PHYS 1444 – Section 501
Lecture #17

Wednesday, Mar. 29, 2006
Dr. Jaehoon Yu

- Solenoid and Toroidal Magnetic Field
- Biot-Savart Law
- Magnetic Materials
- B in Magnetic Materials
- Hysteresis

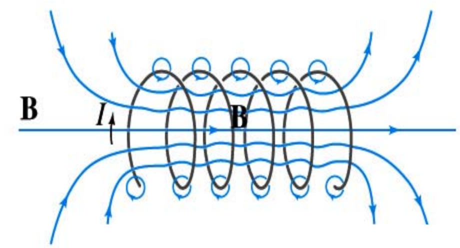
Today's homework is #9, due 7pm, Thursday, Apr. 13!!

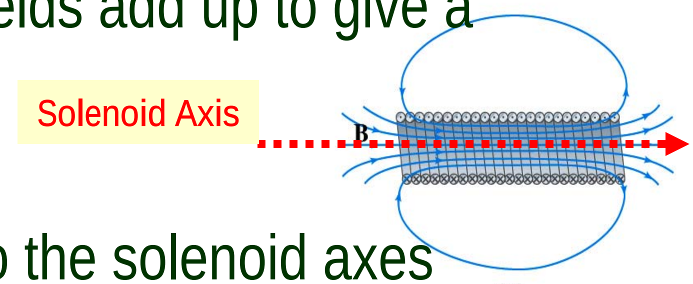
Announcements

- Reading assignments
  - CH28 – 7, 28 – 8, 28 – 9 and 28 – 10
- Two Colloquia you must attend
  - Dr. H. Weerts, director of High Energy Physics Division at Argonne National Laboratory
    - Friday, Apr. 21
    - International Linear Collider: The Physics and Its Challenges
  - Dr. I. Hinchcliff, Lorentz Berkley Laboratory
    - Wednesday, Apr. 26
    - Title: Early Physics with ATLAS at the LHC
- Term exam #2
  - Date and time: 5:30 – 6:50pm, Wednesday, Apr. 5
  - Location: SH103
  - Coverage: Ch. 25 – 4 to Ch. 28

Solenoid and Its Magnetic Field

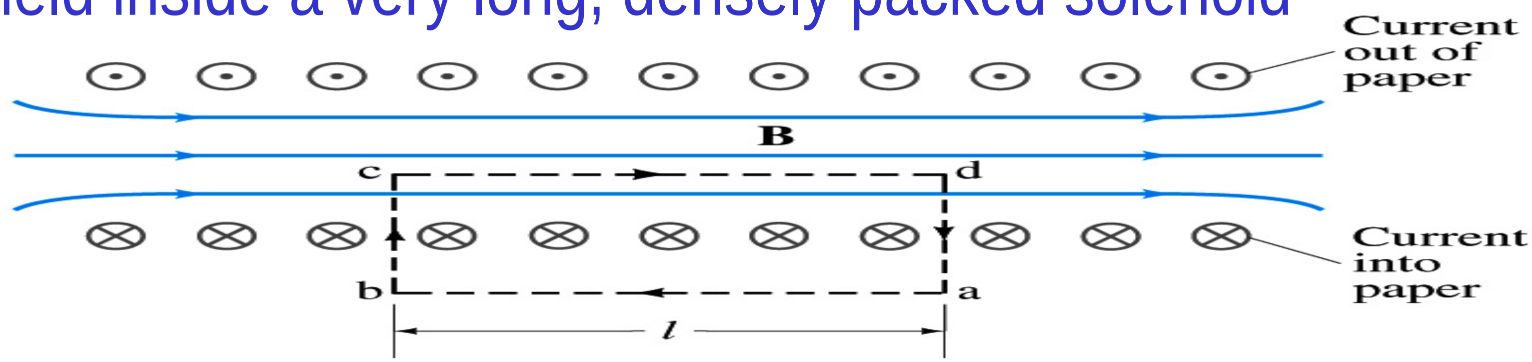
- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops are wide
    - The field near the wires are nearly circular
    - Between any two wires, the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform
  - For a long, densely packed loops
    - The field is nearly uniform and parallel to the solenoid axes within the entire cross section
    - The field outside the solenoid is very small compared to the field inside, except the ends
      - The same number of field lines spread out to an open space





Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid

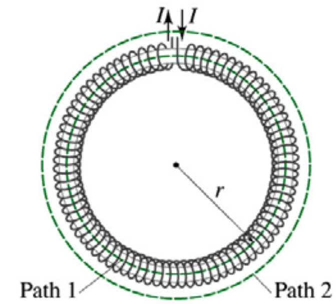


- Let's choose the path *abcd*, far away from the ends
  - We can consider four segments of the loop for integral
  - $\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} \quad \int_d^a \vec{B}\cdot d\vec{l}$
  - The field outside the solenoid is negligible. So the integral on *a*→*b* is 0.
  - Now the field B is perpendicular to the *bc* and *da* segments. So these integrals become 0, also.

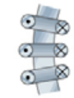
Solenoid Magnetic Field

- So the sum becomes: $\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$
- If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$
  - Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed
- Thus Ampere's law gives us $Bl = \mu_0 NI$
- If we let $n=\mathcal{N}/\ell$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes
- $$B = \mu_0 nI$$
  - B depends on the number of loops per unit length, n, and the current I
    - Does not depend on the position within the solenoid but uniform inside it, like a bar magnet

Example 28 – 8



(a)



(b)

Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle.

(a) How do you think the magnetic field lines inside the toroid look?

Since it is a bent solenoid, it should be a circle concentric with the toroid.

If we choose path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path, we obtain from Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 NI$$

Solving for B

$$B = \frac{\mu_0 NI}{2\pi r}$$

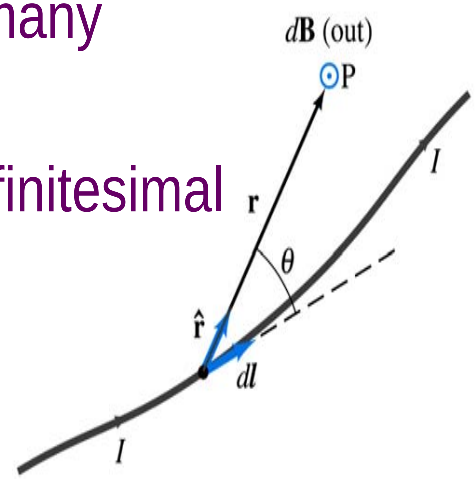
So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin and $B = \mu_0 n I$.

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.

Biot-Savart Law

- Ampere's law is useful in determining magnetic field utilizing symmetry
- But sometimes it is useful to have another method of using infinitesimal current segments for B field
  - Jean Baptiste Biot and Feilx Savart developed a law that a current I flowing in any path can be considered as many infinitesimal current elements
  - The infinitesimal magnetic field d**B** caused by the infinitesimal length dℓ that carries current I is
  - $$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2}$$

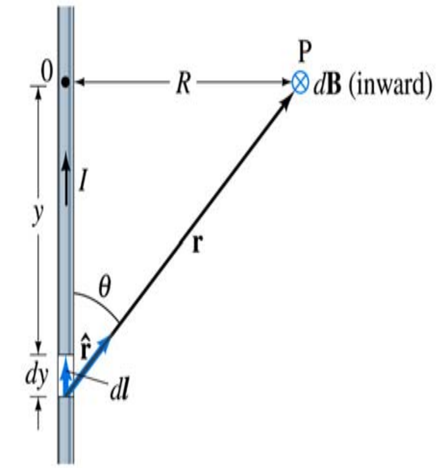
Biot-Savart Law



    - **r** is the displacement vector from the element dℓ to the point P
    - Biot-Savart law is the magnetic equivalent to Coulomb's law

B field in Biot-Savart law is only that by the current, nothing else.

Example 28 – 9



B due to current I in a straight wire. For the field near a long straight wire carrying a current I, show that the Biot-Savarat law gives the same result as the simple long straight wire, $B=\mu_0 I/2\pi R$.

What is the direction of the field **B** at point P? Going into the page.

All dB at point P has the same direction based on right-hand rule.

The magnitude of B using Biot-Savart law is

$$B=\oint dB=\frac{\mu_0 I}{4\pi}\int_{-\infty}^{+\infty}\frac{\left|d\vec{l}\times\hat{r}\right|}{r^2}=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}$$

Where dy=dℓ and $r^2=R^2+y^2$ and since $y=-R\cot\theta$ we obtain

$$dy=+R\csc^2\theta d\theta=\frac{Rd\theta}{\sin^2\theta}=\frac{Rd\theta}{(R/r)^2}=\frac{r^2 d\theta}{R}$$

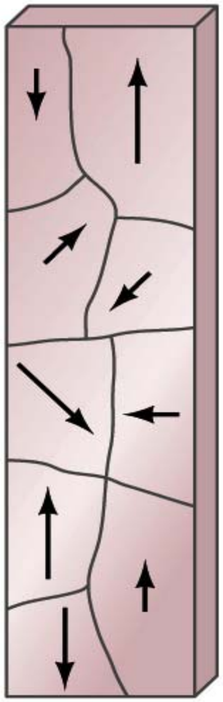
Integral becomes

$$B=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}=\frac{\mu_0 I}{4\pi}\frac{1}{R}\int_{\theta=0}^{\pi}\sin\theta d\theta=-\frac{\mu_0 I}{4\pi}\frac{1}{R}\cos\theta\Big|_0^{\pi}=\frac{\mu_0 I}{2\pi}\frac{1}{R}$$

The same as the simple, long straight wire!! It works!!

Magnetic Materials - Ferromagnetism

- Iron is a material that can turn into a strong magnet
  - This kind of material is called <u>**ferromagnetic**</u> material
- In microscopic sense, ferromagnetic materials consists of many tiny regions called <u>**domains**</u>
  - Domains are like little magnets usually smaller than 1mm in length or width
- What do you think the alignment of domains are like when they are not magnetized?
  - Randomly arranged



- What if they are magnetized?
  - The size of the domains aligned with the external magnetic field direction grows while those of the domains not aligned reduce
  - This gives magnetization to the material
- How do we demagnetize a bar magnet?
  - Hit the magnet hard or heat it over the Curie temperature

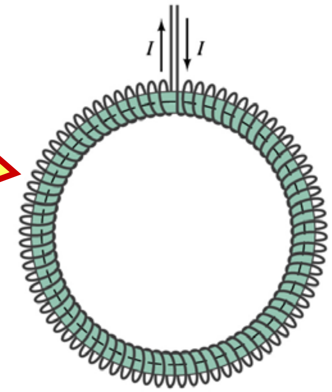
B in Magnetic Materials

- What is the magnetic field inside a solenoid?
- $B_0 = \mu_0 nI$
  - Magnetic field in a long solenoid is directly proportional to the current.
  - This is valid only if air is inside the coil
- What do you think will happen to B if we have something other than the air inside the solenoid?
  - It will be increased dramatically, when the current flows
    - Especially if a ferromagnetic material such as an iron is put inside, the field could increase by several orders of magnitude
- Why?
  - Since the domains in the iron aligns permanently by the external field.
  - The resulting magnetic field is the sum of that due to current and due to the iron

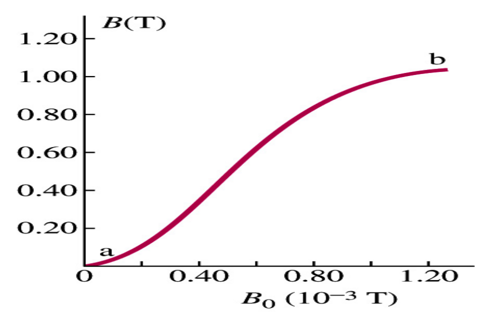
B in Magnetic Materials

- It is sometimes convenient to write the total field as the sum of two terms
- $\vec{B} = \vec{B}_0 + \vec{B}_M$
  - B_0 is the field due only to the current in the wire, namely the external field
    - The field that would be present without a ferromagnetic material
  - B_M is the additional field due to the ferromagnetic material itself; often $B_M >> B_0$
- The total field in this case can be written by replacing μ_0 with another proportionality constant μ, the magnetic permeability of the material $B = \mu n I$
  - μ is a property of a magnetic material
  - μ is not a constant but varies with the external field

Hysteresis

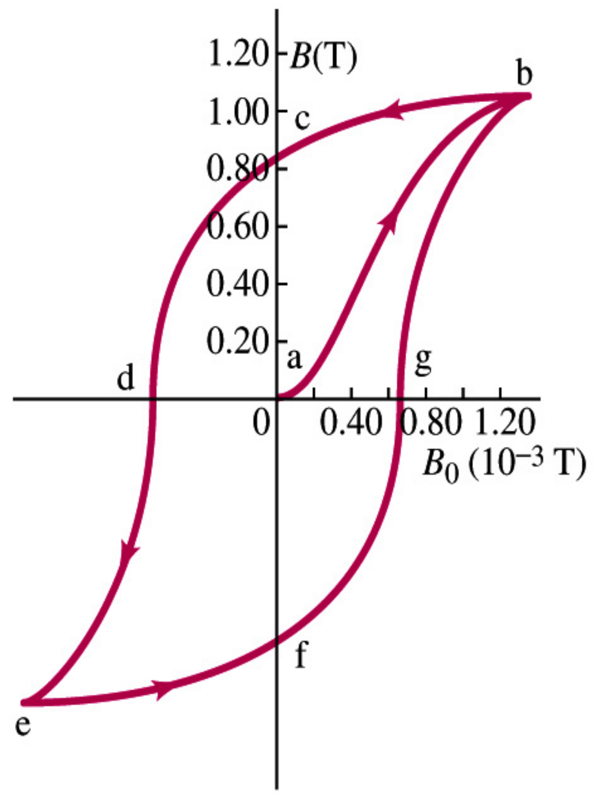


- What is a toroid?
  - A solenoid bent into a shape
- Toroid is used for magnetic field measurement
  - Why?
  - Since it does not leak magnetic field outside of itself, it fully contains all the magnetic field created within it.
- Consider an un-magnetized iron core toroid, without any current flowing in the wire
  - What do you think will happen if the current slowly increases?
  - B_0 increases linearly with the current.
  - And B increases also but follows the curved line shown in the graph
  - As B_0 increases, the domains become more aligned until nearly all are aligned (point b on the graph)
    - The iron is said to be approaching saturation
    - Point b is typically at 70% of the max



Hysteresis

- What do you think will happen to B if the external field B_0 is reduced to 0 by decreasing the current in the coil?
  - ~~Of course it goes to 0!!~~
  - Wrong! Wrong! Wrong! They do not go to 0. Why not?
  - The domains do not completely return to random alignment state
- Now if the current direction is reversed, the external magnetic field direction is reversed, causing the total field B pass 0, and the direction reverses to the opposite side
  - If the current is reversed again, the total field B will increase but never goes through the origin
- This kind of curve whose path does not retrace themselves and does not go through the origin is called the Hysteresis.



Magnetically Soft Material

- In a hysteresis cycle, much energy is transformed to thermal energy. Why?
  - Due to the microscopic friction between domains as they change directions to align with the external field
- The energy dissipated in the hysteresis cycle is proportional to the area of the hysteresis loop
- Ferromagnetic material with large hysteresis area is called magnetically hard while the small ones are called soft
  - Which ones do you think are preferred in electromagnets or transformers?
    - Soft. Why?
    - Since the energy loss is small and much easier to switch off the field
- Then how do we demagnetize a ferromagnetic material?
  - Keep repeating the Hysteresis loop, reducing the range of B_0.