PHYS 1444 – Section 501
Lecture #8

Monday, Feb. 13, 2006
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- Capacitors and Capacitance
- Capacitors in Series or Parallel
- Electric Energy Storage
- Electric Energy Density
- Dielectric
- Effect of Dielectric Material

Announcements

- Distribution list
  - Did you all receive my e-mail on a video clip?
  - Raise your hand if you didn't.
- 1st term exam Wednesday, Feb. 22
  - Covers CH21 – CH25 or whichever we finish
- Reading assignments
  - CH24 – 6

Capacitor Cont'd

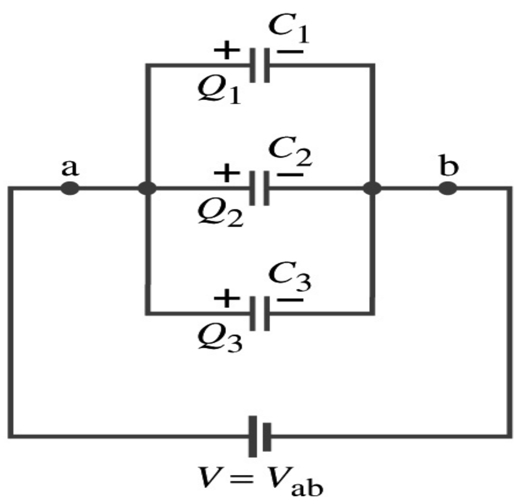
- A single isolated conductor can be said to have a capacitance, C.
- C can still be defined as the ratio of the charge to absolute potential V on the conductor.
  - So Q=CV.
- The potential of a single conducting sphere of radius r_b can be obtained as

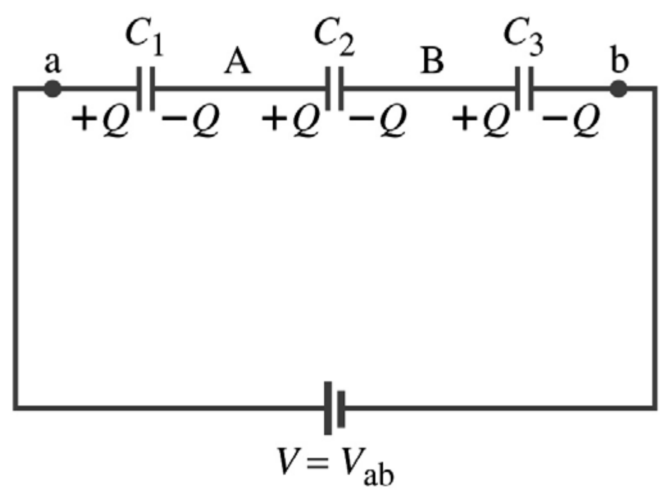
$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0 r_b} \qquad \text{where} \quad r_a \to \infty$$

- So its capacitance is $\boxed{C = \frac{Q}{V} = 4\pi\varepsilon_0 r_b}$
- Single conductor alone is not considered as a capacitor. There must be another object near by to form a capacitor.

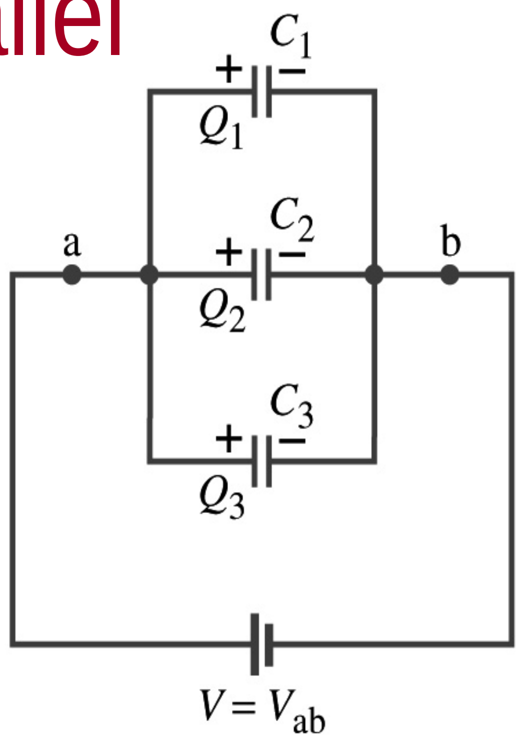
Capacitors in Series or Parallel

- Capacitors are used in may electric circuits
- What is an electric circuit?
  – A closed path of conductors, usually wires connecting capacitors and other electrical devices, in which
    - charges can flow
    - And includes a voltage source such as a battery
- Capacitors can be connected in various ways.
  – In parallel and in Series or in combination





Capacitors in Parallel



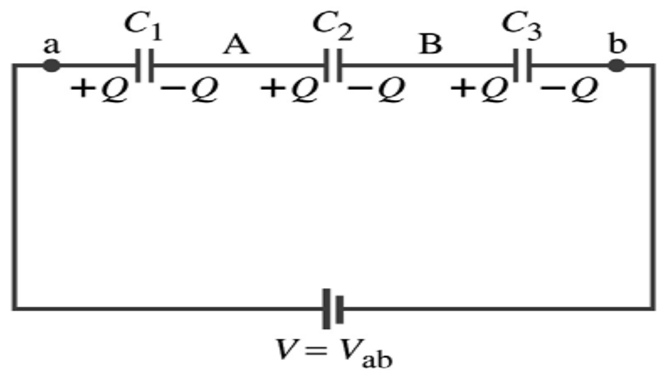
- Parallel arrangement provides the <u>same voltage</u> across all the capacitors.
  - Left hand plates are at V_a and right hand plates are at V_b
  - So each capacitor plate acquires charges given by the formula
    - $Q_1=C_1V$, $Q_2=C_2V$, and $Q_3=C_3V$
- The total charge Q that must leave battery is then
  - $Q=Q_1+Q_2+Q_3=V(C_1+C_2+C_3)$
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V= V(C_1+C_2+C_3)$
- Thus the equivalent capacitance in parallel is $\boxed{C_{eq} = C_1 + C_2 + C_3}$

What is the net effect?

The capacitance increases!!!

Capacitors in Series

- Series arrangement is more interesting
  - When battery is connected, +Q flows to the left plate of C_1 and –Q flows to the right plate of C_3.inducing opposite sign charges on the other plates.
  - Since the capacitor in the middle is originally neutral, charges get induced to neutralize the induced charges
  - So the charge on each capacitor is the same value, Q. (<u>Same charge</u>)

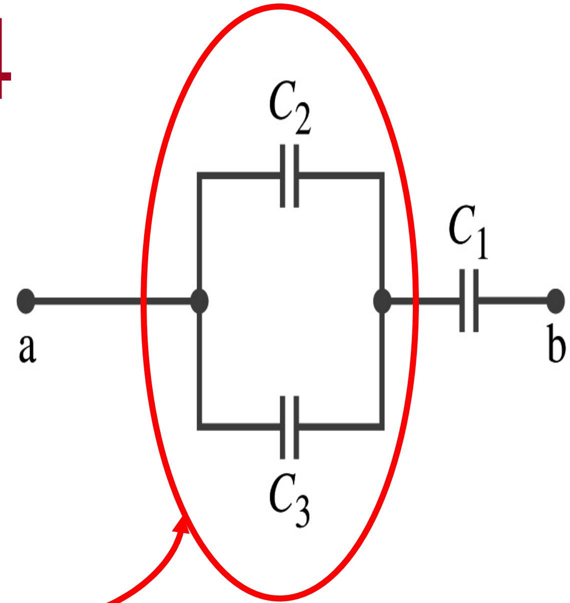


- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V \Rightarrow V=Q/C_{eq}$
- The total voltage V across the three capacitors in series must be equal to the sum of the voltages across each capacitor.
  - $V=V_1+V_2+V_3=(Q/C_1+Q/C_2+Q/C_3)$
- Putting all these together, we obtain:
- $V=Q/C_{eq}=Q(1/C_1+1/C_2+1/C_3)$
- Thus the equivalent capacitance is $\boxed{\dfrac{1}{C_{eq}}=\dfrac{1}{C_1}+\dfrac{1}{C_2}+\dfrac{1}{C_3}}$

What is the net effect?

The capacitance smaller than the smallest C!!!

Example 24 – 4



Equivalent Capacitor: Determine the capacitance of a single capacitor that will have the same effect as the combination shown in the figure. Take $C_1=C_2=C_3=C$.

We should do these first!!

How? These are in parallel so the equivalent capacitance is:

$$C_{eq1} = C_1 + C_2 = 2C$$

Now the equivalent capacitor is in series with C1.

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq1}} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

Solve for C_{eq}

$$C_{eq} = \frac{2C}{3}$$

Electric Energy Storage

- A charged capacitor stores energy.
  - The stored energy is the work done to charge it.
- The net effect of charging a capacitor is removing one type of charge from a plate and put them on to the other.
  - Battery does this when it is connected to a capacitor.
- Capacitors do not charge immediately.
  - Initially when the capacitor is uncharged, no work is necessary to move the first bit of charge. Why?
    - Since there is no charge, there is no field that the external work needs to overcome.
  - When some charge is on each plate, it requires work to add more charge due to electric repulsion.

Electric Energy Storage

- The work needed to add a small amount of charge, dq, when a potential difference across the plate is V: dW=Vdq.
- Since V=q/C, the work needed to store total charge Q is

$$W = \int_0^Q V dq = \frac{1}{C}\int_0^Q q dq = \frac{Q^2}{2C}$$

- Thus, the energy stored in a capacitor when the capacitor carries charges +Q and –Q is

$$\boxed{U = \frac{Q^2}{2C}}$$

- Since Q=CV, we can rewrite

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Example 24 – 7

Energy store in a capacitor: A camera flash unit stores energy in a 150μF capacitor at 200V. How much electric energy can be stored?

Use the formula for stored energy. Umm.. Which one?

What do we know from the problem? C and V

So we use the one with C and V: $U = \frac{1}{2}CV^2$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(150\times10^{-6}F\right)\left(200V\right)^2 = 3.0J$$

How do we get J from FV²? $FV^2 = \left(\frac{C}{V}\right)V^2 = CV = C\left(\frac{J}{C}\right) = J$

Electric Energy Density

- The energy stored in a capacitor can be considered as being stored in the electric field between the two plates
- For a uniform field E between two plates, $V=Ed$ and $C=\varepsilon_0 A/d$
- Thus the stored energy is

$$U=\frac{1}{2}CV^2=\frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2=\frac{1}{2}\varepsilon_0 E^2 Ad$$

- Since Ad is the gap volume V, we can obtain the energy density, stored energy per unit volume, as

$$\boxed{u=\frac{1}{2}\varepsilon_0 E^2}$$

Valid for any space that is vacuum

Electric energy stored per unit volume in any region of space is proportional to the square of E in that region.

Dielectrics

- Capacitors have an insulating sheet of material, called dielectric, between the plates to
  - Increase breakdown voltage than that in the air
  - Higher voltage can be applied without the charge passing across the gap
  - Allow the plates get closer together without touching
    - Increases capacitance (recall $C=\varepsilon_0 A/d$)
  - Also increases the capacitance by the dielectric constant

$$\boxed{C = KC_0}$$

    - Where C_0 is the intrinsic capacitance when the gap is vacuum

Dielectrics

- The value of dielectric constant varies depending on material (Table 24 – 1)
  - K for vacuum is 1.0000
  - K for air is 1.0006 (this is why permittivity of air and vacuum are used interchangeably.)
- Maximum electric field before breakdown occurs is the dielectric strength. What is its unit?
  - V/m
- The capacitance of a parallel plate capacitor with a dielectric (K) filling the gap is

$$\boxed{C = KC_0 = K\varepsilon_0 \frac{A}{d}}$$

Dielectrics

- A new quantity, the permittivity of dielectric, is defined as $\varepsilon = K\varepsilon_0$
- The capacitance of a parallel plate with a dielectric medium filling the gap is

$$\boxed{C = \varepsilon \frac{A}{d}}$$

- The energy density stored in an electric field E in a dielectric is

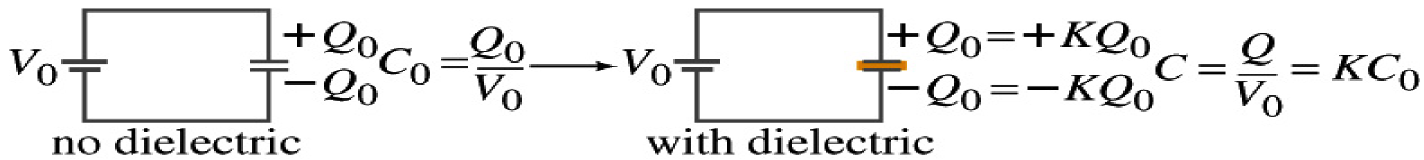
$$\boxed{u = \frac{1}{2} K \varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2}$$

Valid for any space w/ dielectric w/ permittivity ε.

Effect of a Dielectric Material

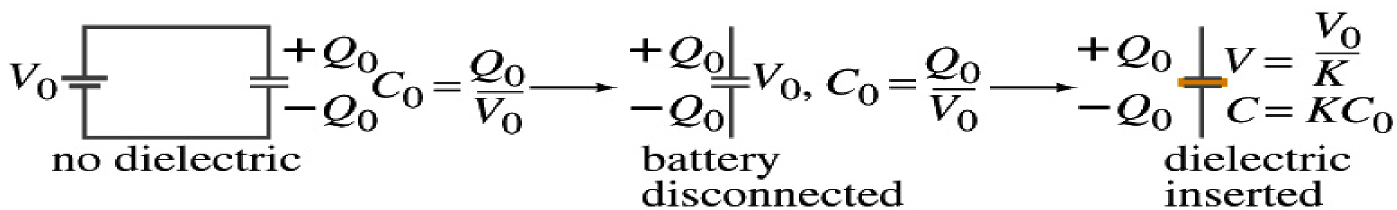
- Let's consider the two cases below:

Case #1 : constant V



(a) Voltage constant

Case #2 : constant Q



(b) Charge constant

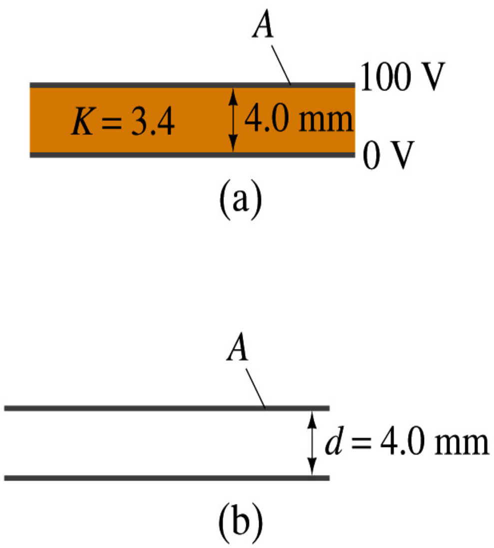
- Constant voltage: Experimentally observed that the total charge on the each plate of the capacitor increases by K as the dielectric material is inserted between the gap ➔ $Q=KQ_0$
  - The capacitance increased to $C=Q/V_0=KQ_0/V_0=KC_0$
- Constant charge: Voltage found to drop by a factor K ➔ $V=V_0/K$
  - The capacitance increased to $C=Q_0/V=KQ_0/V_0=KC_0$

Effect of a Dielectric Material on Field

- What happens to the electric field within a dielectric?
- Without a dielectric, the field is $$E_0 = \frac{V_0}{d}$$
  - What are V_0 and d?
    - V_0: Potential difference between the two plates
    - d: separation between the two plates
- For the constant voltage, the electric field remains the same
- For the constant charge: the voltage drops to $V=V_0/K$, thus the field in the dielectric is $$E = E_D = \frac{V}{d} = \frac{V_0}{dK} = \frac{E_0}{K}$$
  - Reduced. $$E_D = \frac{E_0}{K}$$

Example 24 – 8

Dielectric Removal: A parallel-plate capacitor, filled with a dielectric with K=3.4, is connected to a 100-V battery. After the capacitor is fully charged, the battery is disconnected. The plates have area $A=4.0m^2$, and are separated by d=4.0mm. (a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new value of capacitance, electric field strength, voltage between the plates and the energy stored in the capacitor.



(a) $$C=\frac{\varepsilon A}{d}=\frac{K\varepsilon_0 A}{d}=\left(3.4\times 8.85\times 10^{-12}\, C^2/N\cdot m^2\right)\frac{4.0m^2}{4.0\times 10^{-3}m}=3.0\times 10^{-8}F=30nF$$

$$Q=CV=\left(3.0\times 10^{-8}F\right)\times 100V=3.0\times 10^{-6}C=3.0\mu C$$

$$E=\frac{V}{d}=\frac{100V}{4.0\times 10^{-3}m}=2.5\times 10^{4}\, V/m$$

$$U=\frac{1}{2}CV^2=\frac{1}{2}\left(3.0\times 10^{-8}F\right)\left(100V\right)^2=1.5\times 10^{-4}J$$

Example 24 – 8 cont'd

(b) Since the dielectric has been removed, the effect of dielectric constant must be removed as well.

$$C_0=\frac{C}{K}=\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\frac{4.0m^2}{4.0\times10^{-3}m}=8.8\times10^{-9}F=8.8nF$$

Since charge is the same ($Q_0=Q$) before and after the removal of the dielectric, we obtain

$$V_0=Q/C_0=K\,Q/C=KV=3.4\times100V=340V$$

$$E_0=\frac{V_0}{d}=\frac{340V}{4.0\times10^{-3}m}=8.5\times10^4\,V/m=84\,kV/m$$

$$U_0=\frac{1}{2}C_0V_0^2=\frac{1}{2}\frac{C}{K}(KV)^2=\frac{1}{2}KCV^2=KU=3.4\times1.5\times10^{-4}J=5.1\times10^{-4}J$$

Where did the extra energy come from?.

~~Th[illegible]ergy conservation law is violated in elec[illegible]??~~

Wrong!
Wrong!
Wrong!

External force has done the work of 3.6×10^{-4}J on the system to remove dielectric!!