PHYS 1441 – Section 002
Lecture #8

Monday, Feb. 11, 2008
Dr. Jaehoon Yu

- Components and unit vectors
- Motion in Two Dimensions
  - Projectile Motion
  - Maximum ranges and heights
- Newton's Laws of Motion
  - Force
  - Newton's Law of Inertia & Mass

Today's homework is homework #4, due 9pm, Monday, Feb. 25!!

Announcements

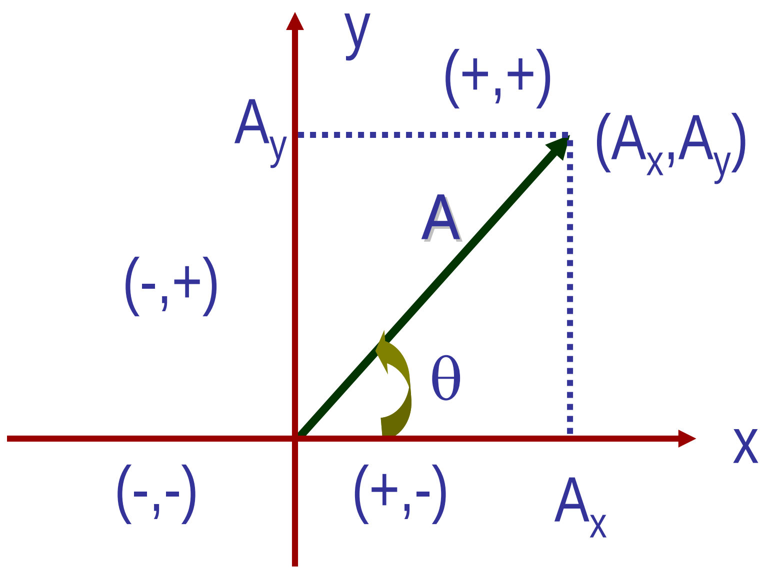
- 1st term exam next Wednesday, Feb. 20
  - Time: 1 – 2:20pm
  - Place: SH103
  - Covers: Appendices, CH 1 – what we learn till this Wednesday
  - Style: mixture of multiple choices and essay problems
  - Class on Monday, Feb. 18: Jason will be here to go over the any problems you would like to review
- Quiz Results
  - Average: 3.5/8
    - Equivalent to 40/100
    - Quiz 1 result: 40/100
  - Top score: 8/8

Special Project

- Show that a projectile motions trajectory is a parabola!!
  - 20 points
  - Due: Wednesday, Feb. 27
  - You MUST show full details of computations to obtain any credit

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$A_x = \left|\vec{A}\right| \cos\theta$$
$$A_y = \left|\vec{A}\right| \sin\theta$$
} Components

$$\left|\vec{A}\right| = \sqrt{A_x^{\ 2} + A_y^{\ 2}}$$
} Magnitude

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2 \left(\cos^2\theta + \sin^2\theta\right)} = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- <u>Dimensionless</u>
- <u>Magnitudes are exactly 1</u>
- Unit vectors are usually expressed in **i**, **j**, **k** or

$\vec{i},\ \vec{j},\ \vec{k}$

So the vector **A** can be re-written as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$

$$= (2.0+2.0)\vec{i} + (2.0-4.0)\vec{j} = 4.0\vec{i} - 2.0\vec{j}\,(m)$$

$$\left|\vec{C}\right| = \sqrt{(4.0)^2 + (-2.0)^2}$$

$$= \sqrt{16+4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements: $\mathbf{d}_1$=(15**i**+30**j** +12**k**)cm, $\mathbf{d}_2$=(23**i**+14**j** -5.0**k**)cm, and $\mathbf{d}_3$=(-13**i**+15**j**)cm

$$\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

$$= (15 + 23 - 13)\vec{i} + (30 + 14 + 15)\vec{j} + (12 - 5.0)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}\,(cm)$$

Magnitude

$$\left|\vec{D}\right| = \sqrt{(25)^2 + (59)^2 + (7.0)^2} = 65(cm)$$

Kinematic Equations in 2-Dim

x-component

$$v_x = v_{xo} + a_x t$$

$$x = \tfrac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x$$

$$x = v_{xo}t + \tfrac{1}{2}a_x t^2$$

y-component

$$v_y = v_{yo} + a_y t$$

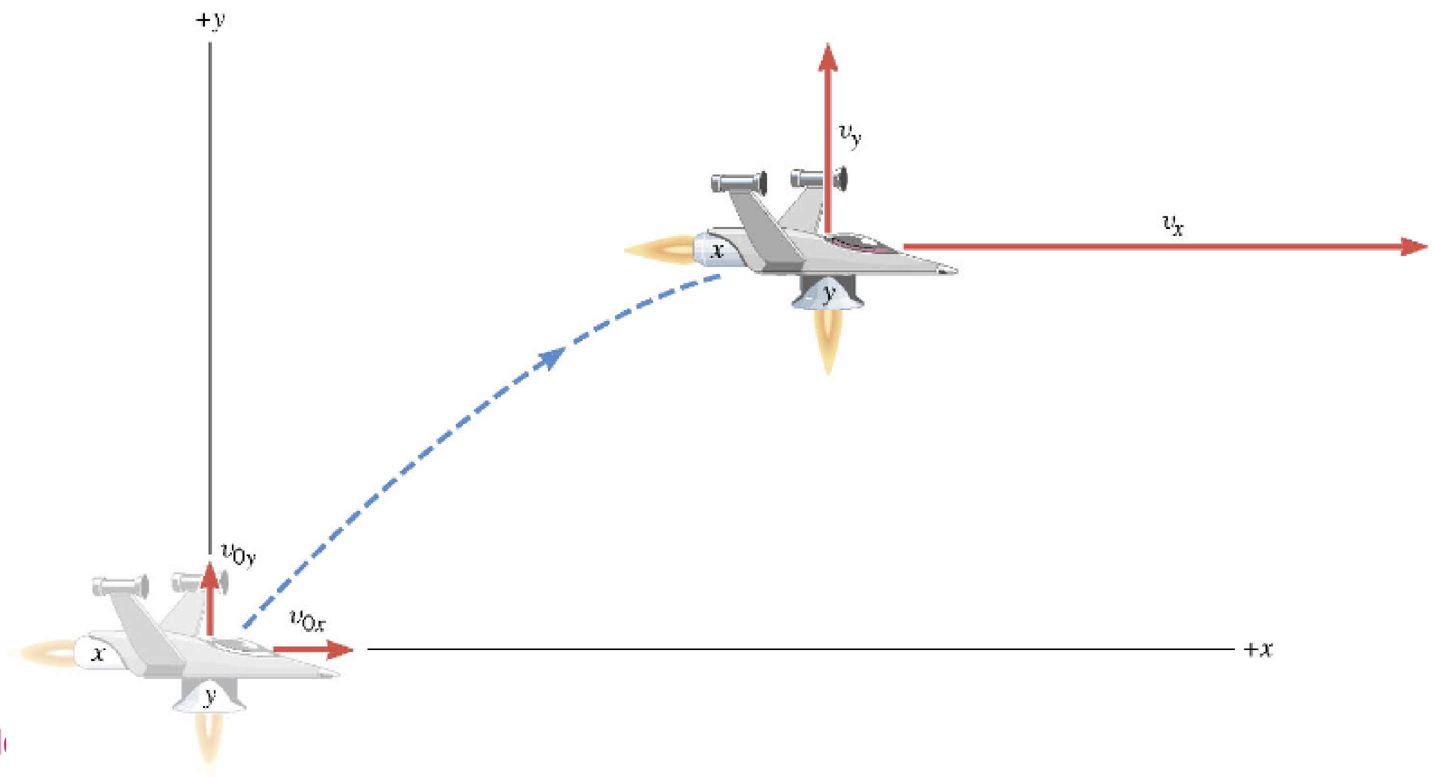
$$y = \tfrac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$y = v_{yo}t + \tfrac{1}{2}a_y t^2$$

Ex.1 A Moving Spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



How do we solve this problem?

1. Visualize the problem → Make a drawing.
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y separately.* Select the appropriate equation.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

Ex.1 continued

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22.0 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14.0 m/s	7.0 s

First, the motion in x-direciton…

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$= (22\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(24\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +740\ \mathrm{m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\mathrm{m/s}) + (24\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +190\,\mathrm{m/s}$$

Now, the motion in y-direction…

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

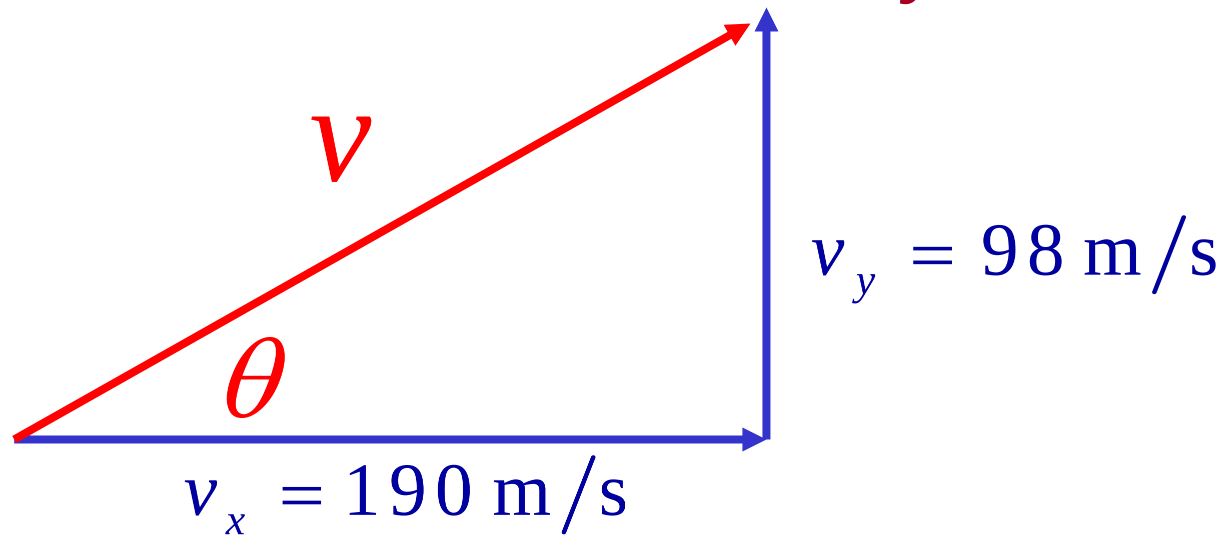
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$= (14\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}\left(12\,\mathrm{m/s^2}\right)(7.0\ \mathrm{s})^2 = +390\ \mathrm{m}$$

$$v_y = v_{oy} + a_y t$$

$$= (14\,\mathrm{m/s}) + \left(12\,\mathrm{m/s^2}\right)(7.0\ \mathrm{s}) = +98\,\mathrm{m/s}$$

The final velocity…



$$v = \sqrt{(190\ \mathrm{m/s})^2 + (98\ \mathrm{m/s})^2} = 210\ \mathrm{m/s}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using components and unit vectors!!

$$\vec{v} = v_x \vec{i} + v_y \vec{j} = (190\vec{i} + 98\vec{j})\ m/s$$

If we visualize the motion…