PHYS 1441 – Section 002
Lecture #6

Monday, Feb. 4, 2008
Dr. Jaehoon Yu

- Examples for 1-Dim kinematic equations
- Free Fall
- Motion in Two Dimensions
  - Maximum ranges and heights

Today's homework is homework #3, due 9pm, Monday, Feb. 11!!

Announcements

- E-mail distribution list
  - Test message sent out last Thursday
  - 24 of you confirmed the receipt
  - If you did not get the message, you could be missing important information related to class
  - Please make sure that you are on the distribution list
- Quiz Wednesday, Feb 6, early in the class
  - Covers from CH1 – what we learn today

Reminder: Special Problems for Extra Credit

- Derive the quadratic equation for $yx^2-zx+v=0$
  ➔ 5 points
- Derive the kinematic equation $v^2 = v_0^2 + 2a(x - x_0)$ from first principles and the known kinematic equations ➔ 10 points
- You must <u>show your work in detail</u> to obtain the full credit
- Due next Wednesday, Feb. 6

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$\boxed{v(t) = v_0 + at}$$

Velocity as a function of time

$$\boxed{x - x_0 = \frac{1}{2}\bar{v}t = \frac{1}{2}(v + v_0)t}$$

Displacement as a function of velocities and time

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

Displacement as a function of time, velocity, and acceleration

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you in specific physical problems!!

How do we solve a problem using a kinematic formula under constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance, initial position or final position?
  - Time?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem wants
- Identify which kinematic formula is **appropriate and easiest** to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equations for the quantity or quantities wanted.

Example 8 An Accelerating Spacecraft

A spacecraft is traveling with a velocity of +3250 m/s. Suddenly the retrorockets are fired, and the spacecraft begins to slow down with an acceleration whose magnitude is 10.0 m/s^2. What is the velocity of the spacecraft when the displacement of the craft is +215 km, relative to the point where the retrorockets began firing?

x	a	v	v_o	t
+215000 m	-10.0 m/s^2	?	+3250 m/s	

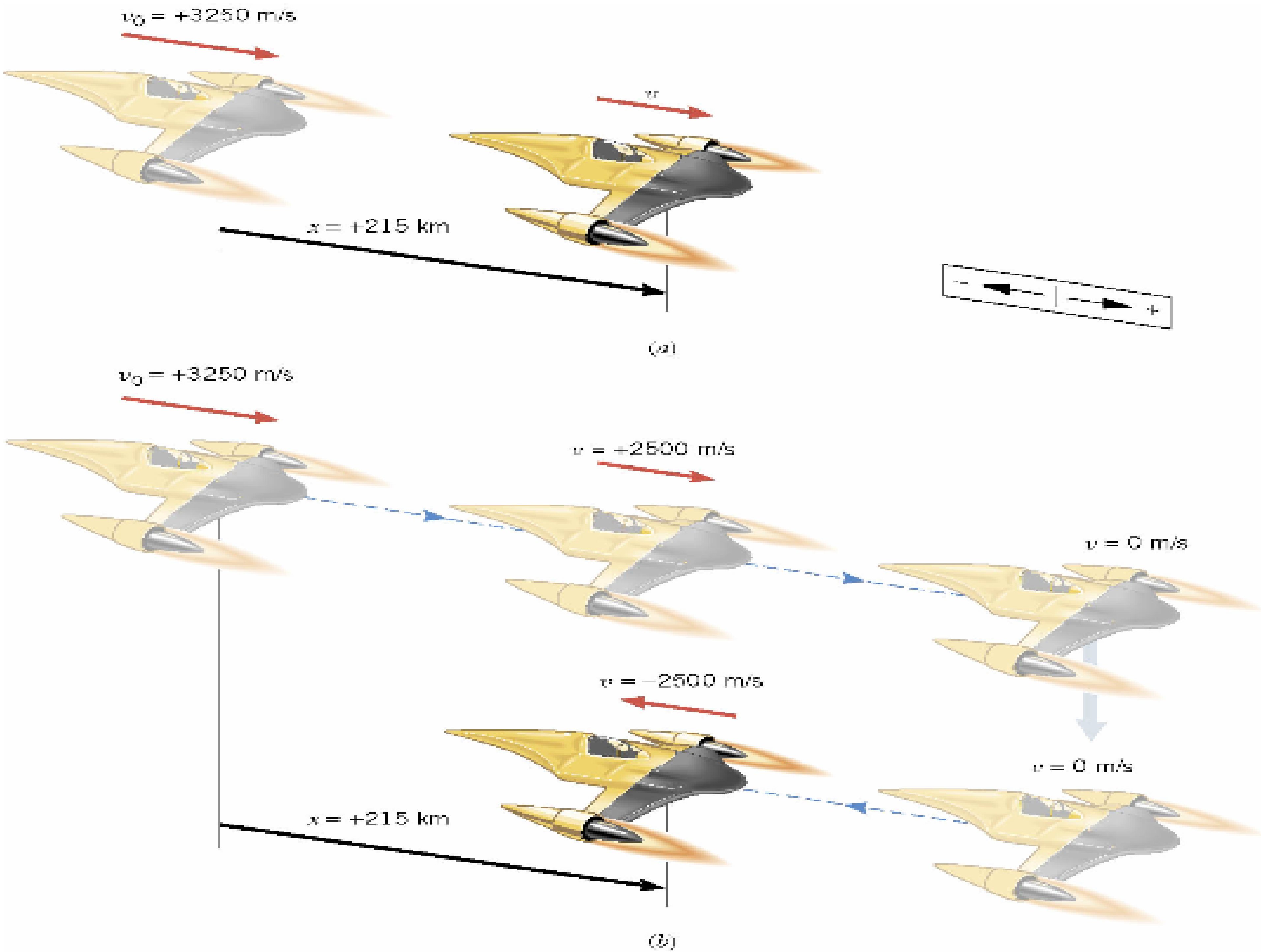
x	a	v	v_o	t
+215000 m	-10.0 m/s^2	?	+3250 m/s	

$$v^2 = v_o^2 + 2ax \quad \xrightarrow{\text{Solve for v}} \quad v = \pm\sqrt{v_o^2 + 2ax}$$

$$v = \pm\sqrt{\left(3250\,\mathrm{m/s}\right)^2 + 2\left(-10.0\,\mathrm{m/s^2}\right)\left(215000\ \mathrm{m}\right)}$$

$$= \pm 2500\,\mathrm{m/s}$$

What do two opposite signs mean?



v0 = +3250 m/s
v
x = +215 km
(a)
v0 = +3250 m/s
v = +2500 m/s
v = 0 m/s
v = −2500 m/s
v = 0 m/s
x = +215 km
(b)

Example

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇨ As long as it takes for it to crumple.

The initial speed of the car is $$v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

The acceleration is $$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \frac{0-\left(28m/s\right)^2}{2\times 1m} = -390m/s^2$$

Thus the time for air-bag to deploy is $$t = \frac{v_{xf} - v_{xi}}{a} = \frac{0-28m/s}{-390m/s^2} = 0.07s$$

Free Fall

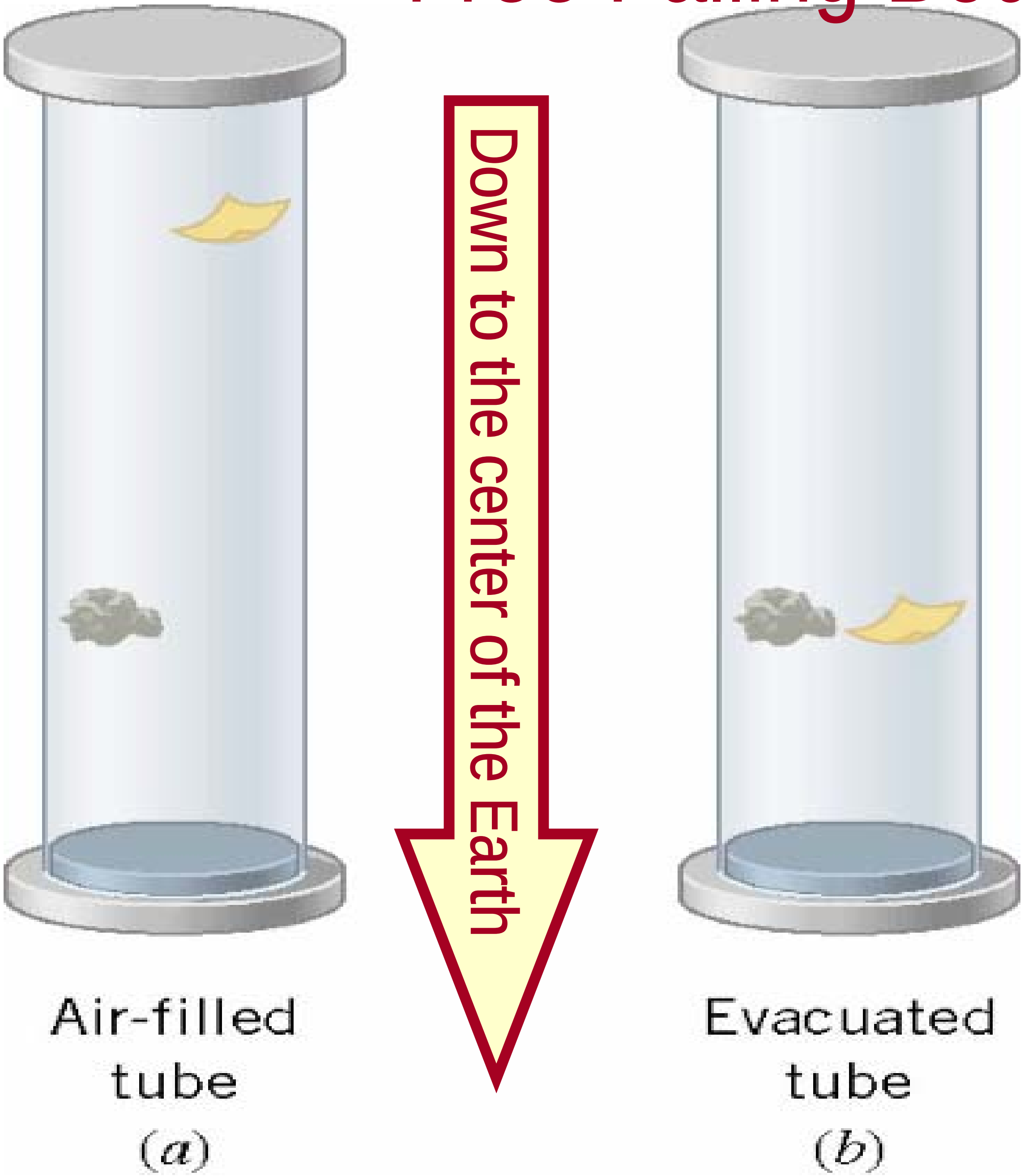
- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
  - A motion under constant acceleration
  - All kinematic formulae we learned can be used to solve for falling motions.

Down to the center of the Earth!

- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $|g|=9.80m/s^2$ on the surface of the Earth, most of the time.
- The direction of gravitational acceleration is <u>ALWAYS</u> toward the center of the earth, which we normally call (-y); when the vertical directions are indicated with the variable "y"
- Thus the correct denotation of the gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$ $g = -32.2\,\mathrm{ft}/\mathrm{s}^2$

Note the negative sign!!

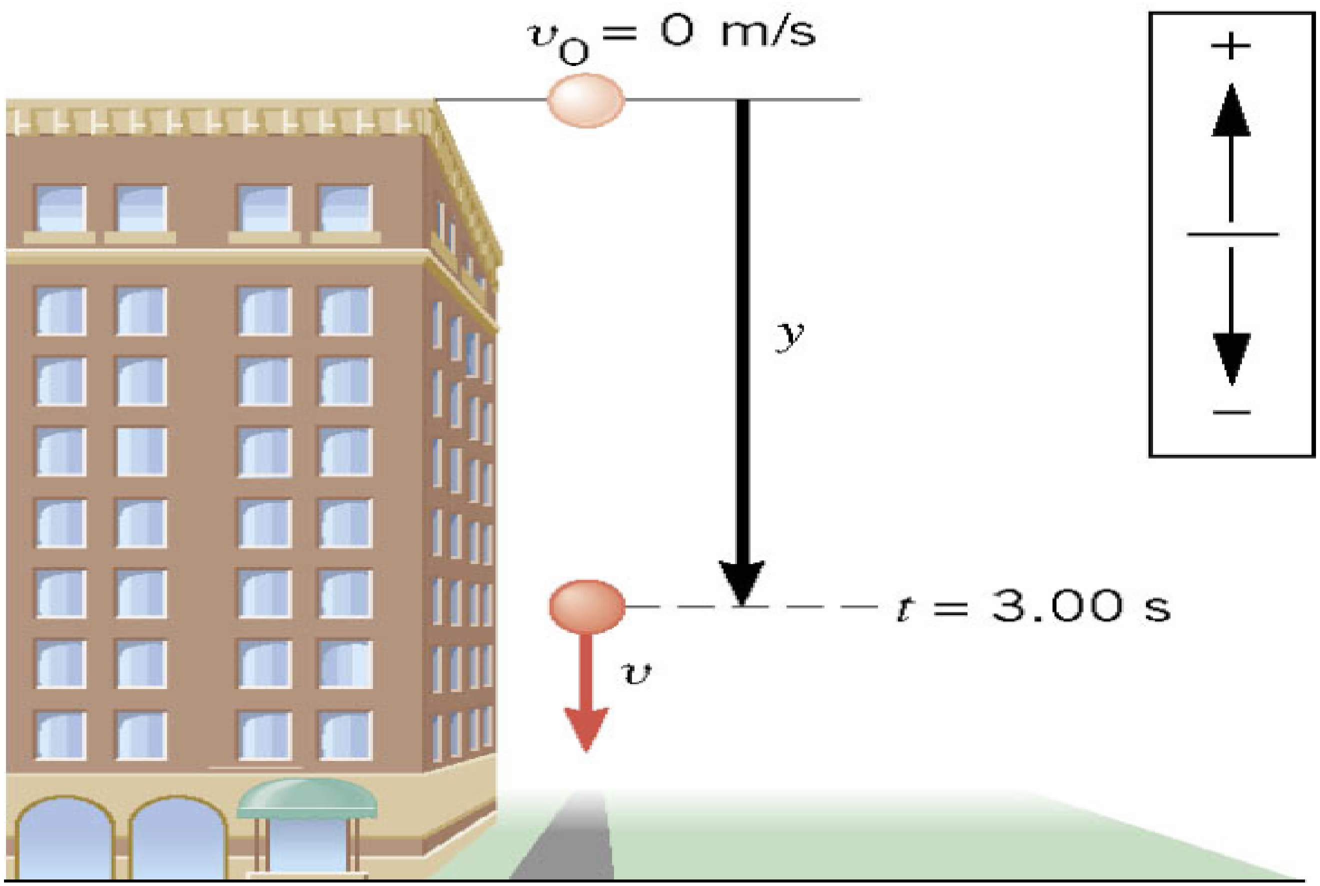
Free Falling Bodies

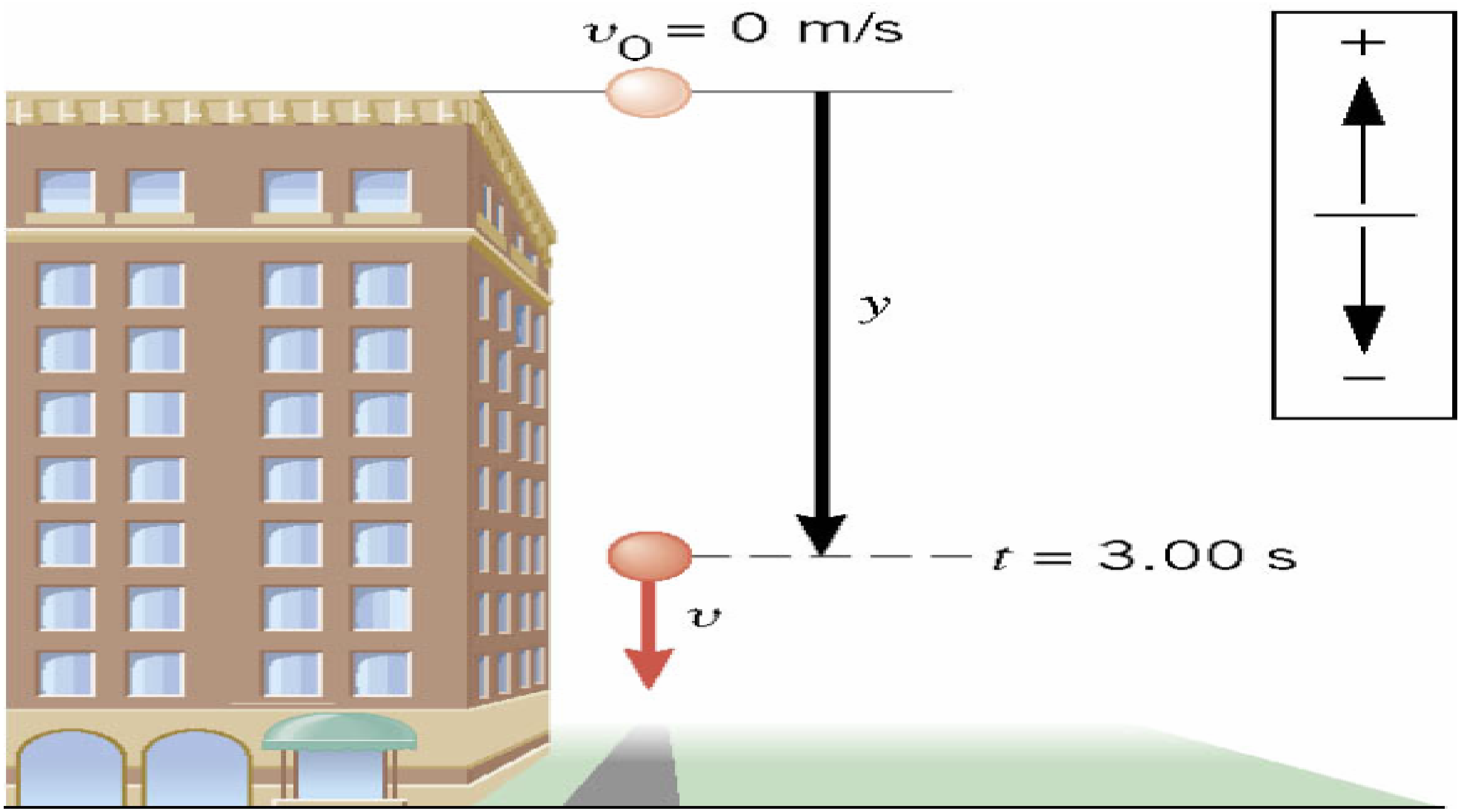


$$|\vec{g}| = 9.80\,\mathrm{m/s^2}$$

Example 10

A stone is dropped from the top of a tall building. After 3.00s of free fall, what is the displacement *y* of the stone?





y	a	v	v_o	t
?	-9.8 m/s^2		0 m/s	3.00 s

y	a	v	v_o	t
?	-9.80 m/s^2		0 m/s	3.00 s

Which formula would you like to use?

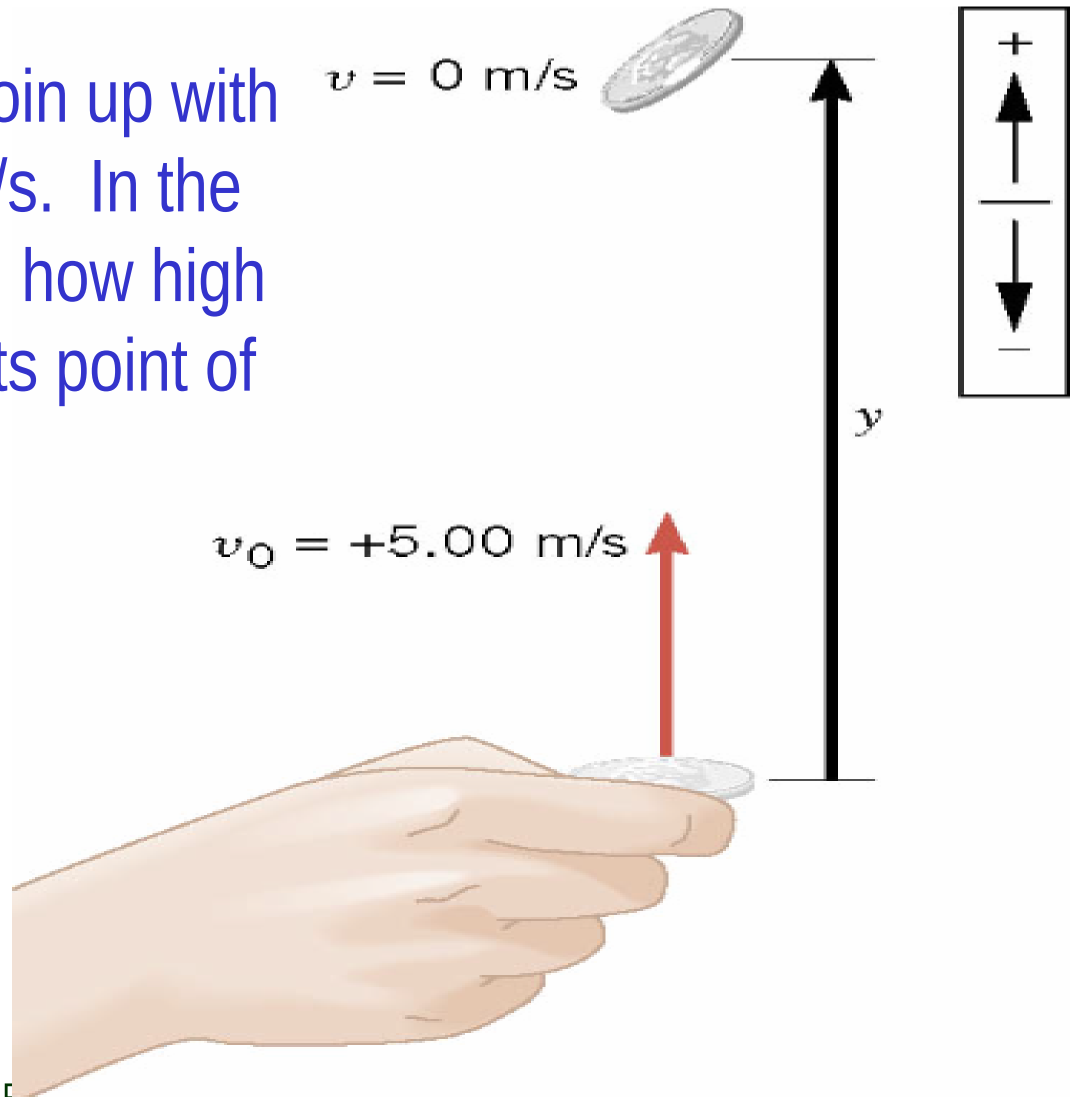
$$y = v_o t + \tfrac{1}{2} a t^2 = v_o t + \tfrac{1}{2} g t^2$$

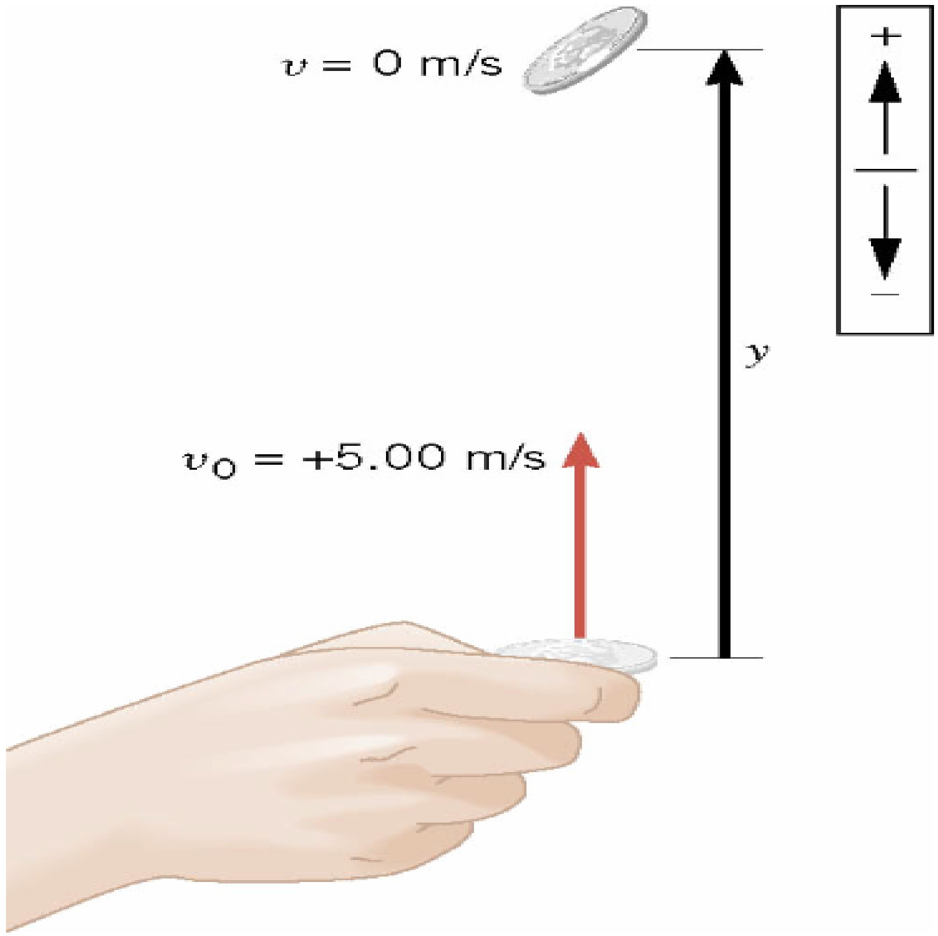
$$= (0\,\text{m/s})(3.00\,\text{s}) + \tfrac{1}{2}\left(-9.80\,\text{m/s}^2\right)(3.00\,\text{s})^2$$

$$= -44.1\,\text{m}$$

Example 12 How high does it go?

The referee tosses the coin up with an initial speed of 5.00m/s. In the absence if air resistance, how high does the coin go above its point of release?





y	a	v	v_o	t
?	-9.8 m/s^2	0 m/s	+ 5.00 m/s	

y	a	v	v_o	t
?	-9.80 m/s^2	0 m/s	+5.00 m/s	

$$v^2 = v_o^2 + 2ay$$

Solve for y

$$y = \frac{v^2 - v_o^2}{2a}$$

$$y = \frac{v^2 - v_o^2}{2a} = \frac{(0\,\mathrm{m/s})^2 - (5.00\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 1.28\ \mathrm{m}$$

Conceptual Example 14 Acceleration Versus Velocity

There are three parts to the motion of the coin. On the way up, the coin has a vector velocity that is directed upward and has decreasing magnitude. At the top of its path, the coin momentarily has zero velocity. On the way down, the coin has downward-pointing velocity with an increasing magnitude.

In the absence of air resistance, does the acceleration of the coin, like the velocity, change from one part to another?

Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $\boxed{g=-9.80m/s^2}$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00 m/s$$ Solve for t $$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

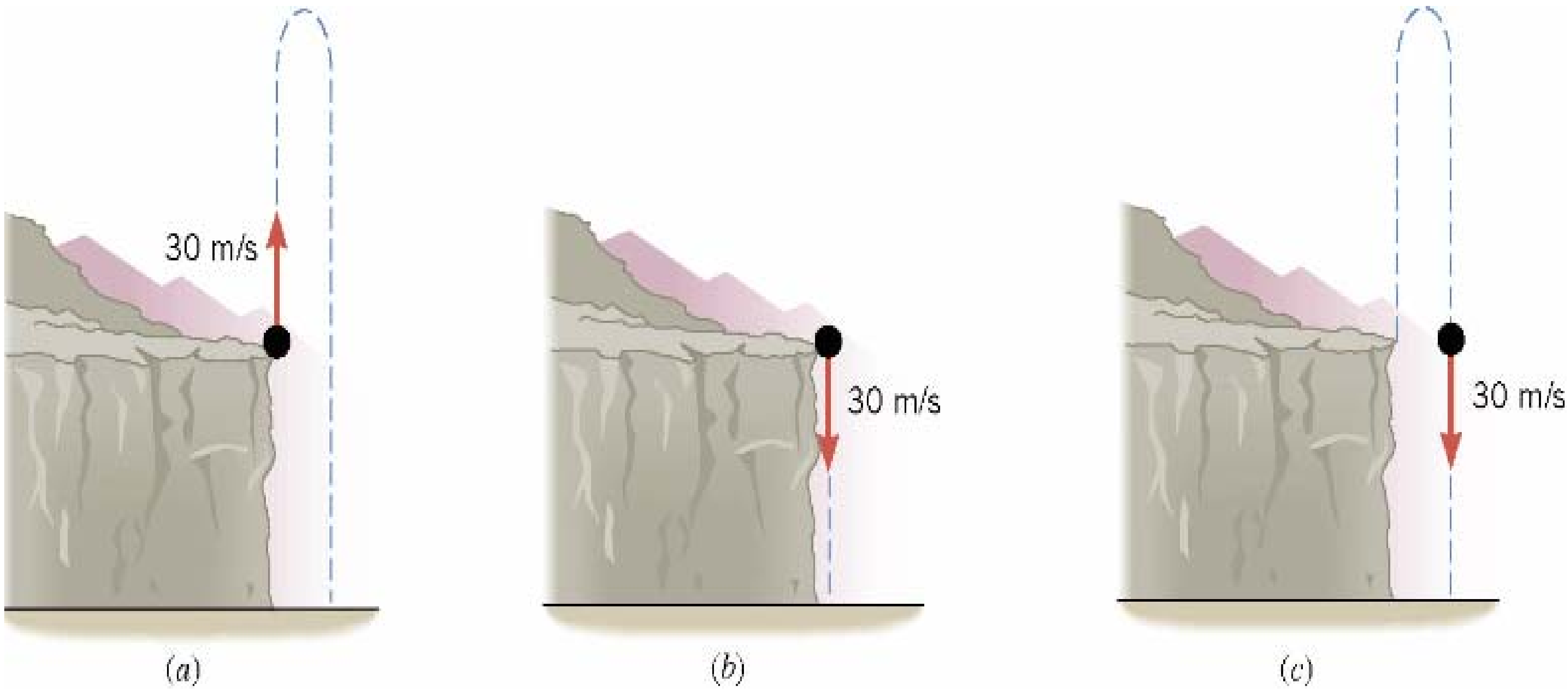
Position

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$

Conceptual Example 15 Taking Advantage of Symmetry

Does the pellet in part *b* strike the ground beneath the cliff with a smaller, greater, or the same speed as the pellet in part *a*?



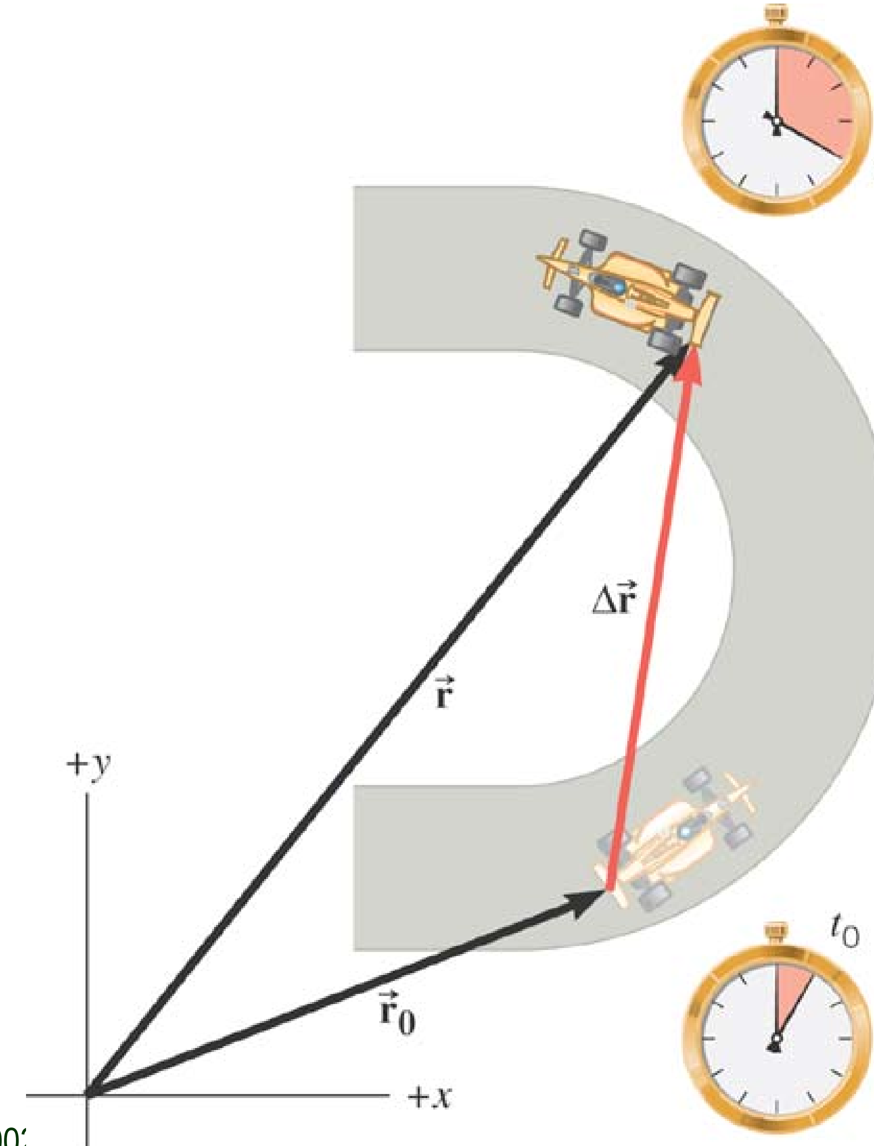
The same!!

2 Dimensional Kinematic Quantities

$$\vec{\mathbf{r}}_o = \text{ initial position}$$

$$\vec{\mathbf{r}} = \text{ final position}$$

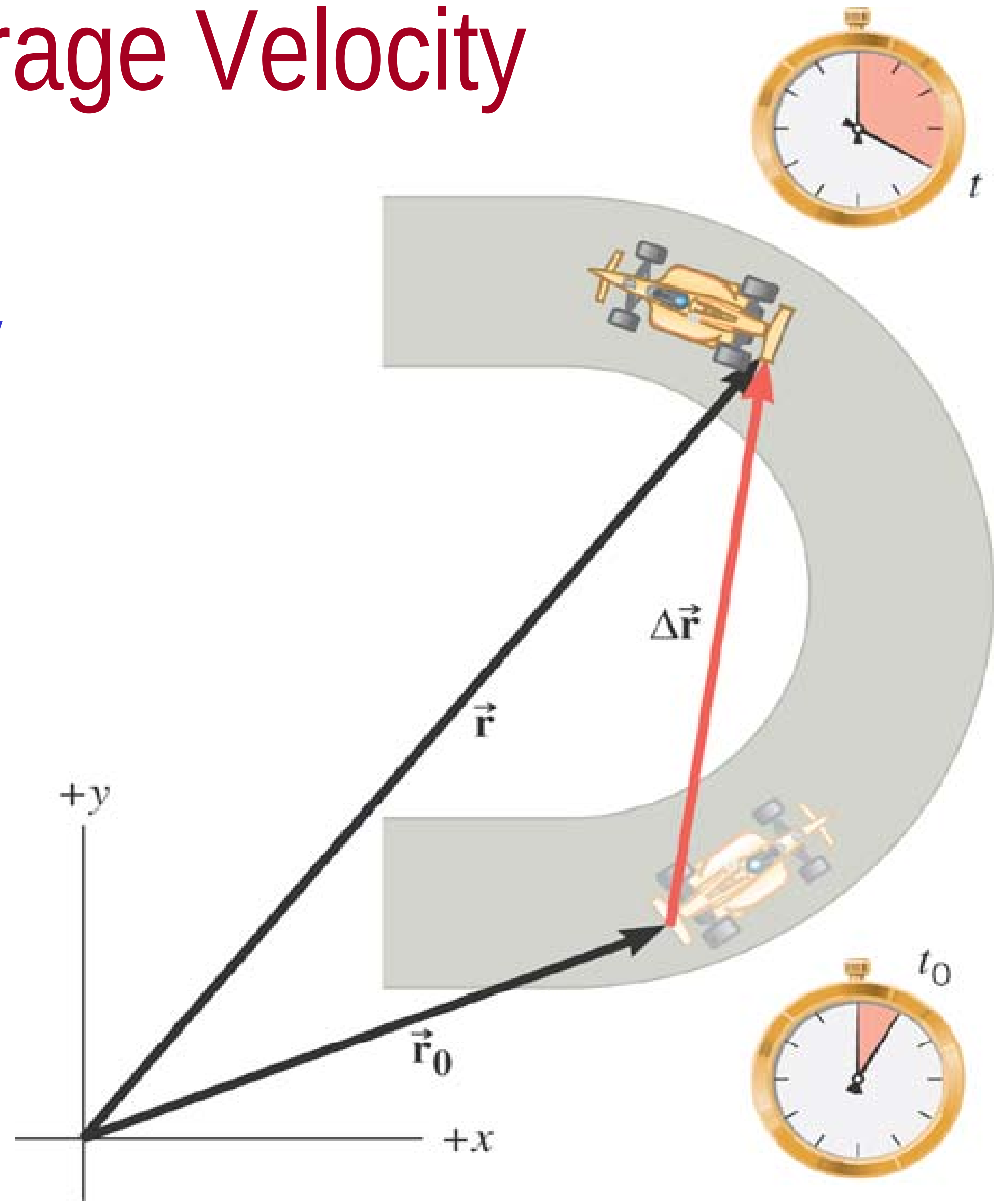
$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o = \text{ displacement}$$



2D Average Velocity

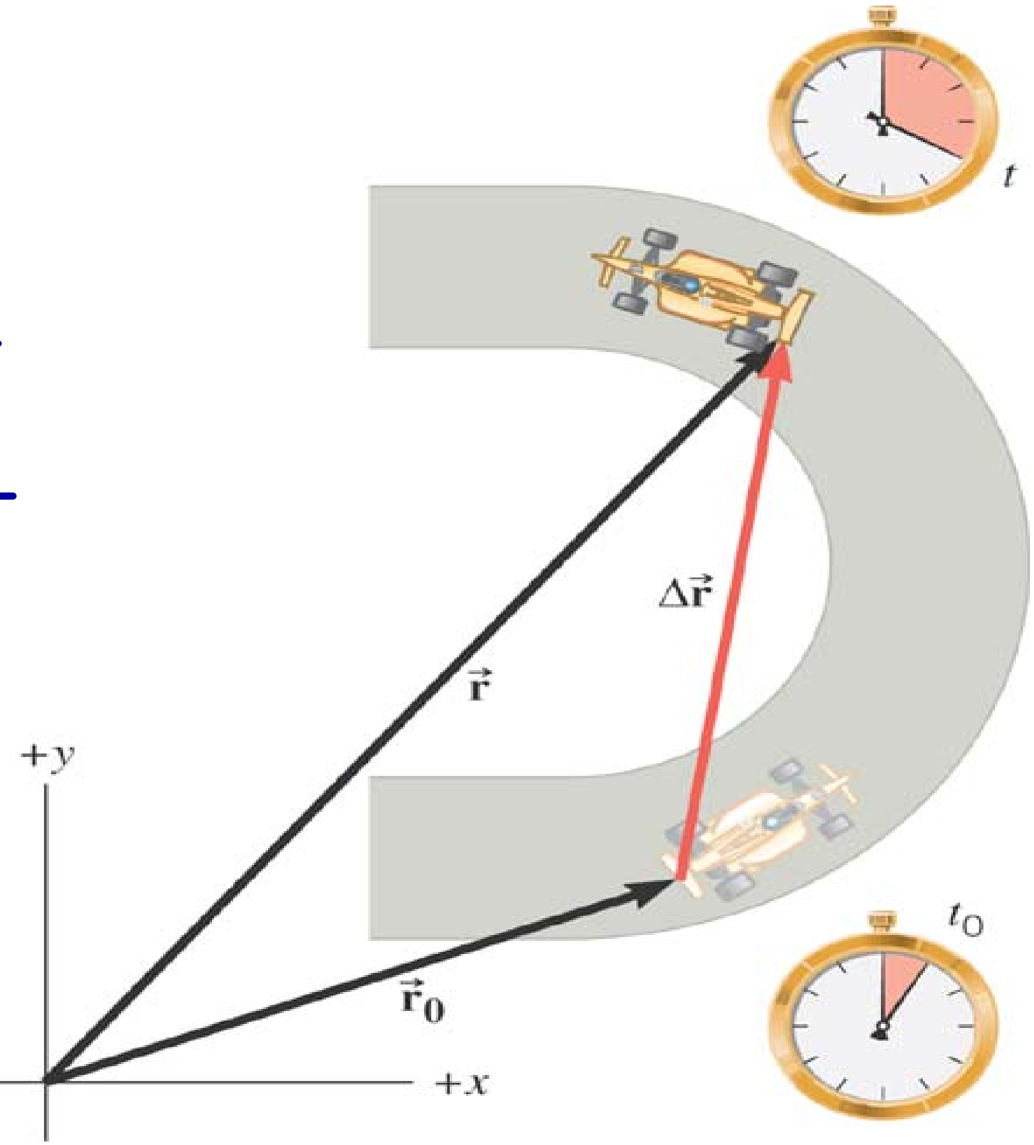
Average velocity is the displacement divided by the elapsed time.

$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$



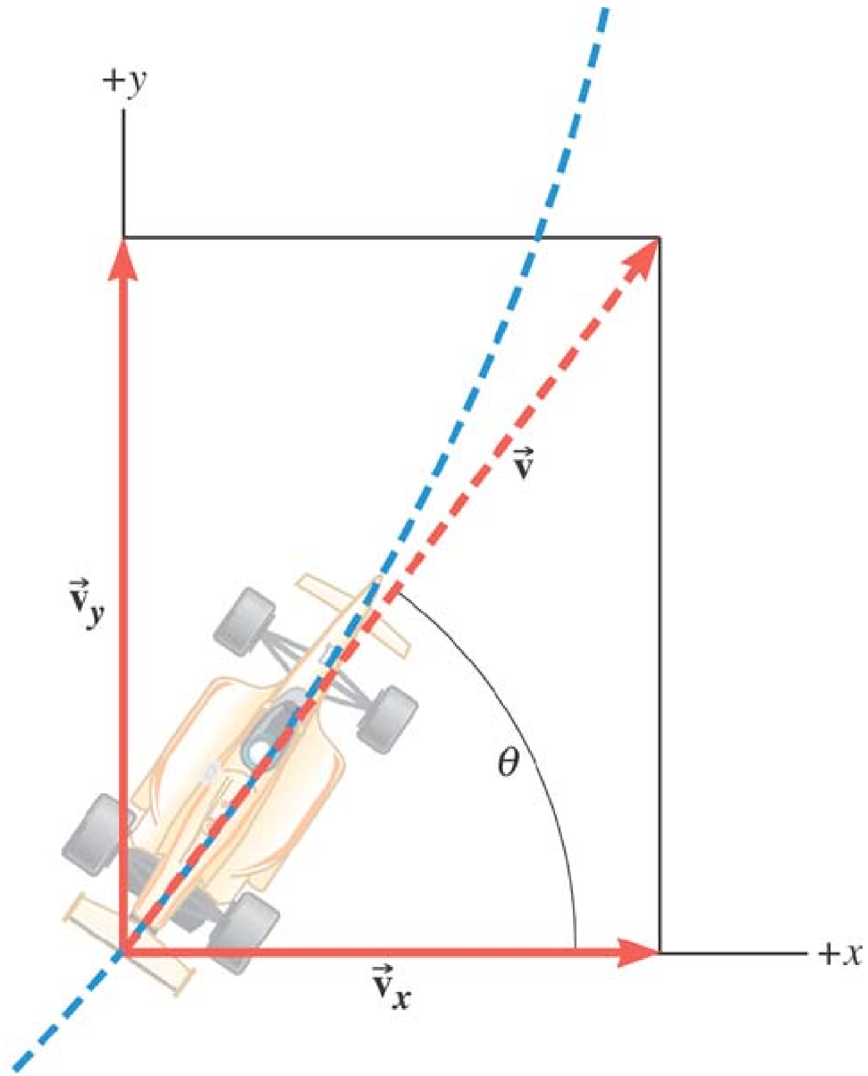
The *instantaneous velocity* indicates how fast the car moves and the direction of motion at each instant of time.

$$\overline{\vec{\mathbf{v}}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$

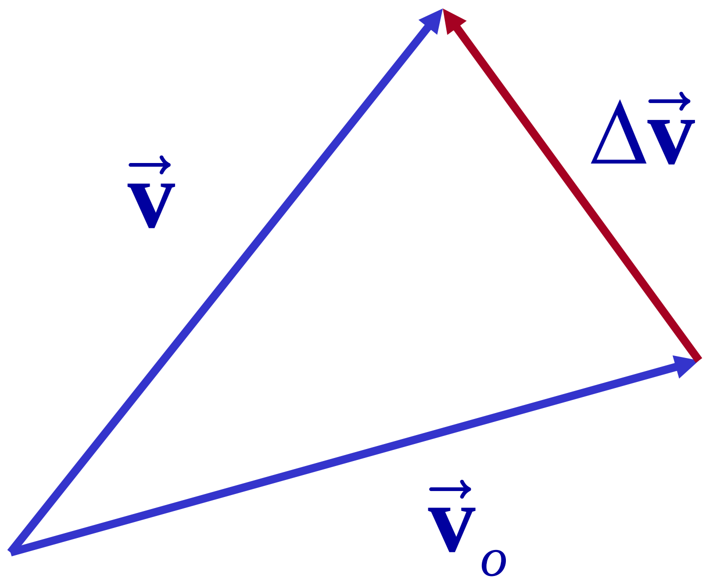


2D Instantaneous Velocity

$$\vec{\bar{\mathbf{v}}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$



2D Average Acceleration



$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

How is each of these quantities defined in 1-D?

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$

What is the difference between 1D and 2D quantities?