PHYS 5326 – Lecture #3

Monday, Jan. 29, 2007
Dr. Jae Yu

1. Neutrino-Nucleon DIS
2. ν-N DIS Formalism
3. Proton Structure Function and Parton Distribution Functions (PDFs)

Neutrino Nucleon Deep Inelastic Scattering

- DIS (Deep Inelastic Scattering) of lepton-nucleon are traditionally used to probe nucleon structures
- Neutrinos (especially) are excellent probes
  - Extremely light
  - Structureless
  - Weak interaction only ➔ Probes helicity
    - ν_μ are normally used for these experiments. Why?
- Nucleons consist of partons
  - Structure of nucleon is described by parton distribution functions (PDF) ➔ Constituents' probability distributions in fractional momentum space
- DIS are viewed as neutrino-parton elastic scattering

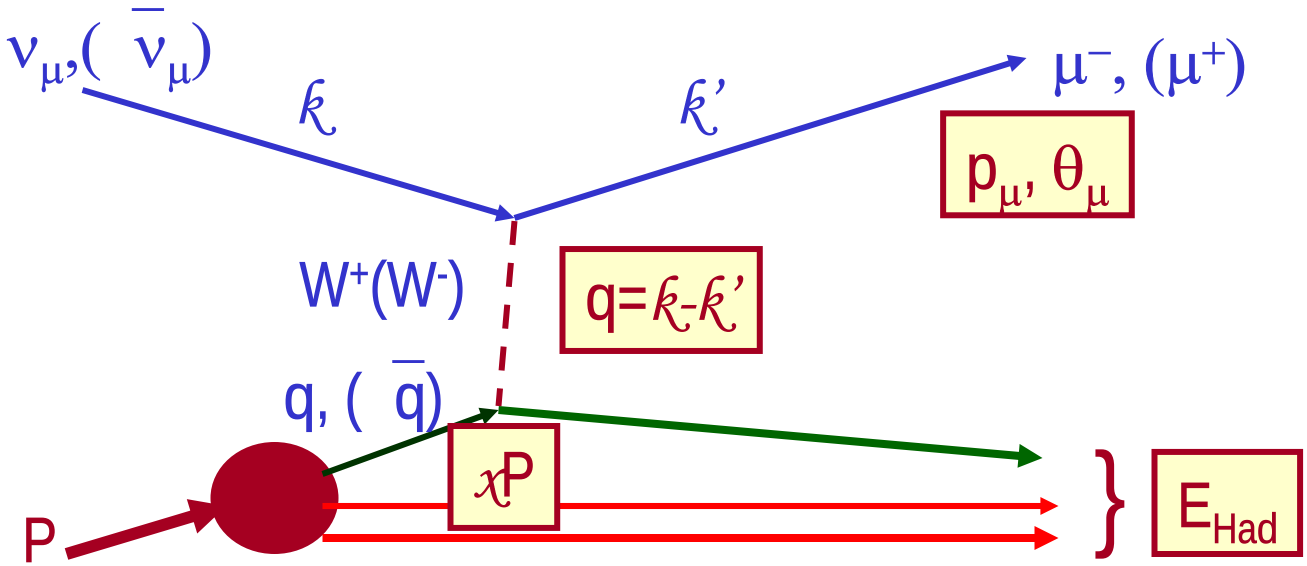
Structure Function Measurements

- A complete set of Lorentz scalars that parameterize the unknown structure of the proton
- Properties of the SF lead to parton model
  - Nucleon is composed of point-like constituents, partons, that elastically scatter with neutrino
- Partons are identified as quarks and gluons of QCD
- QCD theory itself does not provide parton distributions within the proton
  - Then what does QCD provide?
  - The dynamics of strong interactions using color quantum numbers
- QCD analysis of SF provides a determination of nucleon's valence and sea quark and gluon distributions (PDF) along with the strong coupling constant, α_s

Kinematics of ν_μ-N CC Interactions

- DIS is a three dimensional problem
- Three kinematic parameters provide full description of a DIS event are
  - p_μ: Muon momentum
  - θ_μ: Angle of outgoing muon
  - E_{Had}: Observed energy of outgoing hadrons
- Neutrino energy becomes
  - $E_\nu = E_{Had} + E_\mu + M_p$

DIS Kinematics



Leptonic System

$$k = (E_\nu,\ 0,\ 0,\ E_\nu)$$

$$k' = (E_\mu,\ p_\mu \sin\theta_\mu \cos\phi_\mu,\ p_\mu \sin\theta_\mu \sin\phi_\mu,\ p_\mu \cos\theta_\mu)$$

Hadron System

$$p = (M_P,\ 0,\ 0,\ 0)$$

$$p' = p + q = p + (k - k')$$

Why isn't NC used for SF?

DIS Lorentz Invariant Variables

CMS Energy $s = (p+k)^2 = M_P^2 + 2M_P E_\nu$

Energy transferred to the hadronic System

$$\nu = \frac{p \cdot q}{M_P} = E_\nu - E_\mu = E_{Had}$$

Four momentum transfer of the interaction

$$Q^2 = -q^2 = -\left(k - k^{'}\right)^2 = m_\mu^2 + 2E_\nu\left(E_\mu - p_\mu \cos\theta_\mu\right)$$

Invariant mass of the hadronic system

$$W^2 = \left(p^{'}\right)^2 = (p+q)^2 = M_P^2 + 2M_P\nu - Q^2$$

DIS Lorentz Invariant Variables cont'd

Bjorken Scaling Variable = Fractional Momentum of the Struck parton within the nucleon

$$x \equiv \frac{-q^2}{2p \cdot q} = \frac{Q^2}{2M_p \nu}$$

Inelasticity $$y \equiv \frac{p \cdot q}{p \cdot k} = \frac{E_{Had}}{E_\nu} = \frac{\nu}{E_\nu}$$

$$y \approx 1 - \frac{1}{2}\left(1 + \cos\theta^*\right)$$

where θ^* is CMS scattering angle of μ

DIS Formalism

Matrix element for ν-N interaction

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \frac{1}{1+Q^2/M_W^2} \bar{u}_\mu(k', s')\gamma_\alpha(1-\gamma_5)u_\nu(k,s)\langle X|J_{CC}|N(p,s)\rangle$$

Weak Coupling Constant

W (CC) Propagator

Lepton

Hadron

Inclusive Spin-Averaged Cross section

$$\frac{d^2\sigma^{\nu N}}{d\Omega_\mu dE_\mu} = \frac{1}{\left(1+Q^2/M_W^2\right)} \frac{G_F}{2} \frac{m_\nu}{E_\nu} \frac{m_\mu}{E_\mu} \frac{E_\mu^2}{(2\pi)^2} L_{\alpha\beta} W^{\alpha\beta}$$

Leptonic Tensor

Hadronic Tensor

ν-N DIS Cross Sections for SF Extraction

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{2G_F M_P E_\nu}{\pi}\left[\left(1-y-\frac{M_P xy}{2E_\nu}\right)F_2^{\nu(\bar{\nu})}\left(x,Q^2\right)+\frac{y^2}{2}2xF_1^{\nu(\bar{\nu})}\left(x,Q^2\right) \pm y\left(1-\frac{y}{2}\right)xF_3^{\nu(\bar{\nu})}\left(x,Q^2\right)\right]$$

Using ratio of absorption xsec for longitudinal and transversely polarized boson, R

$$R\left(x,Q^2\right) \equiv \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1}\left(1-\frac{Q^2}{\left(2M_P x\right)^2}\right)-1$$

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{2G_F M_P E_\nu}{\pi}\left[\left(1-y-\frac{M_P xy}{2E_\nu}+\frac{y^2}{2}\frac{1+4M_P^2x^2/Q^2}{1+R\left(x,Q^2\right)}\right)F_2^{\nu(\bar{\nu})}\left(x,Q^2\right) \pm y\left(1-\frac{y}{2}\right)\nu F_3^{\nu(\bar{\nu})}\left(x,Q^2\right)\right]$$

Structure Functions and PDF's

- Assuming parton model, ν-N cross section can be rewritten in terms of point-like particle interactions that exchange a intermediate vector boson

$$\frac{d^2\sigma^{\nu T}}{dxdy} = \frac{G_F^2 xs}{\pi\left(1+Q^2/M_W^2\right)^2}\left[q^{\nu T}(x) + \left(1-y^2\right)\bar{q}^{\nu T} + 2(1-y)k^{\nu T}(x)\right]$$

$$\frac{d^2\sigma^{\bar{\nu} T}}{dxdy} = \frac{G_F^2 xs}{\pi\left(1+Q^2/M_W^2\right)^2}\left[\bar{q}^{\bar{\nu} T}(x) + \left(1-y^2\right)q^{\bar{\nu} T} + 2(1-y)k^{\bar{\nu} T}(x)\right]$$

Spin 0 partons

- Comparing the parton-neutrino to proton-neutrino SF and PDF's are related as

$$2xF_1^{\nu(\bar{\nu})T} = 2\left[xq^{\nu(\bar{\nu})T}(x) + x\bar{q}^{\nu(\bar{\nu})T}(x)\right]$$

$$F_2^{\nu(\bar{\nu})T} = 2\left[xq^{\nu(\bar{\nu})T}(x) + x\bar{q}^{\nu(\bar{\nu})T}(x) + 2xk^{\nu(\bar{\nu})T}\right]$$

$$xF_3^{\nu(\bar{\nu})T} = 2\left[xq^{\nu(\bar{\nu})T}(x) - x\bar{q}^{\nu(\bar{\nu})T}(x)\right]$$

Parity violating components

If no spin 0, $2xF_1=F_2$

Linking to Quark Flavors

- ν-N scattering resolves flavor of constituents
  - CC changes the flavor of the struck quark
  - Charge conservation at the vertex constraints
    - Neutrinos to interact with d, s, $\bar{u}$, $\bar{c}$
    - Anti-neutrinos to interact with $\bar{d}$, $\bar{s}$, u, c
- For parton target, the quark densities contribute to SF are

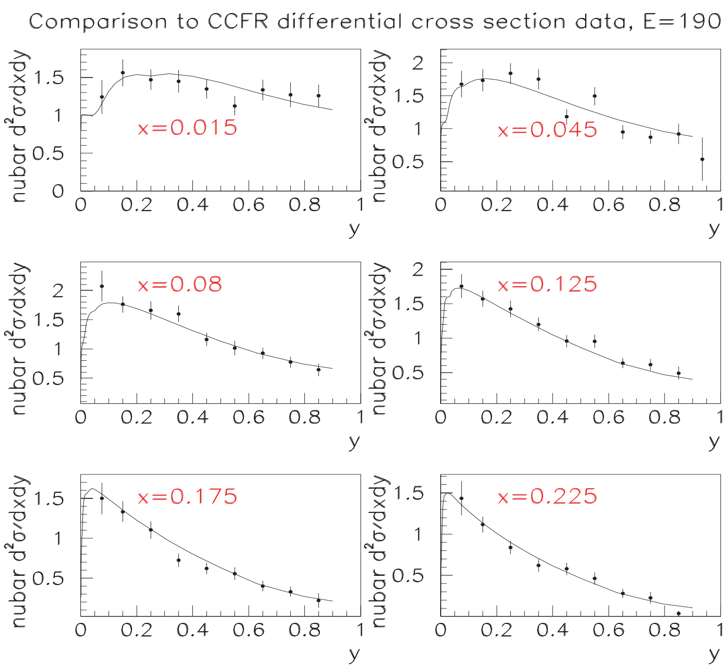
$$q^{\nu p} = d^p(x) + s^p(x)$$
$$\bar{q}^{\nu p} = \bar{u}^p(x) + \bar{c}^p(x)$$
$$q^{\bar{\nu} p} = u^p(x) + c^p(x)$$
$$\bar{q}^{\bar{\nu} p} = \bar{d}^p(x) + \bar{s}^p(x)$$

$$2xF_1^{\nu N}(x) = 2xF_1^{\bar{\nu} N}(x)$$
$$= xu(x) + x\bar{u}(x) + xd(x) + x\bar{d}(x)$$
$$+ xs(x) + x\bar{c}(x) + xc(x) + x\bar{c}(x)$$
$$xF_3^{\nu N}(x) = xu_V(x) + xd_V(x) + 2xs(x) - 2xc(x)$$
$$xF_3^{\bar{\nu} N}(x) = xu_V(x) + xd_V(x) - 2xs(x) + 2xc(x)$$

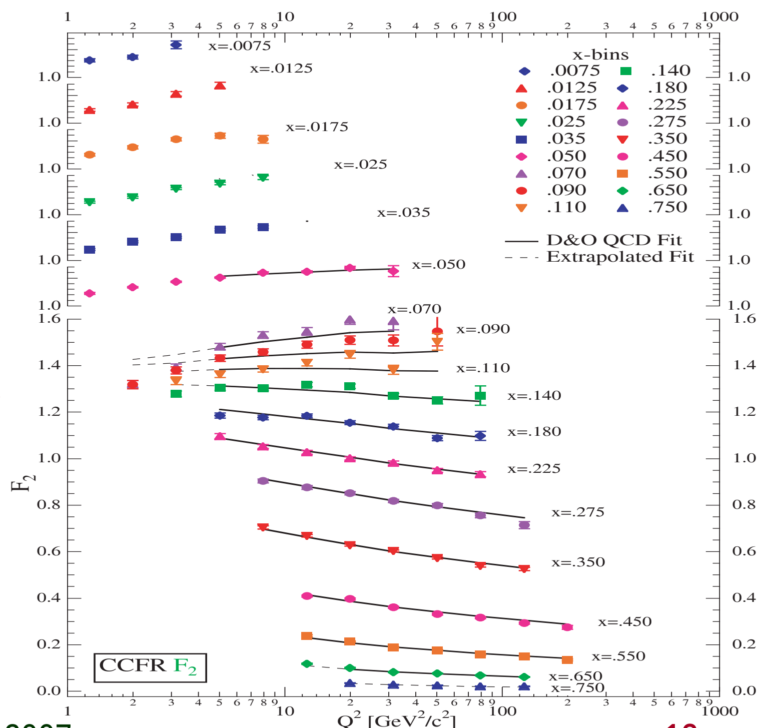
$$u_V(x) \equiv u - \bar{u}; \quad d_V(x) \equiv d - \bar{d}$$

How Are PDFs Determined?

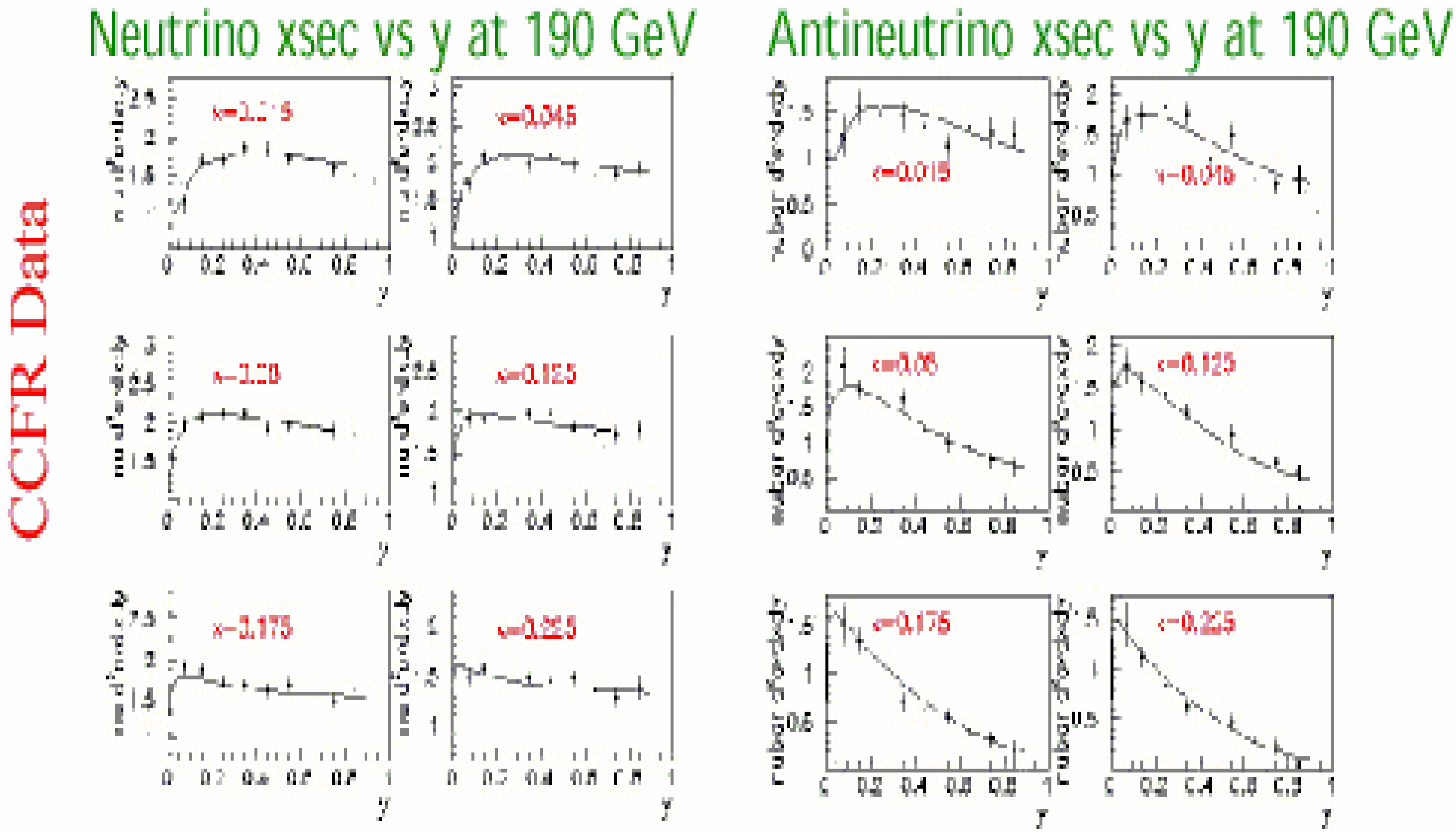
- Measure ν-N differential cross sections, correcting for target
- Compare them to theoretical x-sec
- Fit SF's to measured x-sec
- Extract PDF's from the SF fits➔
  - Different QCD models could generate different sets of PDF's
  - CTEQ, MRST, GRV, etc



Fit to Data for SF



Comparisons of Neutrino and Antineutrino cross sections



What can PDF's depend on?

- Different functional forms of PDF and SF's
- Order of QCD calculations
  - Higher order (NLO or NNLO) calculations require higher order PDF's
- Different assumptions in the protons
  - No intrinsic sea quarks
  - Fixed flavors only
- Approximation at non-perturbative regime
  - Different method of approximating low x behavior

Homework Assignments

- Provide a method to measure the average valence quark distributions in a ν-N scattering experiment
- Derive the Lorentz invariant variables of ν-N scattering, s, Q^2, W^2, x and y on pages 6 and 7 of this lecture.
- Make at least 8 plots (two - 1d histograms, two - 2d scatter plots, four composite variables) using root
- All these are due Monday, Feb. 12