PHYS 5326 – Lecture #4

Wednesday, Jan. 31, 2007
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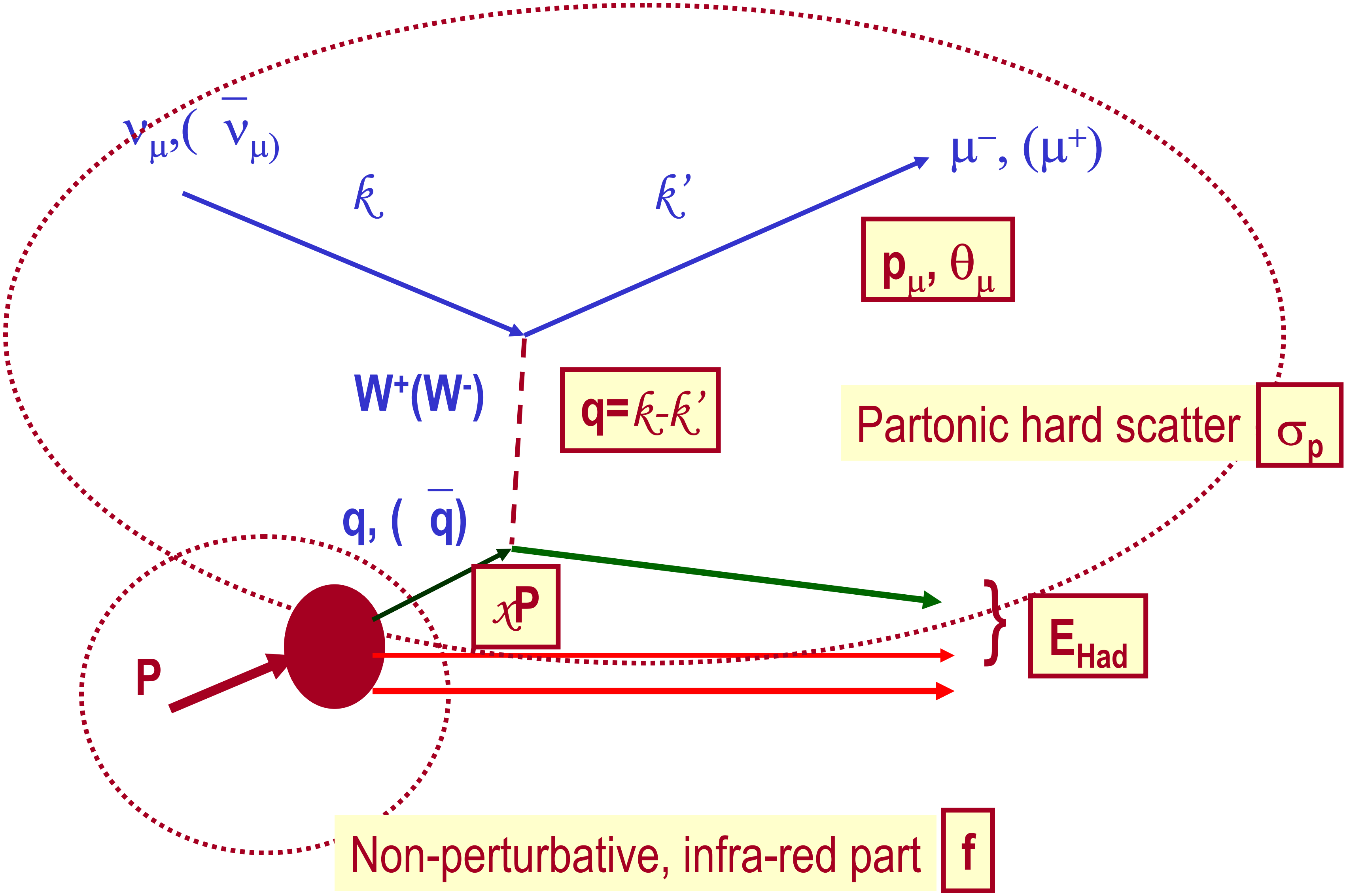
1. QCD Evolution of PDF
2. Measurement of $Sin^2\theta_W$
3. Formalism of $Sin^2\theta_W$ in ν-N DIS
4. Improvements in $Sin^2\theta_W$
5. Interpretation of $Sin^2\theta_W$ and Its Link to Higgs

Factorization

$\sigma = f^{*}\sigma_p$



DGLAP QCD Evolution Equations

- The evolution equations by **D**okshitzer-**G**ribov-**L**ipatov-**A**ltarelli-**P**arisi provide mechanism to evolve PDF's to any kinematic regime or momentum scale

$$\frac{dq^{NS}\left(x,M^2\right)}{d\ln M^2}=\sum_i q^i-\bar{q}^i=u_V+d_V=\frac{\alpha_s\left(\mu^2\right)}{2\pi}\int_x^1\frac{dy}{y}q^{NS}\left(y,M^2\right)P_{qq}\left(\frac{x}{y}\right)$$

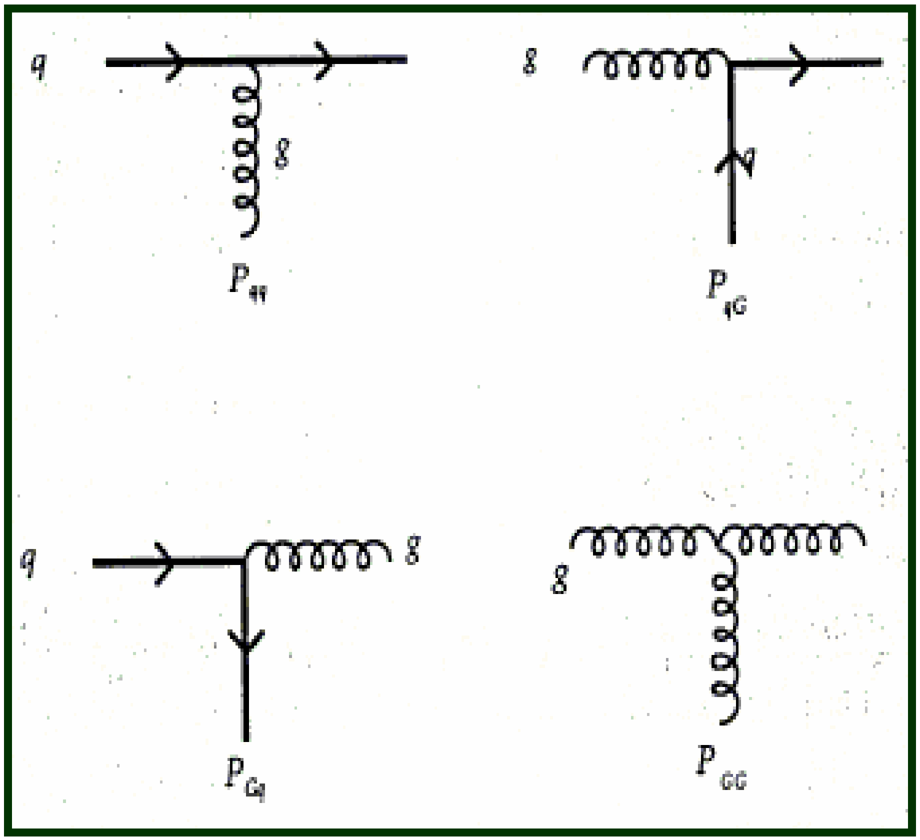
$$\frac{dq^S\left(x,M^2\right)}{d\ln M^2}=\sum_i q^i+\bar{q}^i=\frac{\alpha_s\left(\mu^2\right)}{2\pi}\int_x^1\frac{dy}{y}\left[q^{NS}\left(y,M^2\right)P_{qq}^s\left(\frac{x}{y}\right)+G\left(y,M^2\right)P_{qG}^s\left(\frac{x}{y}\right)\right]$$

$$\frac{dG\left(x,M^2\right)}{d\ln M^2}=\frac{\alpha_s\left(\mu^2\right)}{2\pi}\int_x^1\frac{dy}{y}\left[q^S\left(y,M^2\right)P_{Gq}^s\left(\frac{x}{y}\right)+G\left(y,M^2\right)P_{GG}\left(\frac{x}{y}\right)\right]$$

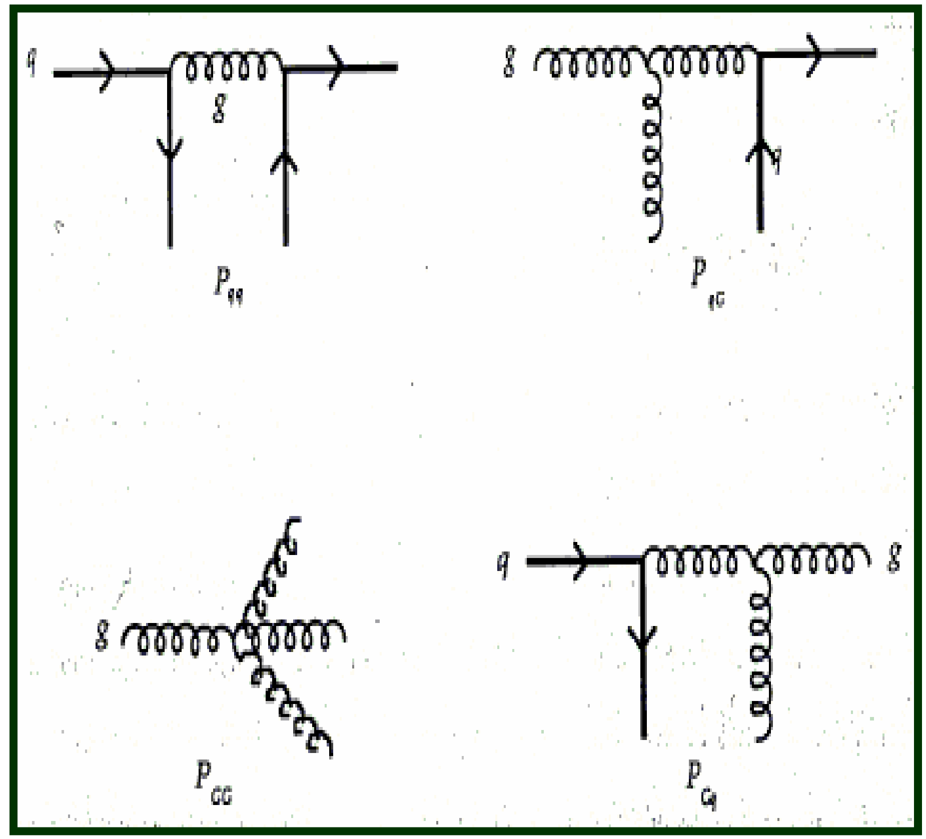
P_{ij}(x/y): Splitting function which describes the probability for a parton i with momentum y get resolved as a parton j with momentum x<y

Feynman Diagrams for Parton Splitting

LO: O(α_s)



NLO: O(α_s^2)



Electroweak Theory

- Standard Model unifies Weak and EM to SU(2)xU(1) gauge theory
  - Weak neutral current interaction
  - Measured physical parameters related to mixing parameters for the couplings

$$g' = g\tan\theta_W \quad e = g\sin\theta_W \quad G_F = \frac{g^2\sqrt{2}}{8M_W^2} \quad \frac{M_W}{M_Z} = \cos\theta_W$$

- Neutrinos in this picture are unique because they only interact through left-handed weak interactions ➔ Probe weak sector only
  - Less complication in some measurements, such as proton structure

$\sin^2\theta_W$ and ν-N scattering

- In the electroweak sector of the Standard Model, it is not known *a priori* what the mixture of electrically neutral electromagnetic and weak mediator is➔ This fractional mixture is given by the mixing angle
- Within the on-shell renormalization scheme, $\sin^2\theta_W$ is:

$$\sin^2\theta_w^{On-Shell} = 1 - \frac{M_W^2}{\rho_0 M_Z^2}$$

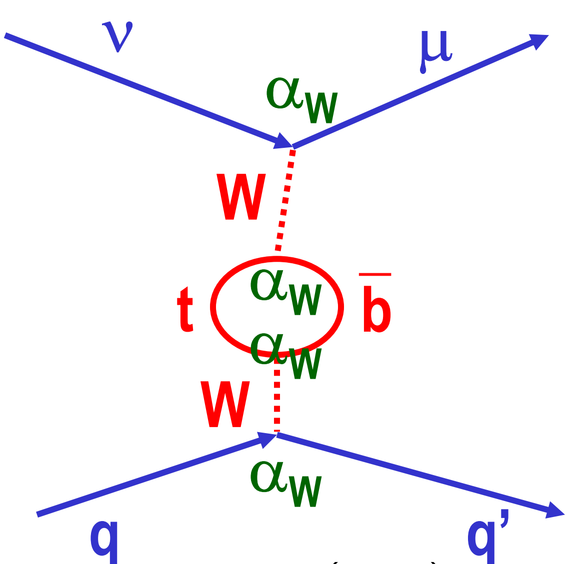
•Provides independent measurement of M_W & information to pin down M_{Higgs} via higher order loop corrections, in comparable uncertainty to direct measurements

•Measures light quark couplings ➔ Sensitive to other types (anomalous) of couplings

•In other words, sensitive to physics beyond SM ➔ New vector bosons, compositeness, ν-oscillations, etc

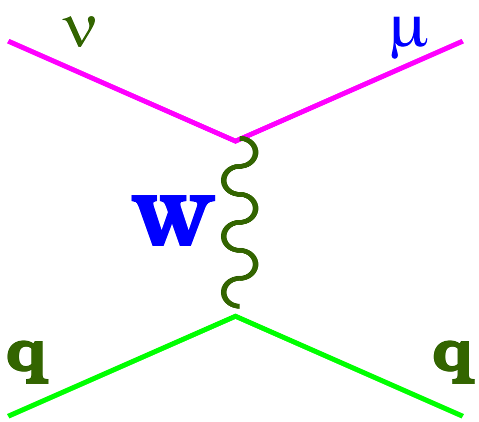
EW Higher Order Corrections

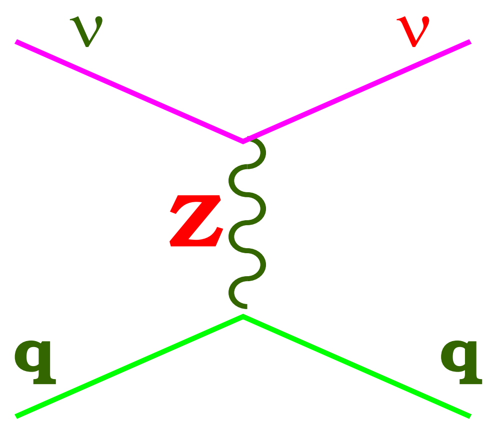
- LO GSW requires three parameters: α, G_F and M_Z
- Higher order corrections bring in dependences to two additional parameters: M_{Top} and M_{Higgs}



$$\sigma \propto O\left(\alpha_W^4\right) \qquad \sigma \propto O\left(\alpha_W^4\right)$$

How is $\sin^2\theta_W$ measured?





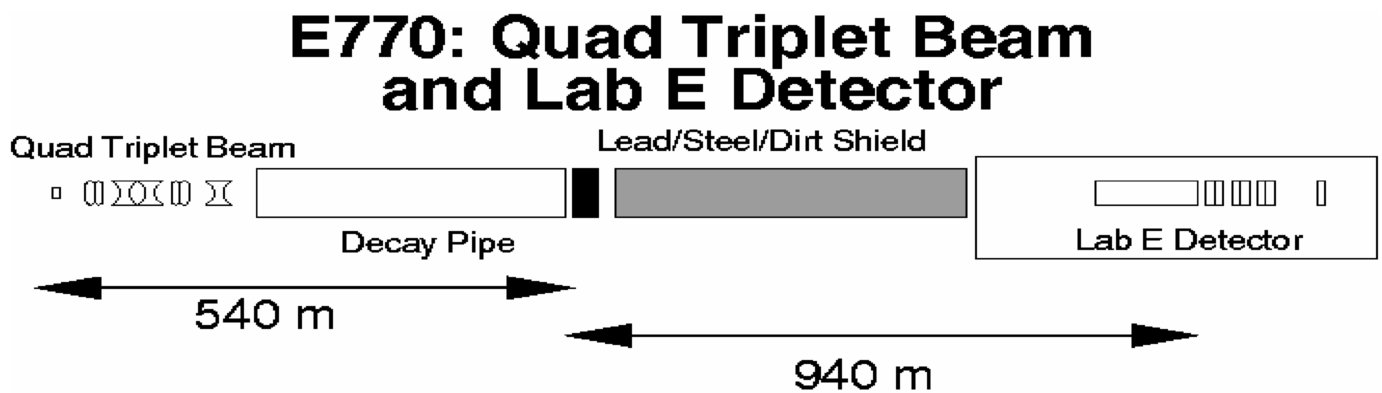
$$coupling \propto I^{(3)}_{weak}$$

$$coupling \propto I^{(3)}_{weak} - Q_{EM}\sin^2\theta_W$$

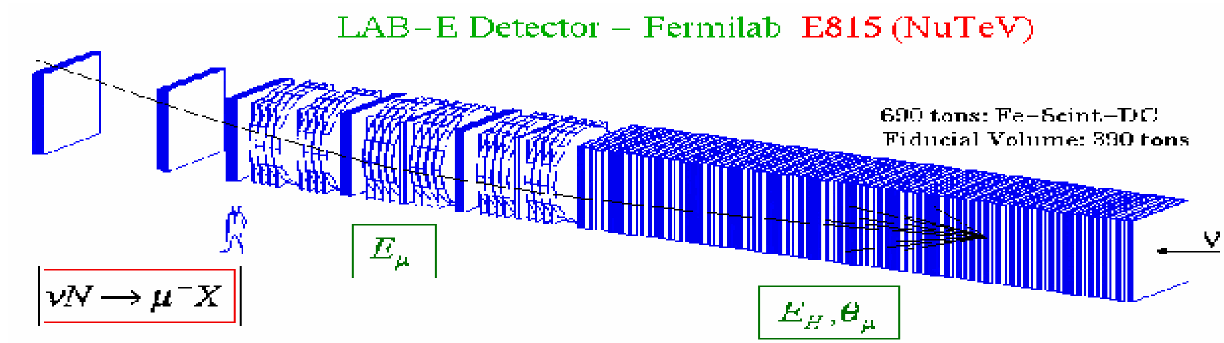
- Cross section ratios between NC and CC proportional to $\sin^2\theta_W$
- Llewellyn Smith Formula:

$$R^{\nu(\bar{\nu})} = \frac{\sigma_{NC}^{\nu(\bar{\nu})}}{\sigma_{CC}^{\nu(\bar{\nu})}} = \rho^2\left(\frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W\left(1 + \frac{\sigma_{CC}^{\bar{\nu}(\nu)}}{\sigma_{CC}^{\nu(\bar{\nu})}}\right)\right)$$

The Original Experiment



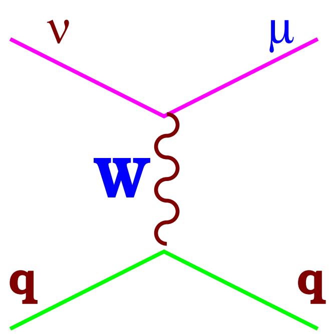
- Conventional neutrino beam from π/k decays
- Focus all signs of π/k for neutrinos and antineutrinos
- Only ν_μ in the beam (NC events are mixed)

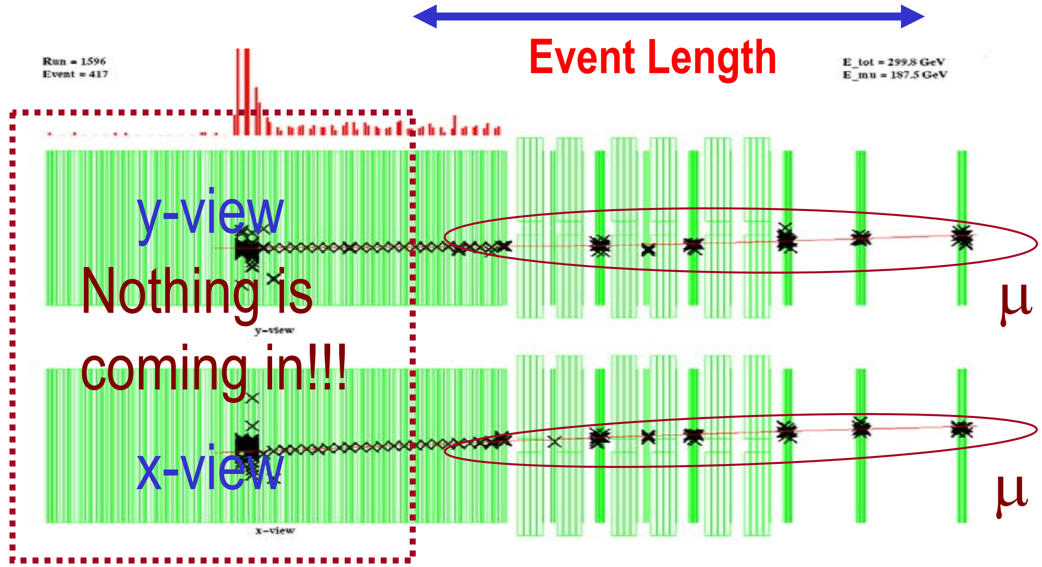


- Very small cross section ➔ Heavy neutrino target
- ν_e are the killers (CC events look the same as NC events)

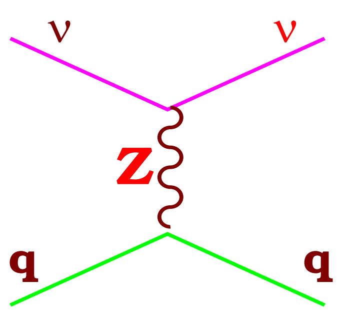
How Can Events be Separated?

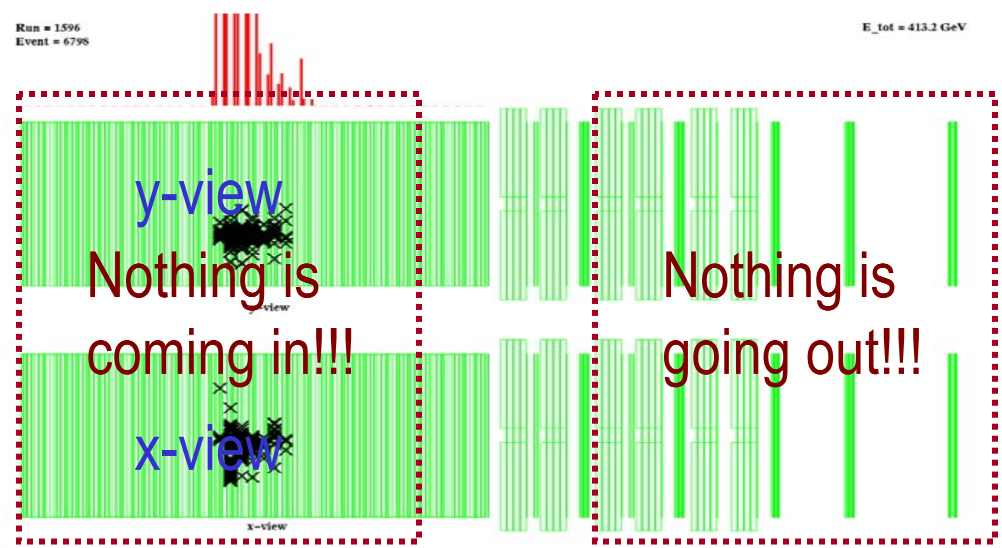
Charged Current Events





Neutral Current Events

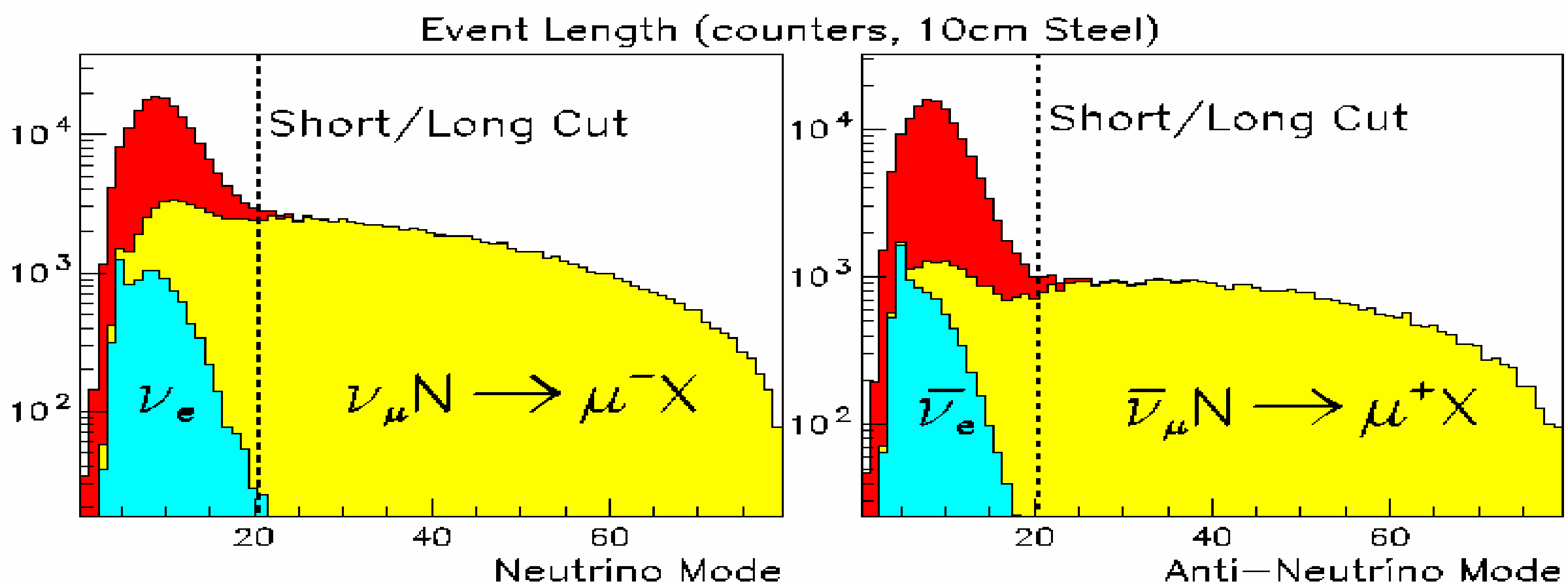




Experimental Variable

Define an Experimental Length variable

➔Distinguishes CC from NC experimentally in statistical manner



Compare experimentally measured ratio

$$R_{Exp} = \frac{N_{Short}}{N_{Long}} = \frac{L < L_{Cut}}{L > L_{Cut}} = \frac{N_{NC\ Candidates}}{N_{CC\ Candidates}}$$

to theoretical prediction of R^ν

Past Experimental Results

$$\sin^2\theta_W^{On-Shell} = 1 - \frac{M_W^2}{M_Z^2} = 0.2277 \pm 0.0036$$

$$\Rightarrow M_W{}^{On-Shell} = 80.14 \pm 0.19 \mathrm{GeV/c^2}$$

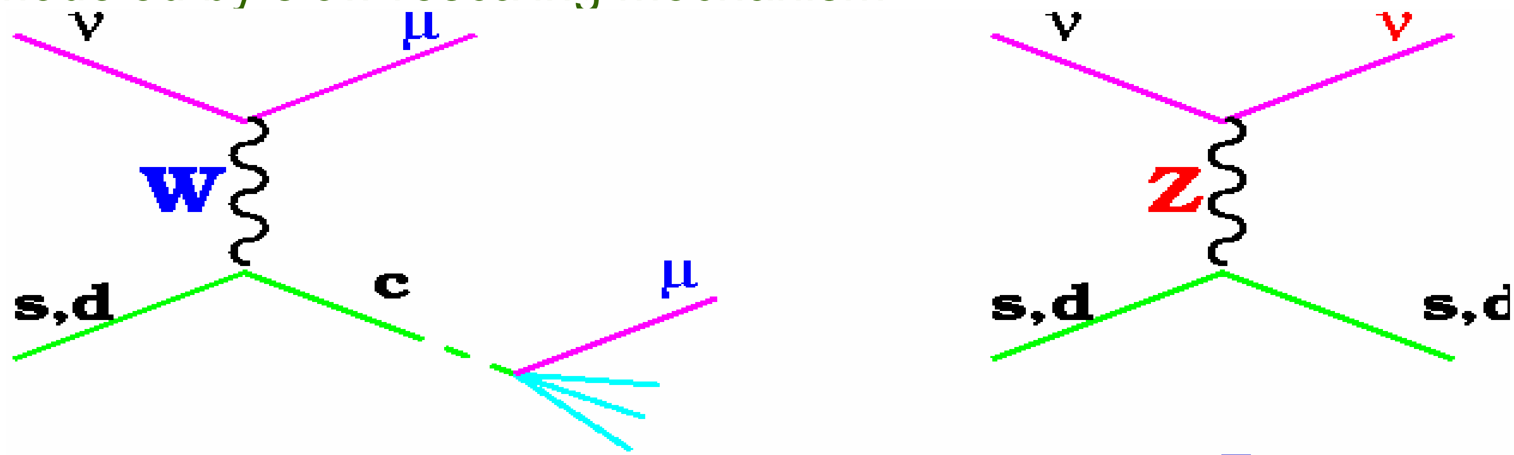
The yellow band represents a correlated uncertainty!!

Improvements on Measurements

- Asses the uncertainties from previous measurements
- Determine what the sources of largest theoretical and experimental uncertainties are
- Provide new methods to reduce large uncertainties

$\sin^2\theta_W$ Theoretical Uncertainty

- Significant correlated error from CC production of charm quark (m_c) modeled by slow rescaling mechanism



- Suggestion by Paschos-Wolfenstein by separating ν and $\bar{\nu}$ beams:

$$R^- = \frac{\sigma^{\nu}_{NC} - \sigma^{\bar{\nu}}_{NC}}{\sigma^{\nu}_{CC} - \sigma^{\bar{\nu}}_{CC}} = \rho^2\left(\frac{1}{2} - \sin^2\theta_W\right) = \frac{R^{\nu} - R^{\bar{\nu}}}{1-r}$$

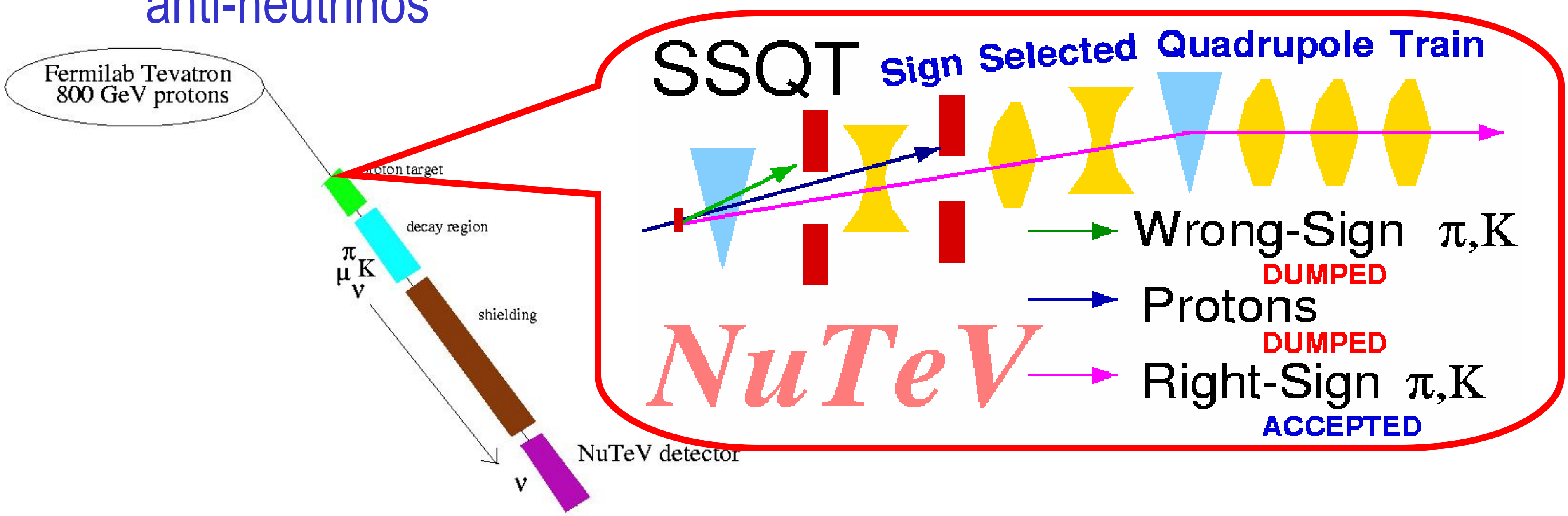
➔Reduce charm CC production error by subtracting sea quark contributions

➔Only valence u, d, and s contributes while sea quark contributions cancel out

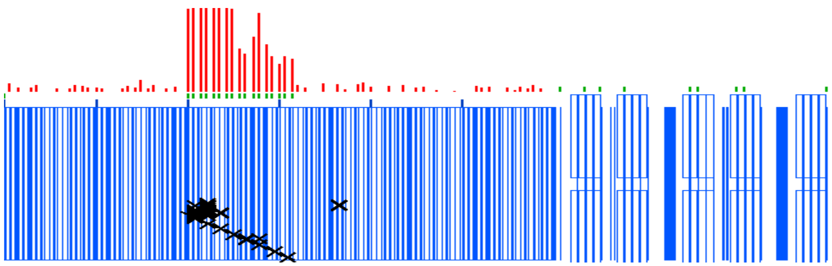
➔Massive quark production through Cabbio suppressed d_v quarks only

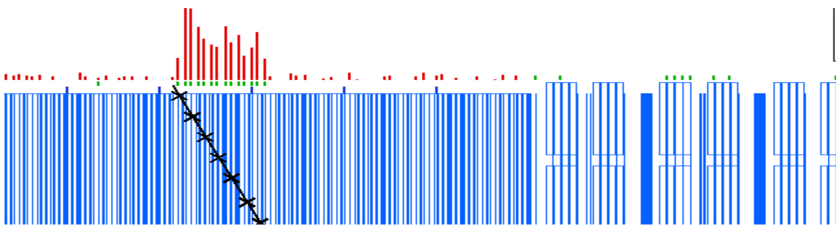
Improving Experimental Uncertainties

- Electron neutrinos, ν_e, in the beam fakes NC events from CC interactions
  - If the production cross section is well known, the effect will be smaller but since majority come from neutral K (K_L) whose x-sec is known only to 20%, this is a source of large experimental uncertainty
- Need to come up with a beamline that separates neutrinos from anti-neutrinos

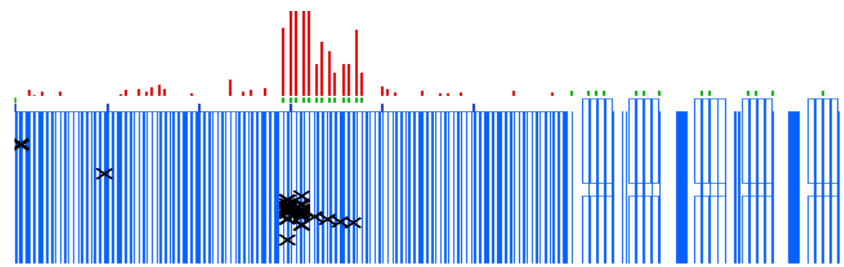


Event Contamination and Backgrounds

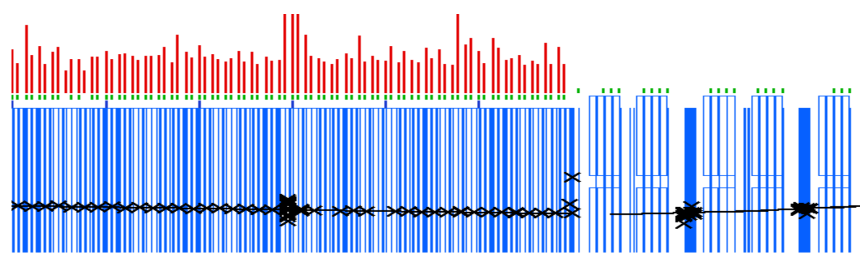




- **SHORT ν_μ CC's (20% ν, 10% $\bar{\nu}$)**
  μ exit and rangeout
- **SHORT ν_e CC's (5%)**
  $\nu_e N \rightarrow eX$
- **Cosmic Rays (0.9%)**

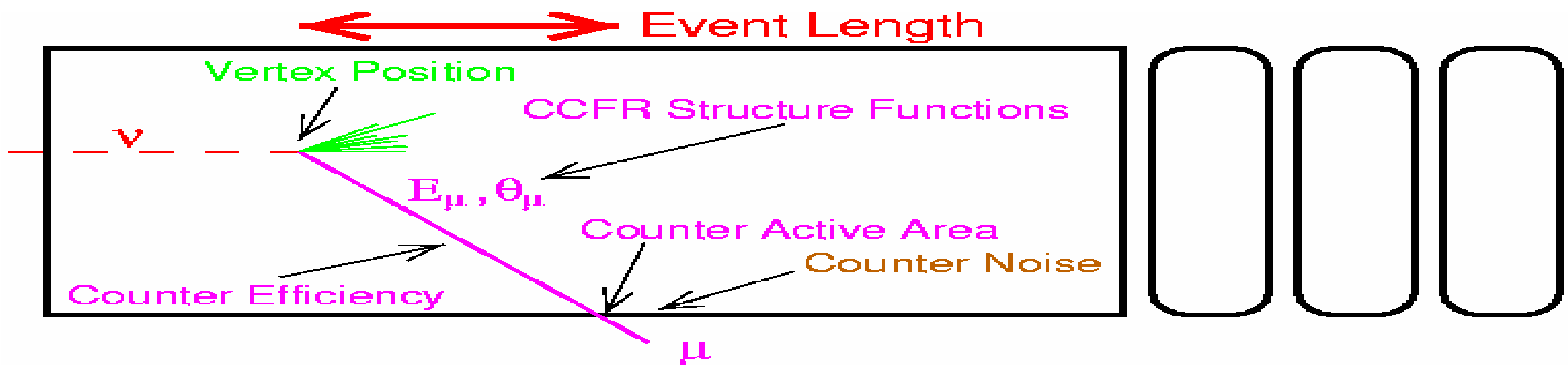


- **LONG ν_μ NC's (0.7%)**
  hadron shower
  punch-through effects



- **Hard μ Brem(0.2%)**
  Deep μ events

Other Detector Effects

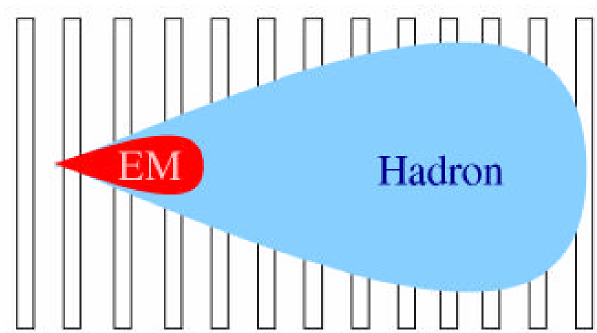


Sources of experimental uncertainties kept small, through modeling using ν and TB data

Effect	Size($\delta\sin^2\theta_W$)	Tools
Z_{vert}	0.001/inch	$\mu^+\mu^-$ events
X_{vert} & Y_{vert}	0.001	MC
Counter Noise	0.00035	TB μ's
Counter Efficiency	0.0002	ν events
Counter active area	0.0025/inch	ν CC, TB
Hadron shower length	0.0015/cntr	TB π's and k's
Energy scale	0.001/1%	TB
Muon Energy Deposit	0.004	ν CC

Measurements of ν_e Flux

- Use well known processes (Ke3: $K^{\pm} \to \pi^0 e^{\pm} \overset{(-)}{\nu}_e$)
- Shower Shape Analysis can provide direct measurement ν_e events, though less precise



N_{meas}/N_{MC}

$$1.05 \pm 0.03\,(\nu_e)$$

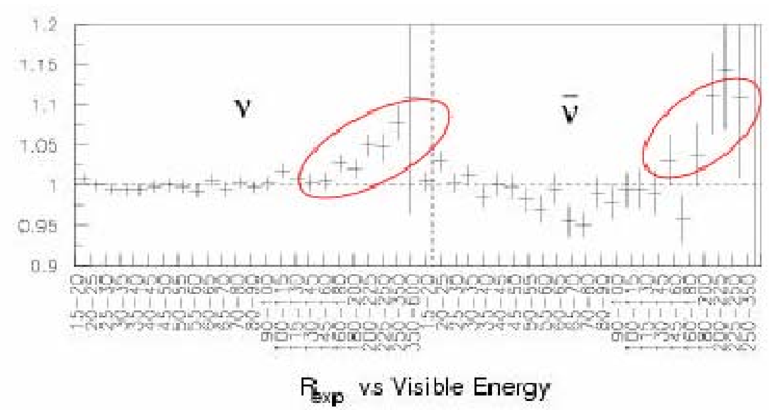
$$1.01 \pm 0.04\,(\bar{\nu}_e)$$

Weighted average used for ν_e
➔ $\delta R_\nu^{exp} \sim 0.0005$

- ν_e from very short events (E_ν>180 GeV)
  - Precise measurement of ν_e flux in the tail region of flux ➔ ~35% more $\bar{\nu}_e$ in $\bar{\nu}$ than predicted
  - Had to require (E_{had}<180 GeV)

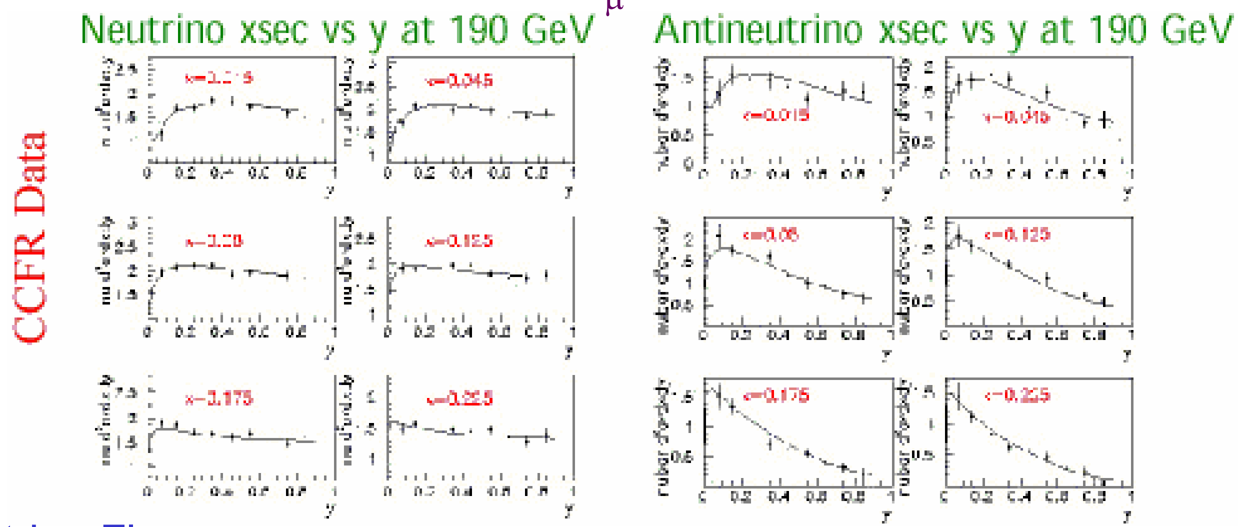
  due to ADC saturation

  Results in $\sin^2\theta_w$ shifts by +0.002

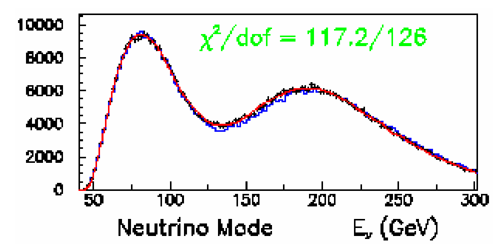


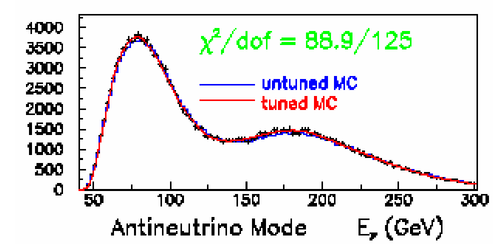
MC to Relate R_ν^{exp} to R^ν and $\sin^2\theta_W$

- Parton Distribution Model
  - Correct for details of PDF model ➔ Used CCFR data for PDF
  - Model cross over from short ν_μ CC events



- Neutrino Fluxes
  - $\nu_\mu, \nu_e, \bar{\nu}_\mu, \bar{\nu}_e$ in the two running modes
  - ν_e CC events always look short
- Shower length modeling
  - Correct for short events that look long
- Detector response vs energy, position, and time
  - Continuous testbeam running minimizes systematics





$\sin^2\theta_W$ Fit to R_ν^{exp} and $R_{\bar{\nu}}^{exp}$

- Thanks to the separate beam → Measure R^ν's separately
- Use MC to simultaneously fit R_ν^{exp} and $R_{\bar{\nu}}^{exp}$ to $\sin^2\theta_W$ and m_c, and $\sin^2\theta_W$ and ρ

$$R^{\nu(\bar{\nu})} = \frac{\sigma_{NC}^{\nu(\bar{\nu})}}{\sigma_{CC}^{\nu(\bar{\nu})}} = \rho^2\left(\frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W\left(1 + \frac{\sigma_{CC}^{\bar{\nu}(\nu)}}{\sigma_{CC}^{\nu(\bar{\nu})}}\right)\right)$$

- R^ν Sensitive to $\sin^2\theta_W$ while $R^{\bar{\nu}}$ isn't, so R^ν is used to extract $\sin^2\theta_W$ and $R^{\bar{\nu}}$ to control systematics
- Single parameter fit, using SM values for EW parameters (ρ_0=1)

$$\sin^2\theta_W = 0.2277 \pm 0.0013\,(\text{stat}) \pm 0.0009\,(\text{syst})$$

$$m_c = 1.32 \pm 0.09\,(\text{stat}) \pm 0.06\,(\text{syst}) \text{ w/ } m_c = 1.38 \pm 0.14\ \text{GeV/c}^2 \text{ as input}$$

- Two parameter fit for $\sin^2\theta_W$ and ρ_0 yields

$$\sin^2\theta_W = 0.2265 \pm 0.0031$$

$$\rho_0 = 0.9983 \pm 0.040$$

Syst. Error dominated since we cannot take advantage of sea quark cancellation

NuTeV $\sin^2\theta_W$ Uncertainties

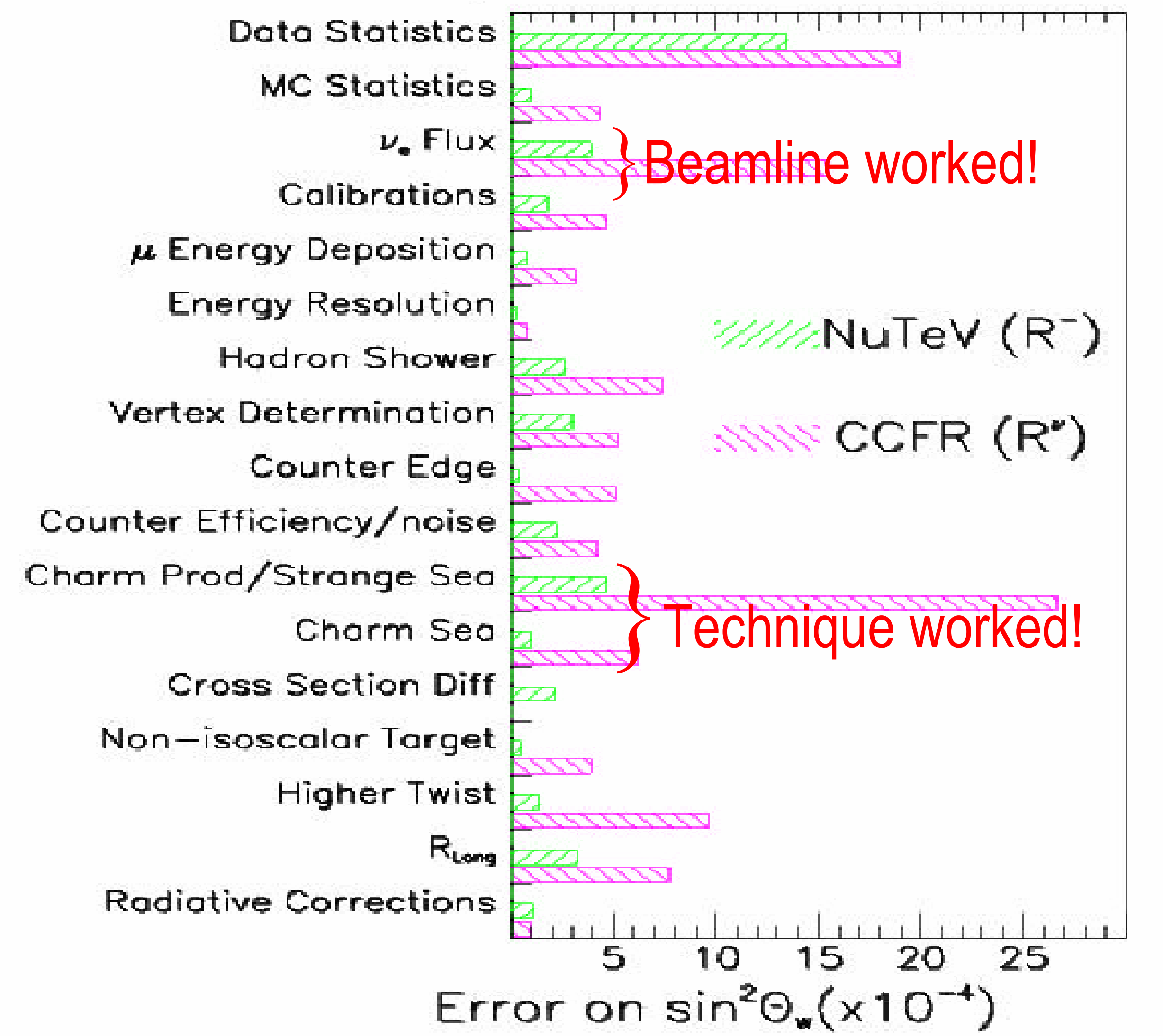
Source of Uncertainty	$\delta \sin^2\theta_W$
Statistical	**0.00135**
ν_e flux	0.00039
Event Length	0.00046
Energy Measurements	0.00018
Total Experimental Systematics	**0.00063**
CC Charm production, sea quarks	0.00047
Higher Twist	0.00014
Non-isoscalar target	0.00005
$\sigma^{\bar{\nu}} / \sigma^{\nu}$	0.00022
RadiativeCorrection	0.00011
R_L	0.00032
Total Physics Model Systmatics	**0.00064**
Total Systematic Uncertainty	**0.00162**
ΔM_W (GeV/c^2)	**0.08**

Dominant uncertainty

1-Loop Electroweak Radiative Corrections based on Bardin, Dokuchaeva **JINR-E2-86-2 60 (1986)**

$$\delta \sin^2\theta_W^{(On-shell)} = -0.00022 \times \left(\frac{M_t^2 - (175\,GeV)^2}{(50\,GeV)^2} \right) + 0.00032 \times \ln\left(\frac{M_H}{150\,GeV} \right)$$

NuTeV vs CCFR Uncertainty Comparisons



Comparison of New $\sin^2\theta_W$

$$\sin^2\theta_W^{On-Shell} = 0.2277 \pm 0.0013\,(stat) \pm 0.0009\,(syst)$$

$$\sin^2\theta_W^{On-shell} = 1 - \frac{M_W^2}{M_Z^2}$$

$$\Rightarrow M_W^{On-Shell} = 80.14 \pm 0.08\ GeV/c^2$$

Comparable precision but value smaller than other measurements