

PHYS 1444 – Section 501

Lecture #13

Wednesday, Mar. 8, 2006

Dr. Jaehoon Yu

- Analysis of RC Circuits
- Discharging of RC Circuits
- Application of RC Circuits
- Magnetism and Magnetic Field
- Electric Current and Magnetism
- Magnetic Forces on Electric Current

Today's homework is #7, due 7pm, Thursday, Mar. 23!!



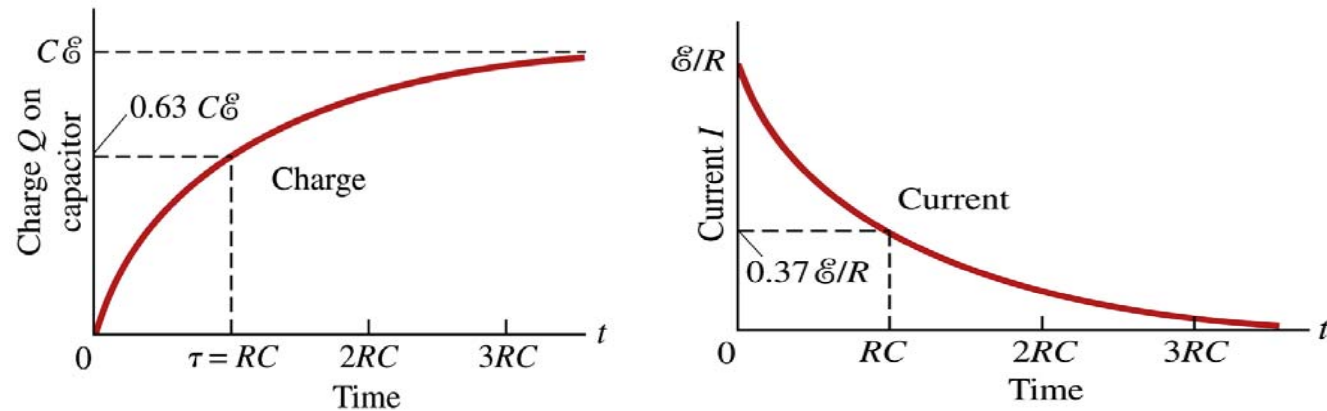
Announcements

- Please bring back your exams to me by Monday, Mar. 20, if you didn't today
- Quiz on Monday, Mar. 20
 - Covers CH 25, 26 and part of 27
- Reminder for the reading assignments
 - CH26 – 5 and 26 – 6
- Spring break next week
 - Have a great break!



RC Circuits

- How does all this look like in graphs?
 - Charge and the current on the capacitor as a function of time



- From energy conservation (Kirchhoff's 2nd rule), the emf $\mathcal{E}$ must be equal to the voltage drop across the capacitor and the resistor
 - $\mathcal{E} = IR + Q/C$
 - R includes all resistance in the circuit, including the internal resistance of the battery, I is the current in the circuit at any instant, and Q is the charge of the capacitor at that same instance.

Analysis of RC Circuits

- From the energy conservation, we obtain
 $\varepsilon = IR + Q/C$
- Which ones are constant in the above equation?
 - ε , R and C are constant
 - Q and I are functions of time
- How do we write the rate at which the charge is accumulated on the capacitor?
 - We can rewrite the above equation as $\varepsilon = R \frac{dQ}{dt} + \frac{1}{C} Q$
 - This equation can be solved by rearranging the terms as $\frac{dQ}{C\varepsilon - Q} = \frac{dt}{RC}$



Analysis of RC Circuits

- Now integrating from $t=0$ when there was no charge on the capacitor to t when the capacitor is fully charged, we obtain

- $\int_0^Q \frac{dQ}{C\mathcal{E} - Q} = \frac{1}{RC} \int_0^t dt \quad \rightarrow$
- $-\ln(C\mathcal{E} - Q) \Big|_0^Q = -\ln(C\mathcal{E} - Q) - (-\ln C\mathcal{E}) = \frac{t}{RC} \Big|_0^t = \frac{t}{RC}$
- So, we obtain $\ln\left(1 - \frac{Q}{C\mathcal{E}}\right) = -\frac{t}{RC} \quad \rightarrow \quad 1 - \frac{Q}{C\mathcal{E}} = e^{-t/RC}$
- Or $Q = C\mathcal{E}(1 - e^{-t/RC})$
- The potential difference across the capacitor is $V=Q/C$, so $V_C = \mathcal{E}(1 - e^{-t/RC})$



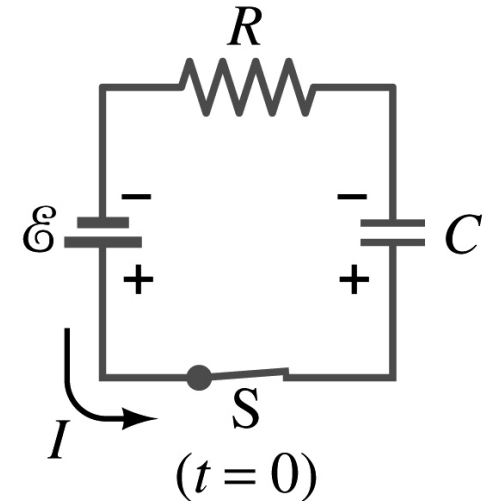
Analysis of RC Circuits

- Since $Q = C\varepsilon(1 - e^{-t/RC})$ and $V_C = \varepsilon(1 - e^{-t/RC})$
- What can we see from the above equations?
 - Q and V_C increase from 0 at $t=0$ to maximum value $Q_{\max}=C\varepsilon$ and $V_C=\varepsilon$.
- In how much time?
 - The quantity RC is called the time constant, τ , of the circuit
 - $\tau=RC$, What is the unit? **Sec.**
 - What is the physical meaning?
 - The time required for the capacitor to reach $(1-e^{-1})=0.63$ or 63% of the full charge
- The current is $I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$



Example 26 – 12

RC circuit, with emf. The capacitance in the circuit of the figure is $C=0.30\mu\text{F}$, the total resistance is $20\text{k}\Omega$, and the battery emf is 12V . Determine (a) the time constant, (b) the maximum charge the capacitor could acquire, (c) the time it takes for the charge to reach 99% of this value, (d) the current I when the charge Q is half its maximum value, (e) the maximum current, and (f) the charge Q when, the current I is 0.20 its maximum value.



(a) Since $\tau = RC$ We obtain $\tau = 20 \times 10^3 \cdot 0.30 \times 10^{-6} = 6.0 \times 10^{-3} \text{ sec}$

(b) Maximum charge is $Q_{\text{max}} = C\mathcal{E} = 0.30 \times 10^{-6} \cdot 12 = 3.6 \times 10^{-6} \text{ C}$

(c) Since $Q = C\mathcal{E}(1 - e^{-t/RC})$ For 99% we obtain $0.99C\mathcal{E} = C\mathcal{E}(1 - e^{-t/RC})$
 $e^{-t/RC} = 0.01$; $-t/RC = -2\ln 10$; $t = RC \cdot 2\ln 10 = 4.6RC = 28 \times 10^{-3} \text{ sec}$

(d) Since $\mathcal{E} = IR + Q/C$ We obtain $I = (\mathcal{E} - Q/C)/R$

The current when Q is $0.5Q_{\text{max}}$ $I = (12 - 1.8 \times 10^{-6} / 0.30 \times 10^{-6}) / 20 \times 10^3 = 3 \times 10^{-4} \text{ A}$

(e) When is I maximum? when $Q=0$: $I = 12 / 20 \times 10^3 = 6 \times 10^{-4} \text{ A}$

(f) What is Q when $I=120\text{mA}$? $Q = C(\mathcal{E} - IR) =$

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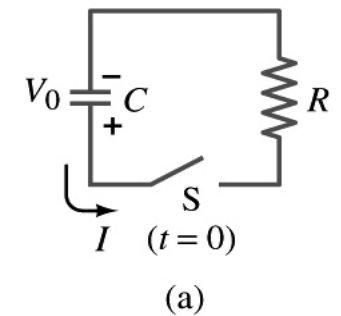


PHY 452 = $0.30 \times 10^{-6} (12 - 1.2 \times 10^{-4} \cdot 2 \times 10^4) = 2.9 \times 10^{-6} \text{ C}$

Dr. J. J. J. J. J.

Discharging RC Circuits

- When a capacitor is already charged, it is allowed to discharge through a resistance R .



- When the switch S is closed, the voltage across the resistor at any instant equals that across the capacitor. Thus $IR=Q/C$.
- The rate at which the charge leaves the capacitor equals the negative the current flows through the resistor
 - $I = -dQ/dt$. Why negative?
 - Since the current is leaving the capacitor
- Thus the voltage equation becomes a differential equation

$$-\frac{dQ}{dt}R = \frac{Q}{C} \quad \xrightarrow{\text{Rearrange terms}} \quad \frac{dQ}{Q} = -\frac{dt}{RC}$$

Discharging RC Circuits

- Now, let's integrate from $t=0$ when the charge is Q_0 to t when the charge is Q

$$\int_{Q_0}^Q \frac{dQ}{Q} = - \int_0^t \frac{dt}{RC}$$

- The result is $\ln Q \Big|_{Q_0}^Q = \ln \frac{Q}{Q_0} = - \frac{t}{RC}$

- Thus, we obtain

$$Q(t) = Q_0 e^{-t/RC}$$

- What does this tell you about the charge on the capacitor?

- It decreases exponentially w/ time and w/ time constant RC
- Just like the case of charging

What is this?

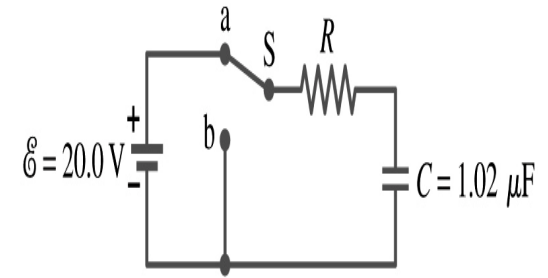
- The current is: $I = - \frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC}$

$$I(t) = I_0 e^{-t/RC}$$

- The current also decreases exponentially w/ time w/ constant RC

Example 26 – 13

Discharging RC circuit. In the RC circuit shown in the figure the battery has fully charged the capacitor, so $Q_0 = C\varepsilon$. Then at $t=0$, the switch is thrown from position a to b. The battery emf is 20.0V, and the capacitance $C=1.02\mu\text{F}$. The current I is observed to decrease to 0.50 of its initial value in $40\mu\text{s}$. (a) what is the value of R ? (b) What is the value of Q , the charge on the capacitor, at $t=0$? (c) What is Q at $t=60\mu\text{s}$?



(a) Since the current reaches to 0.5 of its initial value in $40\mu\text{s}$, we can obtain

$$I(t) = I_0 e^{-t/RC} \xrightarrow{\text{For } 0.5I_0} 0.5I_0 = I_0 e^{-t/RC} \xrightarrow{\text{Rearrange terms}} -t/RC = \ln 0.5 = -\ln 2$$

$$\xrightarrow{\text{Solve for R}} R = t/(C \ln 2) = 40 \times 10^{-6} / (1.02 \times 10^{-6} \cdot \ln 2) = 56.6\Omega$$

(b) The value of Q at $t=0$ is

$$Q_0 = Q_{\max} = C\varepsilon = 1.02 \times 10^{-6} \cdot 20.0 = 20.4\mu\text{C}$$

(c) What do we need to know first for the value of Q at $t=60\mu\text{s}$?

The RC time $\tau = RC = 56.6 \cdot 1.02 \times 10^{-6} = 57.7\mu\text{s}$

Thus $Q(t = 60\mu\text{s}) = Q_0 e^{-t/RC} = 20.4 \times 10^{-6} \cdot e^{-60\mu\text{s}/57.7\mu\text{s}} = 7.2\mu\text{C}$

Application of RC Circuits

- What do you think the charging and discharging characteristics of RC circuits can be used for?

- To produce voltage pulses at a regular frequency

- How?

- The capacitor charges up to a particular voltage and discharges
- A simple way of doing this is to use breakdown of voltage in a gas filled tube

- The discharge occurs when the voltage breaks down at V_0
- After the completion of discharge, the tube no longer conducts
- Then the voltage is at V_0' and it starts charging up
- How do you think the voltage as a function of time look?

» A sawtooth shape

- Pace maker, intermittent windshield wiper, etc

