PHYS 1441 – Section 002
Lecture #3

Wednesday, Jan. 23, 2008

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- Dimensional Analysis
- Trigonometry reminder
- Coordinate system, vector and scalars
- One Dimensional Motion: Average Velocity; Acceleration; Motion under constant acceleration; Free Fall

Dimension and Dimensional Analysis

- An extremely useful concept in solving physical problems
- Good to write physical laws in mathematical expressions
- No matter what units are used the base quantities are the same
  - *Length* (distance) is length whether meter or inch is used to express the size: Usually denoted as *[L]*
  - The same is true for *Mass ([M])* and *Time ([T])*
  - One can say “Dimension of Length, Mass or Time”
  - Dimensions are used as algebraic quantities: Can perform two algebraic operations; multiplication or division

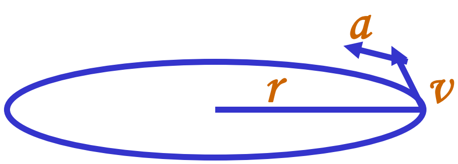
Dimension and Dimensional Analysis

- One can use dimensions only to check the validity of one's expression: Dimensional analysis
  - Eg: Speed $[v] = [L]/[T]=[L][T^{-1}]$
    - *Distance (L) traveled by a car running at the speed V in time T*
    - $L = V*T = [L/T]*[T]=[L]$
- More general expression of dimensional analysis is using exponents: eg. $[v]=[L^n T^m] =[L]\{T^{-1}]$ *where* $n = 1$ *and* $m = -1$

Examples

- Show that the expression $[v] = [at]$ is dimensionally correct
  - Speed: $[v]$ =L/T
  - Acceleration: $[a]$ =L/T^2
  - Thus, $[at]$ = (L/T^2)xT=LT$^{(-2+1)}$ =LT^{-1} =L/T= $[v]$

•Suppose the acceleration a of a circularly moving particle with speed v and radius r is proportional to r^n and v^m. What are n and m?



$$L^1T^{-2} = (L)^n\left(\frac{L}{T}\right)^m = L^{n+m}T^{-m}$$

$$a = kr^n v^m$$

Dimensionless constant

Length

Speed

$$-m = -2 \Rightarrow m = 2$$

$$n + m = n + 2 \equiv 1 \Rightarrow n = -1$$

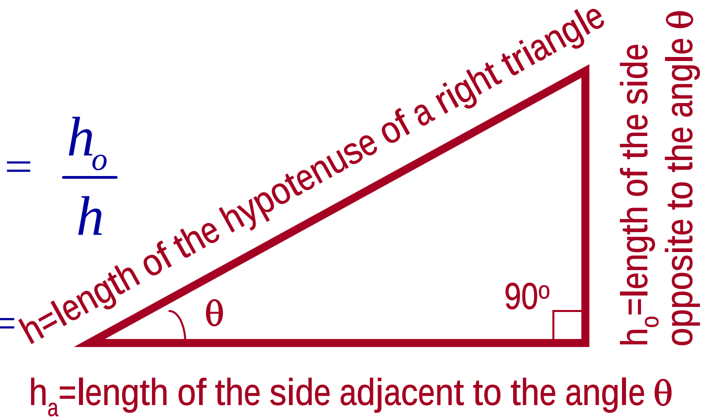
$$a = kr^{-1}v^2 = \frac{v^2}{r}$$

Trigonometry Reminders

- Definitions of sinθ, cosθ and tanθ

$$\sin\theta = \frac{\text{Length of the opposite side to }\theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_o}{h}$$

$$\cos\theta = \frac{\text{Length of the adjacent side to }\theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_a}{h}$$



$$\tan\theta = \frac{\text{Length of the opposite side to }\theta}{\text{Length of the adjacent side to }\theta} = \frac{h_o}{h_a}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\dfrac{h_o}{\cancel{h}}}{\dfrac{h_a}{\cancel{h}}} = \frac{h_o}{h_a}$$

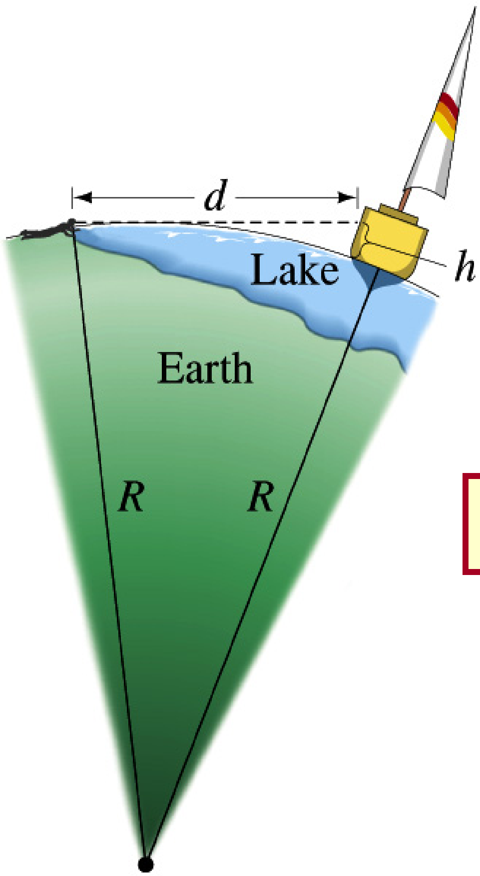
Pythagorian theorem: For right triangles

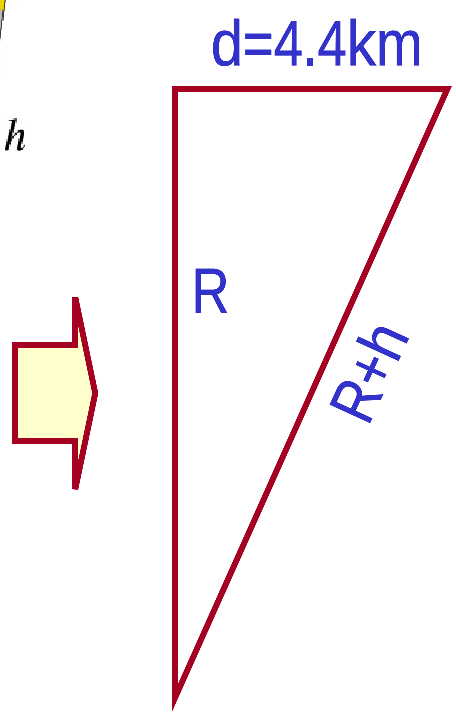
$$h^2 = h_o^2 + h_a^2 \Rightarrow h = \sqrt{h_o^2 + h_a^2}$$

Do Ex. 3 and 4 yourselves…

Example for estimates using trig..

Estimate the radius of the Earth using triangulation as shown in the picture when d=4.4km and h=1.5m.





Pythagorian theorem

$$(R+h)^2 \approx d^2 + R^2$$

$$R^2 + 2hR + h^2 \approx d^2 + R^2$$

Solving for R

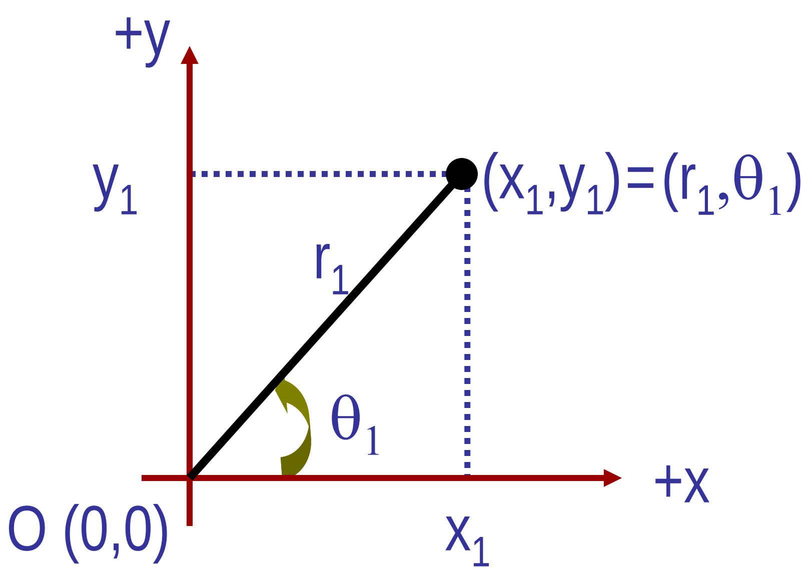
$$R \approx \frac{d^2 - h^2}{2h}$$

$$= \frac{(4400m)^2 - (1.5m)^2}{2 \times 1.5m}$$

$$= 6500km$$

Coordinate Systems

- They make it easy and consistent to express locations or positions
- Two commonly used systems, depending on convenience, are
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in distance from the origin ® and the angle measured from the x-axis, θ (r,θ)
- Vectors become a lot easier to express and compute



How are Cartesian and Polar coordinates related?

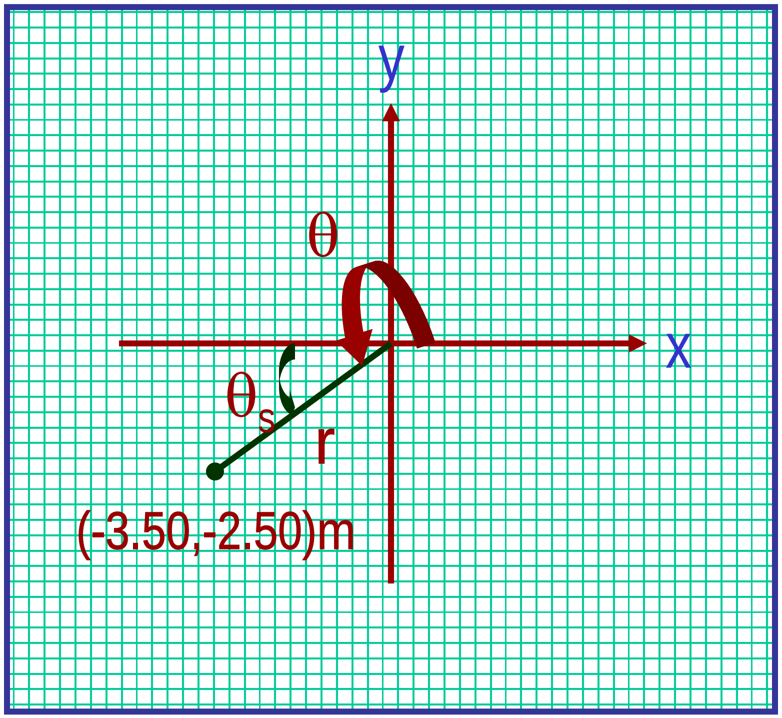
$$x_1 = r_1 \cos\theta_1 \qquad r_1 = \sqrt{\left(x_1^2 + y_1^2\right)}$$

$$y_1 = r_1 \sin\theta_1 \qquad \tan\theta_1 = \frac{y_1}{x_1}$$

$$\theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right)$$

Example

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the equivalent polar coordinates of this point.



$$r = \sqrt{\left(x^2 + y^2\right)}$$

$$= \sqrt{\left((-3.50)^2 + (-2.50)^2\right)}$$

$$= \sqrt{18.5} = 4.30(m)$$

$$\theta = 180 + \theta_s$$

$$\tan\theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ$$

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Force, gravitational acceleration, momentum

Normally denoted in **BOLD** letters, $\mathbf{F}$, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, F, or absolute values: $\left|\vec{F}\right|$ or $\left|\mathbf{F}\right|$

Scalar quantities have magnitudes only

Energy, heat, mass, time

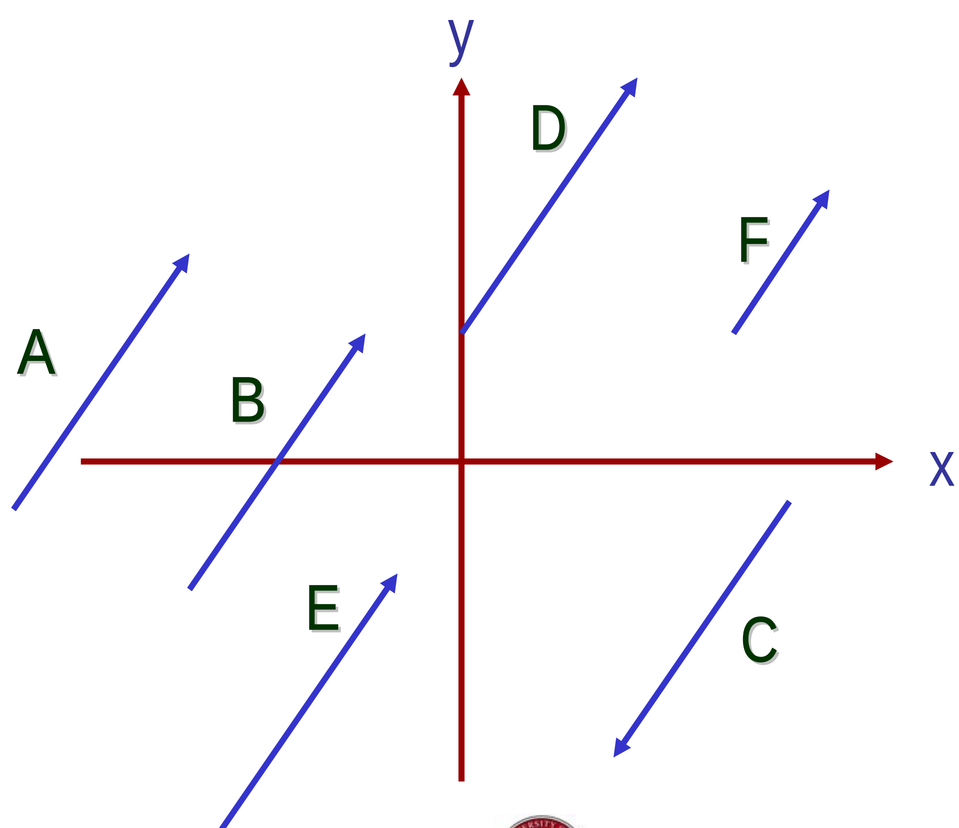
Can be completely specified with a value and its unit

Normally denoted in normal letters, E

Both have units!!!

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!!



Which ones are the same vectors?

A=B=E=D

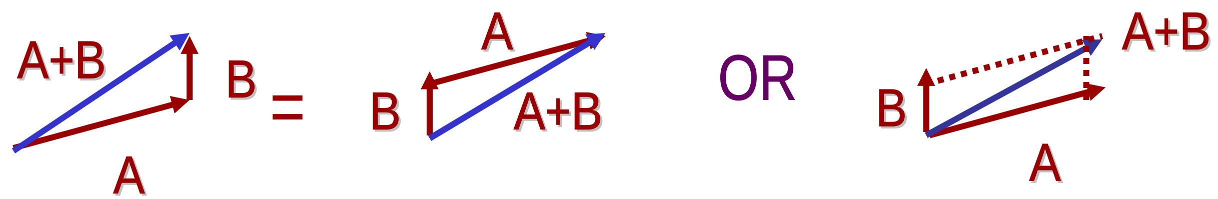
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:** A negative vector

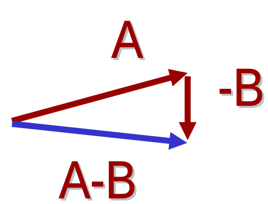
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results A+B=B+A, A+B+C+D+E=E+C+A+B+D



- Subtraction:
  - The same as adding a negative vector: A - B = A + (-B)



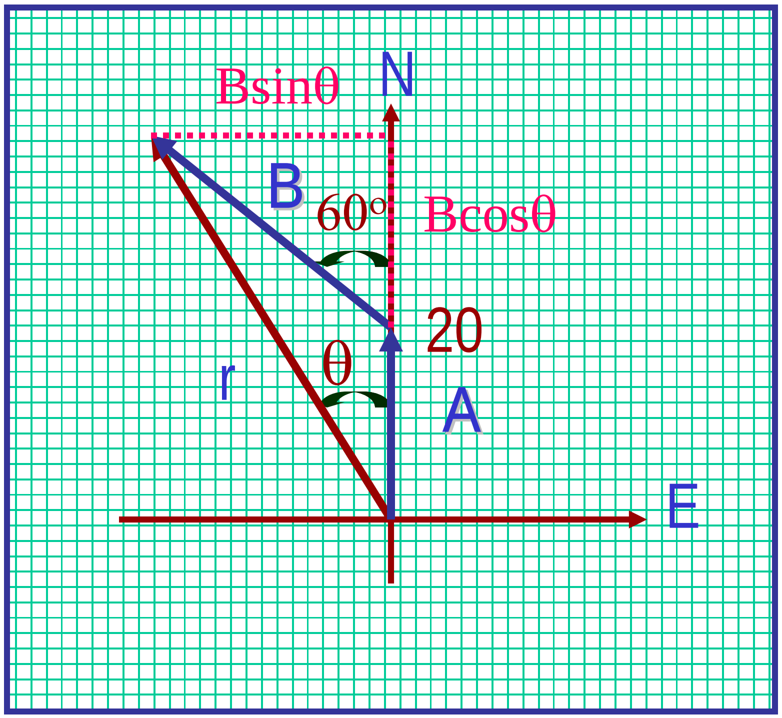
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude A, B=2A

$$\boxed{|\mathcal{B}| = 2|\mathcal{A}|}$$

Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0º west of north. Find the magnitude and direction of resultant displacement.



$$r = \sqrt{\left(A + B\cos\theta\right)^2 + \left(B\sin\theta\right)^2}$$

$$= \sqrt{A^2 + B^2\left(\cos^2\theta + \sin^2\theta\right) + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$

$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{\left|\vec{B}\right|\sin 60}{\left|\vec{A}\right| + \left|\vec{B}\right|\cos 60}$$

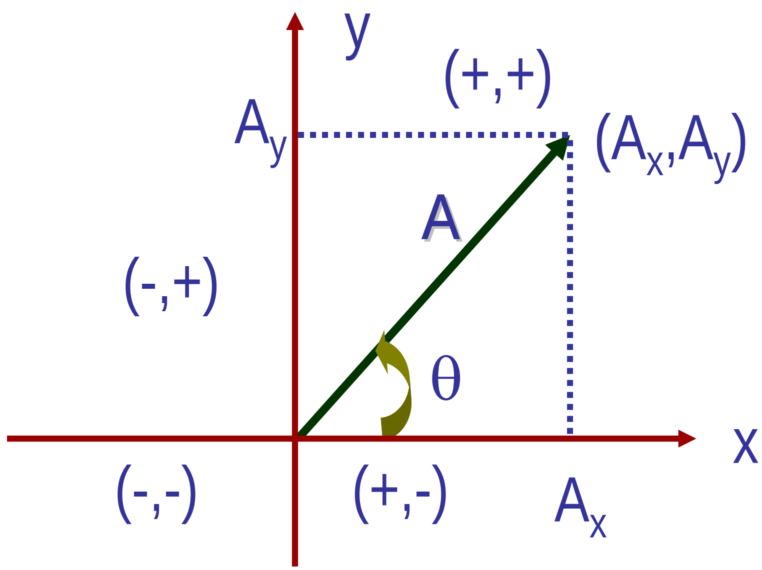
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$

$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^{\circ}\ \text{to W wrt N}$$

Find other ways to solve this problem…

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$\boxed{A_x = \left|\vec{A}\right| \cos\theta}$$
$$\boxed{A_y = \left|\vec{A}\right| \sin\theta}$$

} Components

$$\boxed{\left|\vec{A}\right| = \sqrt{A_x^{\,2} + A_y^{\,2}}}$$

} Magnitude

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right| \cos\theta\right)^2 + \left(\left|\vec{A}\right| \sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2 \left(\cos^2\theta + \sin^2\theta\right)} \quad = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- Dimensionless
- Magnitudes are exactly 1
- Unit vectors are usually expressed in **i**, **j**, **k** or

$$\vec{i},\ \vec{j},\ \vec{k}$$

So the vector **A** can be re-written as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} = \left|\vec{A}\right| \cos\theta \vec{i} + \left|\vec{A}\right| \sin\theta \vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$

$$= \left(2.0 + 2.0\right)\vec{i} + \left(2.0 - 4.0\right)\vec{j} = 4.0\vec{i} - 2.0\vec{j}\,(m)$$

$$\left|\vec{C}\right| = \sqrt{(4.0)^2 + (-2.0)^2}$$

$$= \sqrt{16 + 4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements: **d**$_1$=(15**i**+30**j** +12**k**)cm, **d**$_2$=(23**i**+14**j** -5.0**k**)cm, and **d**$_3$=(-13**i**+15**j**)cm

$$\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

$$= \left(15 + 23 - 13\right)\vec{i} + \left(30 + 14 + 15\right)\vec{j} + \left(12 - 5.0\right)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}\,(cm)$$

Magnitude

$$\left|\vec{D}\right| = \sqrt{(25)^2 + (59)^2 + (7.0)^2} = 65(cm)$$