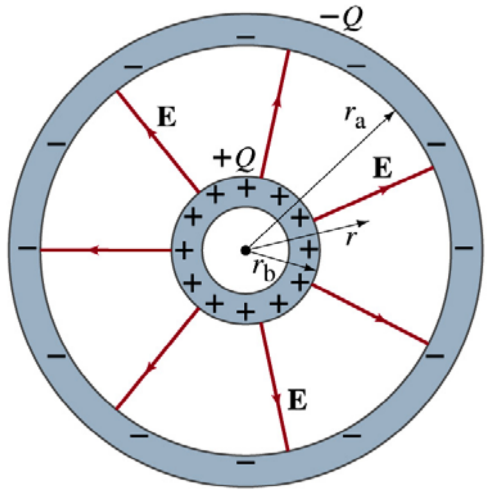
Solve for A

$$A = \frac{Cd}{\varepsilon_0} = \frac{1F\cdot1\times10^{-3}m}{\left(9\times10^{-12}C^2/N\cdot m^2\right)} \approx 10^8 m^2 \approx 100 km^2$$

About 40% the area of Arlington ($256km^2$).

Example 24 – 3

Spherical capacitor: A spherical capacitor consists of two thin concentric spherical conducting shells, of radius r_a and r_b, as in the figure. The inner shell carries a uniformly distributed charge Q on its surface and the outer shell and equal but opposite charge –Q. Determine the capacitance of the two shells.



Using Gauss' law, the electric field outside a uniformly charged conducting sphere is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So the potential difference between a and b is

$$V_{ba} = -\int_a^b \vec{E}\cdot d\vec{l} =$$

$$= -\int_a^b E\cdot dr = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)_{r_a}^{r_b} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a - r_b}{r_b r_a}\right)$$

Thus capacitance is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a - r_b}{r_b r_a}\right)} = \frac{4\pi\varepsilon_0 r_b r_a}{r_a - r_b}$$