PHYS 5326 – Lecture #12, 13, 14

Monday, Apr. 2, 2007
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1. Local Gauge Invariance
2. U(1) Gauge Invariance
3. SU(2) Gauge Invariance
4. Yang-Mills Lagrangian
5. Introduction of Massless Vector Gauge Fields

Klein-Gordon Largangian for scalar (S=0) Field

For a single, scalar field ϕ, the Lagrangian is

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \phi^2$$

Since $\dfrac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi_i\right)} = \partial^\mu \phi_i$ and $\dfrac{\partial \mathcal{L}}{\partial \phi_i} = -\left(\dfrac{mc}{\hbar}\right)^2 \phi_i$

From the Euler-Largange equation, we obtain

$$\partial_\mu \partial^\mu \phi + \left(\frac{mc}{\hbar}\right)^2 \phi = 0$$

This equation is the Klein-Gordon equation describing a free, scalar particle (spin 0) of mass m.

Dirac Largangian for Spinor (S=1/2) Field

For a spinor field ψ, the Lagrangian

$$\mathcal{L} = i(\hbar c)\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - (mc^2)\overline{\psi}\psi$$

Since $\dfrac{\partial\mathcal{L}}{\partial(\partial_{\mu}\overline{\psi})} = 0$ and $\dfrac{\partial\mathcal{L}}{\partial\overline{\psi}} = i(\hbar c)\gamma^{\mu}\partial_{\mu}\psi - mc^2\psi$

From the Euler-Largange equation for $\overline{\psi}$, we obtain

$$i\gamma^{\mu}\partial_{\mu}\psi - \left(\frac{mc}{\hbar}\right)\psi = 0$$

Dirac equation for a particle of spin ½ and mass m.

Proca Largangian for Vector (S=1) Field

Suppose we take the Lagrangian for a vector field A^μ

$$\mathcal{L} = -\frac{1}{16\pi}\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) + \frac{1}{8\pi}\left(\frac{mc}{\hbar}\right)^2 A^\nu A_\nu$$

$$= -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{mc}{\hbar}\right)^2 A^\nu A_\nu$$

Since $\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu A_\nu\right)} = -\frac{1}{4\pi}\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)$ and $\frac{\partial \mathcal{L}}{\partial A_\nu} = \frac{1}{4\pi}\left(\frac{mc}{\hbar}\right)^2 A^\nu$

From the Euler-Largange equation for A^μ, we obtain

$$\partial_\mu\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) + \left(\frac{mc}{\hbar}\right)^2 A^\nu = \partial_\mu F^{\mu\nu} + \left(\frac{mc}{\hbar}\right)^2 A^\nu = 0$$

Proca equation for a particle of spin 1 and mass m.

For m=0, this equation is for an electromagnetic field.

Lagrangians

- Lagrangians we discussed are concocted to produce desired field equations
  - $\mathcal{L}$ derived (L=T-V) in classical mechanics
  - $\mathcal{L}$ taken as axiomatic in field theory
- The Lagrangian for a particular system is not unique
  - Can always multiply by a constant
  - Or add a divergence
  - Since these do not affect field equations due to cancellations

Local Gauge Invariance - I

Dirac Lagrangian for free particle of spin ½ and mass m;

$$\mathcal{L} = i(\hbar c)\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - (mc^2)\overline{\psi}\psi$$

is invariant under a global phase transformation (global gauge transformation) $\psi \rightarrow e^{i\theta}\psi$ since $\overline{\psi} \rightarrow e^{-i\theta}\overline{\psi}$

Why? θ is a constant and is not subject to the derivative.

If the phase θ varies as a function of space-time coordinate, x^{μ}, is $\mathcal{L}$ still invariant under the local gauge transformation, $\psi \rightarrow e^{i\theta(x)}\psi$?

No, because it adds an extra term from derivative of θ.

Local Gauge Invariance - II

The derivative becomes

$$\partial_\mu \psi = i\left(\partial_\mu \theta\right) e^{i\theta(x)} \psi + e^{i\theta(x)} \partial_\mu \psi$$

So the Lagrangian becomes

$$\mathcal{L}' = i(\hbar c)\overline{\psi}\gamma^\mu \left[i\left(\partial_\mu \theta\right) e^{i\theta(x)} \psi + e^{i\theta(x)} \partial_\mu \psi \right] - \left(mc^2\right)\overline{\psi}\psi$$

$$= i(\hbar c)\overline{\psi}\gamma^\mu \partial_\mu \psi - \left(mc^2\right)\overline{\psi}\psi - (\hbar c)\overline{\psi}\gamma^\mu \left(\partial_\mu \theta\right)\psi$$

Since the original $\mathcal{L}$ is $\mathcal{L} = i(\hbar c)\overline{\psi}\gamma^\mu \partial_\mu \psi - \left(mc^2\right)\overline{\psi}\psi$

$\mathcal{L}'$ is $\mathcal{L}' = \mathcal{L} - (\hbar c)\overline{\psi}\gamma^\mu \left(\partial_\mu \theta\right)\psi$

Thus, this Lagrangian is not invariant under local gauge transformation!!

Local Gauge Invariance - III

Defining a local gauge phase, λ(x), as

$$\lambda(x) \equiv -\frac{\hbar c}{q}\theta(x) \Rightarrow \theta(x) = -\frac{q}{\hbar c}\lambda(x)$$

where q is the charge of the particle involved, $\mathcal{L}$ becomes

$$\mathcal{L}' = \mathcal{L} + \left(q\overline{\psi}\gamma^{\mu}\psi\right)\partial_{\mu}\lambda$$

Under the local gauge transformation:

$$\psi \rightarrow e^{-iq\lambda(x)/\hbar c}\psi$$

Local Gauge Invariance - IV

Requiring the complete Lagrangian be invariant under λ(x) local gauge transformation will require additional terms to the free Dirac Lagrangian to cancel the extra term

$$\mathcal{L} = \left[i(\hbar c)\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - \left(mc^{2}\right)\overline{\psi}\psi \right] - \left(q\overline{\psi}\gamma^{\mu}\psi\right)A_{\mu}$$

Where A_μ is a new vector gauge field that transforms under local gauge transformation as follows:

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\lambda$$

Addition of this vector field to $\mathcal{L}$ keeps $\mathcal{L}$ invariant under local gauge transformation, but…

Local Gauge Invariance - V

The new vector field couples with the spinor through the last term. In addition, the full Lagrangian must include a “free” term for the gauge field. Thus, Proca Largangian needs to be added.

Invariant

Not-invariant

$$\mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{m_A c}{\hbar}\right)^2 A^\nu A_\nu$$

This Lagrangian is not invariant under the local gauge transformation, $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$, because

$$A^\nu A_\nu \Rightarrow \left(A^\nu + \partial_\mu \lambda\right)\left(A_\nu + \partial^\mu \lambda\right)$$

$$= A^\nu A_\nu + \left(A^\nu \partial^\mu \lambda + A_\nu \partial_\mu \lambda\right) + \left(\partial_\mu \lambda\right)\left(\partial^\mu \lambda\right)$$

In what ways can we make this $\mathcal{L}$ invariant?

U(1) Local Gauge Invariance

The requirement of local gauge invariance forces the introduction of a massless vector field into the free Dirac Lagrangian➔ QED Lagrangian – Interaction between Dirac fields (e^{+-}) and Maxwell fields (photons)

$$\mathcal{L} = \left[i\left(\hbar c\right)\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - \left(mc^{2}\right)\bar{\psi}\psi \right] + \left[\frac{-1}{16\pi} F^{\mu\nu}F_{\mu\nu} \right] - \left(q\bar{\psi}\gamma^{\mu}\psi\right)A_{\mu}$$

Free $\mathcal{L}$ for gauge field.

Vector field for gauge invariance

A_{μ} is an electromagnetic potential.

And $A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\lambda$ is a gauge transformation of an electromagnetic potential.

U(1) Local Gauge Invariance

The last two terms in the modified Dirac Lagrangian form the Maxwell Lagrangian

$$\mathcal{L}_{\mathcal{M}axwell} = \left[\frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu}\right] - \frac{1}{c} J^{\mu} A_{\mu}$$

$$= \left[\frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu}\right] - \left(q\overline{\psi}\gamma^{\mu}\psi\right) A_{\mu}$$

with the current density $J = cq\left(\overline{\psi}\gamma^{\mu}\psi\right)$

U(1) Local Gauge Invariance

Local gauge invariance is preserved if all the derivatives in the Lagrangian are replaced by the covariant derivative

$$\mathcal{D}_\mu \equiv \partial_\mu + i\frac{q}{\hbar c}A_\mu$$

Minimal Coupling Rule

The gauge transformation preserves local invariance

$$\mathcal{D}_\mu \psi \to \left(\partial_\mu + \frac{iq}{\hbar c}A_\mu\right)e^{-iq\lambda/\hbar c}\psi$$

$$= e^{-iq\lambda/\hbar c}\left[\partial_\mu + \frac{iq}{\hbar c}\left(A_\mu + \partial_\mu \lambda\right)\right]\psi = e^{-iq\lambda/\hbar c}\mathcal{D}_\mu \psi$$

Since the gauge transformation, transforms the covariant derivative $\mathcal{D}_\mu \to \partial_\mu + i\frac{q}{\hbar c}\left(A_\mu + \partial_\mu \lambda\right)$

U(1) Gauge Invariance

The global gauge transformation $\psi \to e^{i\theta}\psi$ is the same as multiplication of ψ by a unitary 1x1 matrix

$$\psi \to U\psi \quad \text{where} \quad U^{+}U = 1 \; \left(U = e^{i\theta}\right)$$

The group of all such matrices as U is U(1).

The symmetry involved in gauge transformation is called the "U(1) gauge invariance".

Lagrangian for Two Spin ½ fields

Free Lagrangian for two Dirac fields ψ_1 and ψ_2 with masses m_1 and m_2 is

$$\mathcal{L} = \left[i(\hbar c)\overline{\psi}_1 \gamma^\mu \partial_\mu \psi_1 - (m_1 c^2)\overline{\psi}_1 \psi_1 \right] + \left[i(\hbar c)\overline{\psi}_2 \gamma^\mu \partial_\mu \psi_2 - (m_2 c^2)\overline{\psi}_2 \psi_2 \right]$$

Applying Euler-Lagrange equation to $\mathcal{L}$, we obtain Dirac equations for two fields

$$i\gamma^\mu \partial_\mu \psi_1 - \left(\frac{m_1 c}{\hbar}\right)\psi_1 = 0 \qquad i\gamma^\mu \partial_\mu \psi_2 - \left(\frac{m_2 c}{\hbar}\right)\psi_2 = 0$$

Lagrangian for Two Spin ½ fields

By defining a two-component column vector

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Where ψ_1 and ψ_2 are four component Dirac spinors

The Lagrangian can be compactified as

$$\mathcal{L} = i(\hbar c)\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - c^2\overline{\psi}M\psi$$

With the mass matrix $M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$

Lagrangian for Two Spin ½ fields

If $m_1=m_2$, the Lagrangian looks the same as one particle free Dirac Lagrangian

$$\mathcal{L} = i(\hbar c)\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi$$

However, ψ now is a two component column vector.

Global gauge transformation of ψ is $\psi \to U\psi$.

Where U is any 2x2 unitary matrix $U^{+}U = 1$

Since $\overline{\psi} \to \overline{\psi}U^{+}$, $\psi\overline{\psi}$ is invariant under the transformation.

SU(2) Gauge Invariance

Any 2x2 unitary matrix can be written, $U = e^{iH}$, where H is a hermitian matrix ($H^{+}=H$).

The matrix H can be generalized by expressing in terms of four real numbers, a_1, a_2, a_3 and θ as;

$$H = \theta\mathbf{1} + \boldsymbol{\tau}\cdot\mathbf{a}$$

where 1 is the 2x2 unit matrix and τ is the Pauli matrices.

Thus, any unitary 2x2 matrix can be expressed as

$$U = e^{i\theta\mathbf{1}} e^{i\boldsymbol{\tau}\cdot\mathbf{a}}$$

U(1) gauge

SU(2) gauge

SU(2) Gauge Invariance

The global SU(2) gauge transformation takes the form

$$\psi \to e^{i\tau\cdot\mathbf{a}}\psi$$

Since the determinant of the matrix $e^{i\tau\cdot\mathbf{a}}$ is 1, the extended Dirac Lagrangian for two spin ½ fields is invariant under SU(2) global transformations.

Yang and Mills took this global SU(2) invariance to local invariance.

SU(2) Local Gauge Invariance

The local SU(2) gauge transformation by taking the parameter a dependent on the position x_μ and defining

$$\lambda \equiv -\frac{\hbar c}{q}\mathbf{a}(x)$$

Where q is a coupling constant analogous to electric charge

is $\psi \to S\psi$ where $S \equiv e^{-iq\tau\cdot\lambda(x)/\hbar c}$

$\mathcal{L}$ is not invariant under this transformation, since the derivative becomes $\partial_\mu \psi \to S\partial_\mu \psi + (\partial_\mu S)\psi$

SU(2) Local Gauge Invariance

Local gauge invariance can be preserved by replacing the derivatives with covariant derivative

$$\mathcal{D}_{\mu} \equiv \partial_{\mu} + i\frac{q}{\hbar c}\boldsymbol{\tau} \cdot \mathbf{A}_{\boldsymbol{\mu}}$$

where the vector gauge field follows the transformation rule $\mathcal{D}_{\mu}\psi \rightarrow S\left(\mathcal{D}_{\mu}\psi\right)$

With a bit more involved manipulation, the resulting $\mathcal{L}$ that is local gauge invariant is

$$\mathcal{L} = i\left(\hbar c\right)\overline{\psi}\gamma^{\mu}\mathcal{D}_{\mu}\psi - mc^{2}\overline{\psi}\psi$$
$$= \left[i\left(\hbar c\right)\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^{2}\overline{\psi}\psi\right] - \left(q\overline{\psi}\gamma^{\mu}\boldsymbol{\tau}\psi\right)\cdot\mathbf{A}_{\mu}$$

SU(2) Local Gauge Invariance

Since the intermediate $\mathcal{L}$ introduced three new vector fields $A^{\mu}=(A_1^{\mu}, A_2^{\mu}, A_3^{\mu})$, and the $\mathcal{L}$ requires free $\mathcal{L}$ for each of these vector fields

$$\mathcal{L}_{\mathcal{A}} = -\frac{1}{16\pi}F_1^{\mu\nu}F_{\mu\nu 1} - \frac{1}{16\pi}F_2^{\mu\nu}F_{\mu\nu 2} - \frac{1}{16\pi}F_3^{\mu\nu}F_{\mu\nu 3} = -\frac{1}{16\pi}\mathbf{F}_3^{\mu\nu}\cdot\mathbf{F}_{\mu\nu 3}$$

Set the Proca mass terms in $\mathcal{L}$, $\frac{1}{8\pi}\left(\frac{mc}{\hbar}\right)^2 \mathbf{A}^{\nu}\cdot\mathbf{A}_{\nu}=0$ to preserve local gauge invariance, making the vector bosons massless.

This time $F^{\mu\nu}=\left(\partial^{\mu}A^{\nu}-\partial^{\nu}A^{\mu}\right)$ also does not make the $\mathcal{L}$ local gauge invariant due to cross terms.

SU(2) Local Gauge Invariance

By redefining

$$\mathbf{F}^{\mu\nu} \equiv \partial^\mu \mathbf{A}^\nu - \partial^\nu \mathbf{A}^\mu - \frac{2q}{\hbar c}\left(\mathbf{A}^\mu \times \mathbf{A}^\nu\right)$$

The complete Yang-Mills Lagrangian $\mathcal{L}$ becomes

$$\mathcal{L} = \left[i(\hbar c)\overline{\psi}\gamma^\mu \partial_\mu \psi - mc^2 \overline{\psi}\psi \right] - \frac{1}{16\pi}\mathbf{F}^{\mu\nu} \cdot \mathbf{F}_{\mu\nu} - \left(q\overline{\psi}\gamma^\mu \tau \psi\right) \cdot \mathbf{A}_\mu$$

This $\mathcal{L}$

- is invariant under SU(2) local gauge transformation.
- describes two equal mass Dirac fields interacting with three massless vector gauge fields.

Yang-Mills Lagrangian

The Dirac fields generates three currents

$$\mathbf{J} \equiv c\left(q\overline{\psi}\gamma^{\mu}\tau\psi\right)$$

These act as the sources for the gauge fields whose lagrangian is

$$\mathcal{L}_{gauge} = -\frac{1}{16\pi}\mathbf{F}^{\mu\nu}\cdot\mathbf{F}_{\mu\nu} - \left(q\overline{\psi}\gamma^{\mu}\tau\psi\right)\cdot\mathbf{A}_{\mu}$$

The complication in SU(2) gauge symmetry stems from the fact that U(2) group is non-Abelian (non-commutative).

Epilogue

Yang-Mills gauge symmetry did not work due to the fact that no-two Dirac particles are equal mass and the requirement of massless iso-triplet vector particle.

This was solved by the introduction of Higgs mechanism to give mass to the vector fields, thereby causing EW symmetry breaking.

Introducing Mass Terms

Consider a free Lagrangian for a scalar field, ϕ:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) + e^{-(\alpha\phi)}$$

No apparent mass terms unless we expand the second term and compare this $\mathcal{L}$ with the Klein-Gordon $\mathcal{L}$:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) + 1 - (\alpha\phi)^2 + \frac{1}{2}(\alpha\phi)^4 - \frac{1}{6}(\alpha\phi)^6 + \ldots.$$

$$\mathcal{L}_{KG} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \phi^2$$

where $m = \sqrt{2}\alpha\hbar / c$

Introducing Mass Terms in Potential

Consider a Lagrangian for a scalar field, ϕ, in a potential:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

Mass term (ϕ^2 term) has the wrong sign unless mass is imaginary. How do we interpret this $\mathcal{L}$?

In Feynman calculus, the fields are the fluctuation (perturbation) from the ground state (vacuum).

Expressing $\mathcal{L}$ =T-U, the potential energy U is

$$U = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$$

Introducing Mass Terms in Interactions

The field that minimizes U is $\phi = \pm \mu / \lambda$

To shift the ground state to occur at 0, we introduce a new variable, η: $\eta \equiv \phi \pm \mu / \lambda$

Replacing field, ϕ, with the new field, η, the $\mathcal{L}$ becomes

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \eta\right)\left(\partial^\mu \eta\right) - \mu^2\eta^2 \pm \mu\lambda\eta^3 - \frac{1}{4}\lambda^2\eta^4 + \frac{1}{4}\left(\mu^2/\lambda\right)^2$$

Mass term

3point vtx

4point vtx

Comparing this modified $\mathcal{L}$ with $\mathcal{L}_{KG}$, the mass of the vector field is $m = \sqrt{2}\mu\hbar / c$

Spontaneous Symmetry Breaking

The original lagrangian, $\mathcal{L}$,

$$\mathcal{L} = \frac{1}{2}\left(\partial_{\mu}\phi\right)\left(\partial^{\mu}\phi\right) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$

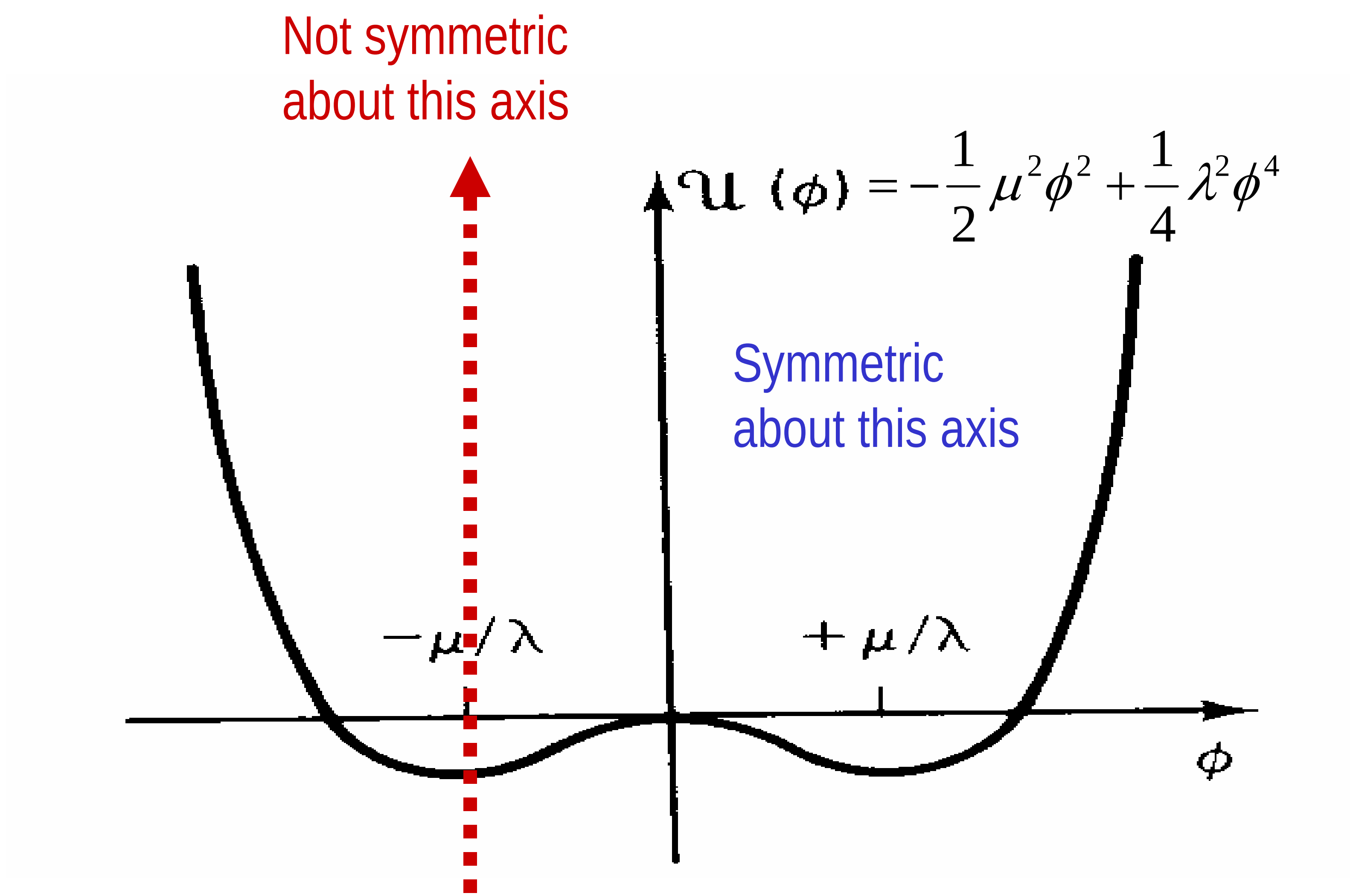
is even and thus invariant under $\phi \rightarrow -\phi$.

However, the new $\mathcal{L}$

$$\mathcal{L} = \frac{1}{2}\left(\partial_{\mu}\eta\right)\left(\partial^{\mu}\eta\right) - \mu^{2}\eta^{2} \pm \mu\lambda\eta^{3} - \frac{1}{4}\lambda^{2}\eta^{4} + \frac{1}{4}\left(\mu^{2}/\lambda\right)^{2}$$

has an odd term that causes this symmetry to break since any one of the ground states (vacuum) does not share the same symmetry as $\mathcal{L}$.

Potential and Symmetry Breaking



Spontaneous Symmetry Breaking

While the collection of ground states does preserve the symmetry in $\mathcal{L}$, the Feynman formalism allows to work with only one of the ground states. ➔ Causes the symmetry to break.

This is called “spontaneous” symmetry breaking, because symmetry breaking is not externally caused.

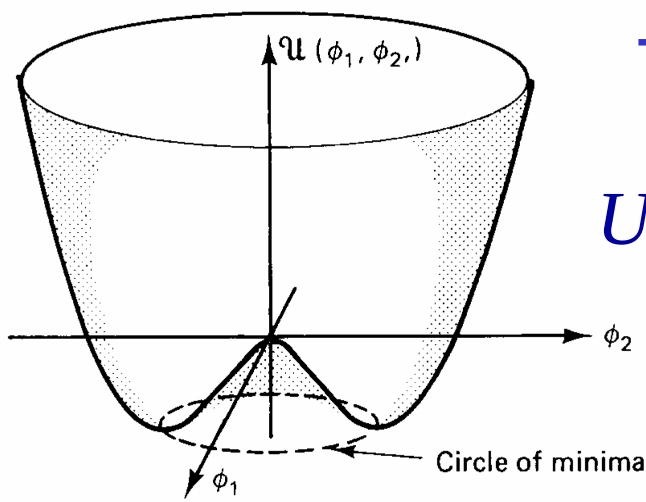
The true symmetry of the system is hidden by an arbitrary choice of a particular ground state. This is a case of discrete symmetry w/ 2 ground states.

Spontaneous Breaking of a Continuous Symmetry

A lagrangian, $\mathcal{L}$, for two fields, ϕ_1 and ϕ_2 can be written

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi_1\right)\left(\partial^\mu \phi_1\right) + \frac{1}{2}\left(\partial_\mu \phi_2\right)\left(\partial^\mu \phi_2\right) + \frac{1}{2}\mu^2\left(\phi_1^2 + \phi_2^2\right) - \frac{1}{4}\lambda^2\left(\phi_1^4 + \phi_2^4\right)$$

is even and thus invariant under $\phi_1, \phi_2 \rightarrow -\phi_1, -\phi_2$.



The potential energy term becomes

$$U = -\frac{1}{2}\mu^2\left(\phi_1^2 + \phi_2^2\right) + \frac{1}{4}\lambda^2\left(\phi_1^4 + \phi_2^4\right)$$

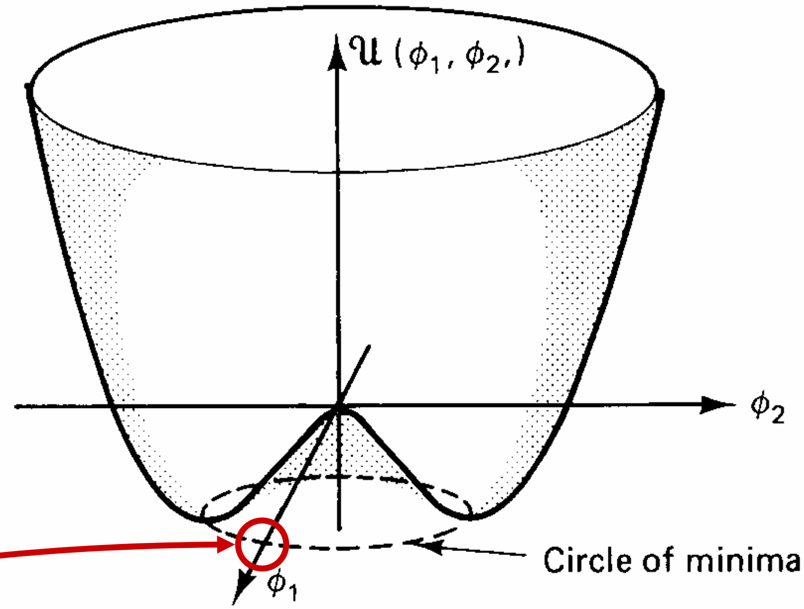
w/ the minima on the circle:

$$\phi_{1,\min}^2 + \phi_{2,\min}^2 = \mu^2 / \lambda^2$$

Spontaneous Breaking of a Continuous Symmetry

To apply Feynman calculus, we need to expand about a particular ground state (the "vacuum"). Picking

$$\phi_{1,min} = \mu/\lambda \text{ and } \phi_{2,min} = 0$$



And introducing two new fields, η and ξ, which are fluctuations about the vacuum:

$$\eta \equiv \phi_1 - \mu/\lambda \quad \text{and} \quad \xi \equiv \phi_2$$

Spontaneous Breaking of a Continuous Symmetry

The new $\mathcal{L}$ becomes

$$\mathcal{L} = \left[\frac{1}{2}\left(\partial_\mu \eta\right)\left(\partial^\mu \eta\right) - \mu^2\eta^2\right] + \left[\frac{1}{2}\left(\partial_\mu \xi\right)\left(\partial^\mu \xi\right)\right]$$
$$+ \left[\mu\lambda\left(\eta^3 + \eta\xi^2\right) - \frac{\lambda^2}{4}\left(\eta^4 + \xi^4 + 2\eta^2\xi^2\right)\right] + \frac{1}{4}\left(\mu^2/\lambda\right)^2$$

3point vtx

$\mathcal{L}_{KG}$ of the field η w/ the mass $m_\eta = \sqrt{2}\mu\hbar/c$

4point vtx

$\mathcal{L}_{free}$ of the massless field ξ

Spontaneous Breaking of Continuous Global Symmetry

- One of the fields is automatically massless.
- Goldstone's theorem says that breaking of continuous global symmetry is always accompanied by one or more massless scalar (spin=0) bosons, called Goldstone Bosons.
- This again poses a problem because the effort to introduce mass to weak gauge fields introduces a massless scalar boson which has not been observed.
- This problem can be addressed if spontaneous SB is applied to the case of local gauge invariance.

Homework

- Prove that the new Dirac Lagrangian with an addition of a vector field A_μ, as shown on page 9, is invariant under local gauge transformation.
- Describe the reason why the local gauge invariance forces the vector field to be massless
- Due is Wednesday, Apr. 11