

PHYS 1444 – Section 003

Lecture #23

Wednesday, Apr. 26, 2006

Dr. Jaehoon Yu

- Phasor Diagram
- Achievements of Maxwell's Equations
- Extension of Ampere's Law
- Gauss' Law of Magnetism
- Maxwell's Equations
- Production of Electromagnetic Waves

Today's homework is homework #12, 7pm, next Thursday, May 4!!

Wednesday, Apr. 26, 2006



PHYS 1444-501, Spring 2006
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Announcements

- Quiz results
 - Average: 50.5
 - Previous averages: 66, 68.7 and 60.8
 - Top score: 80
- Final term exam
 - Time: 5:30pm – 7:00pm, Monday May. 8
 - Location: SH103
 - Covers: CH 29 – whichever chapter we finish Monday, May 1
 - Please do not miss the exam
 - Two best of the three exams will be used for your grades

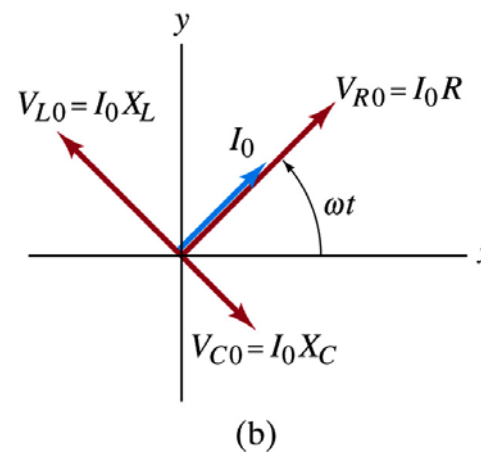
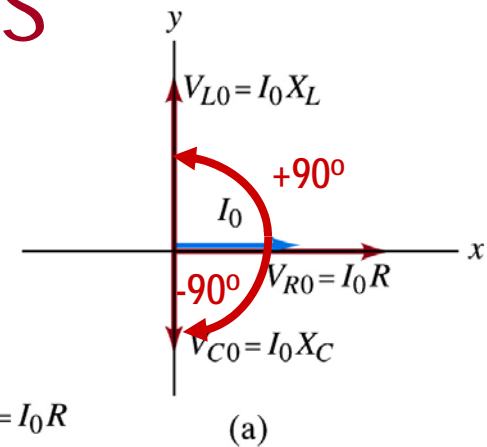
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Phasor Diagrams

- At $t=0$, $I=0$.
 - Thus $V_{R0}=0$, $V_{L0}=I_0X_L$, $V_{C0}=I_0X_C$
- At $t=t$, $I = I_0 \sin \omega t$

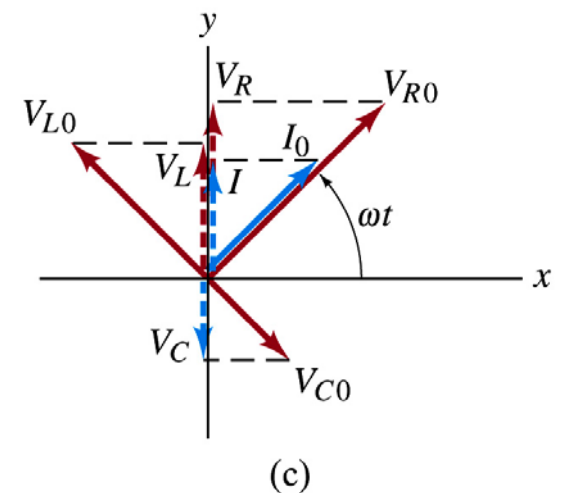


- Thus, the voltages (y-projections) are

$$V_R = V_{R0} \sin \omega t$$

$$V_L = V_{L0} \sin(\omega t + 90^\circ)$$

$$V_C = V_{C0} \sin(\omega t - 90^\circ)$$



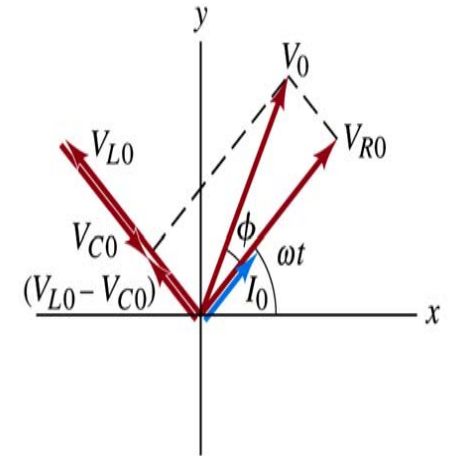
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AC Circuit w/ LRC

- Since the sum of the projections of the three vectors on the y axis is equal to the projection of their sum,
 - The sum of the projections represents the instantaneous voltage across the whole circuit which is the source voltage
 - So we can use the sum of all vectors as the representation of the peak source voltage V_0 .



- V_0 forms an angle ϕ to V_{R0} and rotates together with the other vectors as a function of time, $V = V_0 \sin(\omega t + \phi)$
- We determine the total impedance Z of the circuit defined by the relationship $V_{rms} = I_{rms} Z$ or $V_0 = I_0 Z$
- From Pythagorean theorem, we obtain

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = \sqrt{I_0^2 R^2 + I_0^2 (X_L - X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} = I_0 Z$$

- Thus the total impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

AC Circuit w/ LRC

- The phase angle ϕ is

$$\tan \phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0 (X_L - X_C)}{I_0 R} = \frac{(X_L - X_C)}{R}$$

- or

$$\cos \phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$

- What is the power dissipated in the circuit?

- Which element dissipates the power?
- Only the resistor

- The average power is $\bar{P} = I_{rms}^2 R$

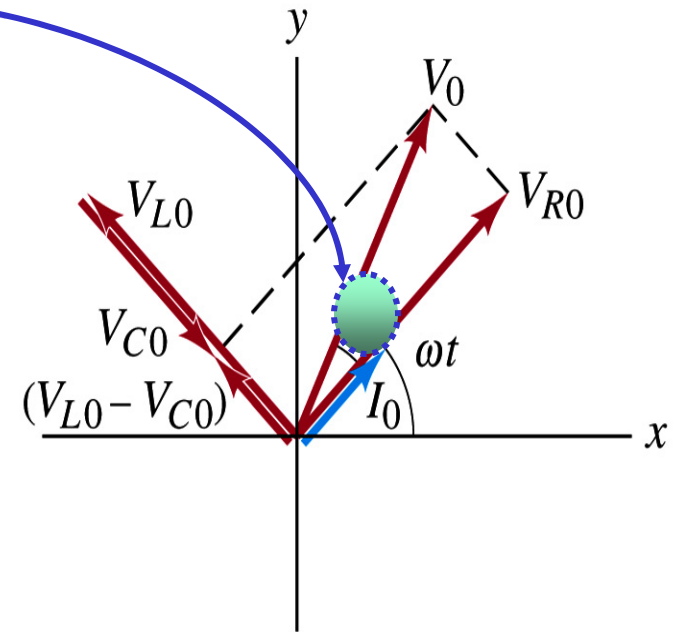
- Since $R = Z \cos \phi$

- We obtain $\bar{P} = I_{rms}^2 Z \cos \phi = I_{rms} V_{rms} \cos \phi$

- The factor $\cos \phi$ is referred as the power factor of the circuit

- For a pure resistor, $\cos \phi = 1$ and $\bar{P} = I_{rms} V_{rms}$

- For a capacitor or inductor alone $\phi = -90^\circ$ or $+90^\circ$, so $\cos \phi = 0$ and $\bar{P} = 0$.



Maxwell's Equations

- The development of EM theory by Oersted, Ampere and others was not done in terms of EM fields
 - The idea of fields was introduced somewhat by Faraday
- Scottish physicist James C. Maxwell unified all the phenomena of electricity and magnetism in one theory with only four equations (Maxwell's Equations) using the concept of fields
 - This theory provided the prediction of EM waves
 - As important as Newton's law since it provides dynamics of electromagnetism
 - This theory is also in agreement with Einstein's special relativity
- The biggest achievement of 19th century electromagnetic theory is the prediction and experimental verification that the electromagnetic waves can travel through the empty space
 - What do you think this accomplishment did?
 - Open a new world of communication
 - It also yielded the prediction that the light is an EM wave
- Since all of Electromagnetism is contained in the four Maxwell's equations, this is considered as one of the greatest achievements of human intellect



Ampere's Law

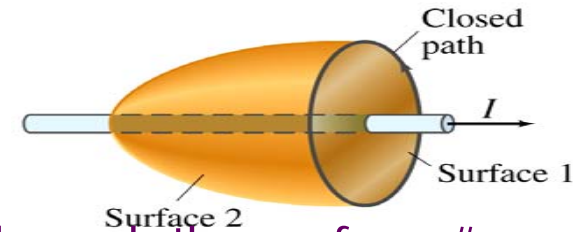
- Do you remember the mathematical expression of Oersted discovery of a magnetic field produced by an electric current, given by Ampere?

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

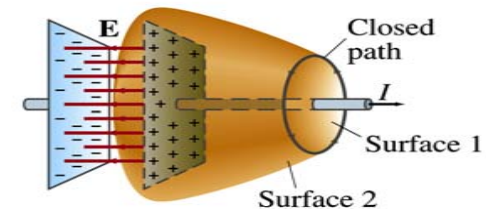
- We've learned that a varying magnetic field produces an electric field
- Then can the reverse phenomena, that a changing electric producing a magnetic field, possible?
 - If this is the case, it would demonstrate a beautiful symmetry in nature between electricity and magnetism



Expanding Ampere's Law



- Let's consider a wire carrying current I
 - The current that is enclosed in the loop passes through the surface # 1 in the figure
 - We could imagine a different surface # 2 that shares the same enclosed path but cuts through the wire in a different location. What is the current that passes through the surface?
 - Still I .
 - So the Ampere's law still works
- We could then consider a capacitor being charged up or being discharged.
 - The current I enclosed in the loop passes through the surface #1
 - However the surface #2 that shares the same closed loop do not have any current passing through it.
 - There is magnetic field present since there is current ➔ In other words there is a changing electric field in between the plates
 - Maxwell resolved this by adding an additional term to Ampere's law involving the changing electric field



Modifying Ampere's Law

- To determine what the extra term should be, we first have to figure out what the electric field between the two plates is
 - The charge Q on the capacitor with capacitance C is $Q=CV$
 - Where V is the potential difference between the plates
 - Since $V=Ed$
 - Where E is the uniform field between the plates, and d is the separation of the plates
 - And for parallel plate capacitor $C=\epsilon_0 A/d$
 - We obtain

$$Q = CV = \left(\epsilon_0 \frac{A}{d} \right) Ed = \epsilon_0 AE$$



Modifying Ampere's Law

- If the charge on the plate changes with time, we can write

$$\frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt}$$

- Using the relationship between the current and charge we obtain

$$I = \frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \frac{d(AE)}{dt} = \epsilon_0 \frac{d\Phi_E}{dt}$$

- Where $\Phi_E = EA$ is the electric flux through the surface between the plates

- So in order to make Ampere's law work for the surface 2 in the figure, we must write it in the following form

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Extra term
by Maxwell

- This equation represents the general form of Ampere's law
 - This means that a magnetic field can be caused not only by an ordinary electric current but also by a changing electric flux

Example 32 – 1

Charging capacitor. A 30-pF air-gap capacitor has circular plates of area $A=100\text{cm}^2$. It is charged by a 70-V battery through a $2.0\text{-}\Omega$ resistor. At the instant the battery is connected, the electric field between the plates is changing most rapidly. At this instance, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume \mathbf{E} is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Since this is an RC circuit, the charge on the plates is: $Q = CV_0(1 - e^{-t/RC})$

For the initial current ($t=0$), we differentiate the charge with respect to time.

$$I_0 = \left. \frac{dQ}{dt} \right|_{t=0} = \frac{CV_0}{RC} e^{-t/RC} \Big|_{t=0} = \frac{V_0}{R} = \frac{70\text{V}}{2.0\Omega} = 35\text{A}$$

The electric field is $E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$

Change of the electric field is $\frac{dE}{dt} = \frac{dQ/dt}{A\epsilon_0} = \frac{35\text{A}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \cdot (1.0 \times 10^{-2} \text{ m}^2)} = 4.0 \times 10^{14} \text{ V/m} \cdot \text{s}$



Example 32 – 1

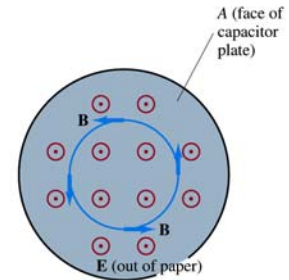
(c) Determine the magnetic field induced between the plates. Assume \mathbf{E} is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

The magnetic field lines generated by changing electric field is perpendicular to \mathbf{E} and is circular due to symmetry

Whose law can we use to determine B ?

Extended Ampere's Law w/ $I_{\text{encl}}=0$!

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$



We choose a circular path of radius r , centered at the center of the plane, following the B .

For $r < r_{\text{plate}}$, the electric flux is $\Phi_E = EA = E\pi r^2$ since E is uniform throughout the plate

So from Ampere's law, we obtain $B \cdot (2\pi r) = \mu_0 \epsilon_0 \frac{d(E\pi r^2)}{dt} = \mu_0 \epsilon_0 \pi r^2 \frac{dE}{dt}$

Solving for B

$$B = \mu_0 \epsilon_0 \frac{r}{2} \frac{dE}{dt}$$

For $r < r_{\text{plate}}$

Since we assume $E=0$ for $r > r_{\text{plate}}$, the electric flux beyond the plate is fully contained inside the surface.

$$\Phi_E = EA = E\pi r_{\text{plate}}^2$$

So from Ampere's law, we obtain $B \cdot (2\pi r) = \mu_0 \epsilon_0 \frac{d(E\pi r_{\text{plate}}^2)}{dt} = \mu_0 \epsilon_0 \pi r_{\text{plate}}^2 \frac{dE}{dt}$

Solving for B

$$B = \frac{\mu_0 \epsilon_0 r_{\text{plate}}^2}{2r} \frac{dE}{dt}$$

For $r > r_{\text{plate}}$