PHYS 1444 – Section 501
Lecture #20

Monday, Apr. 17, 2006
Dr. Jaehoon Yu

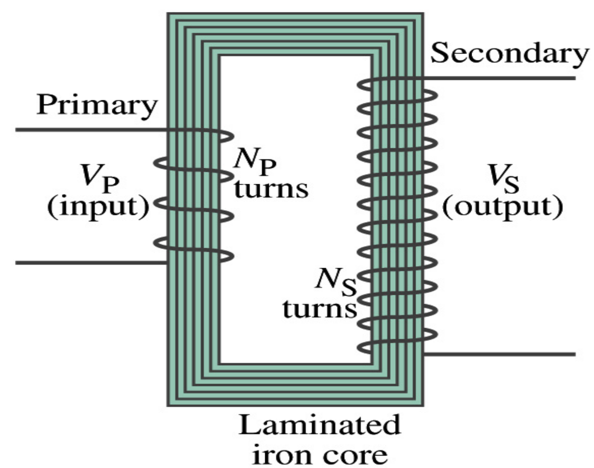
- Transformer
- Generalized Faraday's Law
- Inductance
- Mutual Inductance
- Self Inductance
- Inductor
- Energy Stored in the Magnetic Field

Announcements

- Quiz Monday, Apr. 24 early in class
  - Covers: CH 29 to what we get to on Wednesday
- Exam recounting
  - Average: 76/102 → 75/102
- A colloquium at 4pm this Wednesday in the planetarium
  - Dr. D. Dahlberg from University of Minnesota
    - A UTA graduate
  - Title: Magnetism at the Nanoscale
  - Extra credit opportunity
- Reading assignments:
  - CH29-8

Transformer

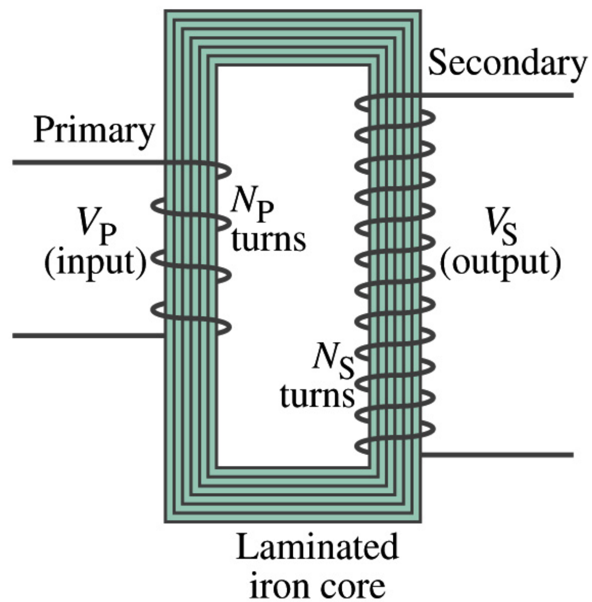
- What is a transformer?
  - A device for increasing or decreasing an AC voltage
  - A few examples?
    - TV sets to provide High Voltage to picture tubes, portable electronic device converters, transformers on the pole, etc
- A transformer consists of two coils of wires known as primary and secondary
  - The two coils can be interwoven or linked by a laminated soft iron core to reduce eddy current losses
- Transformers are designed so that all magnetic flux produced by the primary coil pass through the secondary



How does a transformer work?

- When an AC voltage is applied to the primary, the changing B it produces will induce voltage of the same frequency in the secondary wire
- So how would we make the voltage different?
  - By varying the number of loops in each coil
  - From Faraday's law, the induced emf in the secondary is
  - $V_S = N_S \frac{d\Phi_B}{dt}$
  - The input primary voltage is
  - $V_P = N_P \frac{d\Phi_B}{dt}$
  - Since $d\Phi_B/dt$ is the same, we obtain
  - $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

Transformer Equation



Transformer Equation

- The transformer equation does not work for DC current since there is no change of magnetic flux
- If $N_S > N_P$, the output voltage is greater than the input so it is called a step-up transformer while $N_S < N_P$ is called step-down transformer
- Now, it looks like energy conservation is violated since we can get more emf from smaller ones, right?
  - Wrong! Wrong! Wrong! Energy is always conserved!
  - A well designed transformer can be more than 99% efficient
  - The power output is the same as the input:
  - $V_P I_P = V_S I_S$
  - $$\frac{I_S}{I_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S}$$

The output current for step-up transformer will be lower than the input, while it is larger for step-down x-former than the input.

Example 29 – 8

Portable radio transformer. A transformer for home use of a portable radio reduces 120-V ac to 9.0V ac. The secondary contains 30 turns, and the radio draws 400mA. Calculate (a) the number of turns in the primary; (b) the current in the primary; and (c) the power transformed.

(a) What kind of a transformer is this? A step-down x-former

Since $\frac{V_P}{V_S} = \frac{N_P}{N_S}$ We obtain $N_P = N_S \frac{V_P}{V_S} = 30 \frac{120V}{9V} = 400turns$

(b) Also from the transformer equation $\frac{I_S}{I_P} = \frac{V_P}{V_S}$ We obtain

$$I_P = I_S \frac{V_S}{V_P} = 0.4A \frac{9V}{120V} = 0.03A$$

(c) Thus the power transformed is

$$P = I_S V_S = (0.4A) \cdot (9V) = 3.6W$$

How about the input power? The same assuming 100% efficiency.

Example 29 – 9: Power Transmission

Transmission lines. An average of 120kW of electric power is sent to a small town from a power plant 10km away. The transmission lines have a total resistance of 0.4Ω. Calculate the power loss if the power is transmitted at (a) 240V and (b) 24,000V.

We cannot use $P=V^2/R$ since we do not know the voltage along the transmission line. We, however, can use $P=I^2R$.

(a) If 120kW is sent at 240V, the total current is

$$I=\frac{P}{V}=\frac{120\times10^3}{240}=500A.$$

Thus the power loss due to transmission line is

$$P=I^2R=(500A)^2\cdot(0.4\Omega)=100kW$$

(b) If 120kW is sent at 24,000V, the total current is

$$I=\frac{P}{V}=\frac{120\times10^3}{24\times10^3}=5.0A.$$

Thus the power loss due to transmission line is

$$P=I^2R=(5A)^2\cdot(0.4\Omega)=10W$$

The higher the transmission voltage, the smaller the current, causing less loss of energy. This is why power is transmitted w/ HV, as high as 170kV.

Electric Field due to Magnetic Flux Change

- When electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
  - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- Recall the relation between electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field
- $\varepsilon = \oint \vec{E} \cdot d\vec{l}$
- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

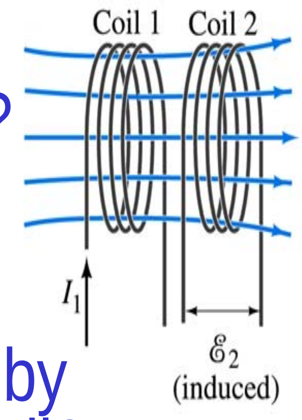
- The integral is taken around a path enclosing the area through which the magnetic flux Φ_B is changing.

Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - Or induce an emf in itself ➔ Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.



- What does the induced emf, ε_2, in coil2 proportional to?
  – Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21}I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined by $M_{21} = N_2\Phi_{21}/I_1$.
- The emf induced in coil2 due to the changing current in coil1 is

$$\varepsilon_2 = -N_2\frac{d\Phi_{21}}{dt} = -\frac{d\left(N_2\Phi_{21}\right)}{dt} = -M_{21}\frac{dI_1}{dt}$$

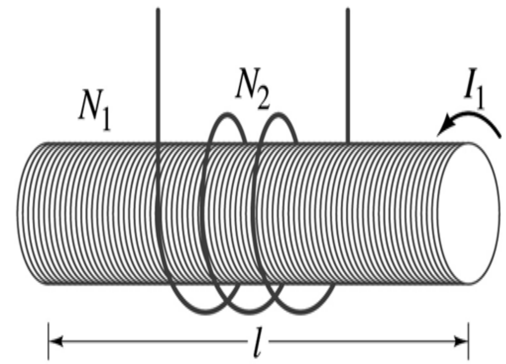
Mutual Inductance

- The mutual induction of coil2 with respect to coil1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

    What? Does this make sense?
    - The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
  - Most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain $\varepsilon_1 = -M\frac{dI_2}{dt}$ and $\varepsilon_2 = -M\frac{dI_1}{dt}$
  - SI unit for mutual inductance is henry (H) $1H = 1V \cdot s/A = 1\Omega \cdot s$

Example 30 – 1

Solenoid and coil. A long thin solenoid of length ℓ and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assume all the flux from coil1 (the solenoid) passes through coil2, and calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $B = \frac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils 2. Thus the flux through coil2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil2 is

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this?
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impedes its increase, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = L I$
- We define self-inductance, $\mathcal{L}$: $$L = \frac{N\Phi_B}{I}$$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V \cdot s/A = 1\Omega \cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

- It is an impediment onto the electrical current due to the existence of changing flux
- So what?
- In other words, it behaves like a resistance to the varying current, such as AC, that causes the constant change of flux
- But it also provides means to store energy, just like the capacitance

Inductor

- An electrical circuit always contain some inductance but is normally negligibly small
  – If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  – Precision resisters are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a "non-inductive winding"
- If an inductor has negligible resistance, inductance controls a changing current
- For an AC current, the greater the inductance the less the AC current
  – An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  – The quality of an inductor is indicated by the term reactance or impedance

Example 30 – 3

Solenoid inductance. (a) Determine a formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing N turns of wire in its length ℓ and whose cross-sectional area is A. (b) Calculate the value of $\mathcal{L}$ if N=100, ℓ=5.0cm, A=0.30cm^2 and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with $\mu=4000\mu_0$.

What is the magnetic field inside a solenoid? $B = \mu_0 nI = \mu_0 NI/l$

The flux is, therefore, $\Phi_B = BA = \mu_0 NIA/l$

Using the formula for self inductance: $L = \dfrac{N\Phi_B}{I} = \dfrac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\left(4\pi \times 10^{-7}\, T\cdot m/m\right)100^2\left(0.30\times 10^{-4}\, m^2\right)}{5.0\times 10^{-2}\, m} = 7.5\mu H$$

(c) The magnetic field with an iron core solenoid is $B = \mu NI/l$

$$L = \frac{\mu N^2 A}{l} = \frac{4000\left(4\pi \times 10^{-7}\, T\cdot m/m\right)100^2\left(0.30\times 10^{-4}\, m^2\right)}{5.0\times 10^{-2}\, m} = 0.030H = 30mH$$