PHYS 1441 – Section 002
Lecture #13

Wednesday, Mar. 5, 2008
Dr. Jaehoon Yu

• Static and Kinetic Frictional Forces
• The Tension Force
• Equilibrium Applications of Newton’s Laws
• Non-equilibrium Applications of Newton’s Laws
• Uniform Circular Motion
Announcements

• Term exam #2
  – Wednesday, March 26, in class
  – Will cover CH4.1 – whatever we finish Monday, Mar. 24

• Colloquium today
  – 4pm in SH101
  – Dr. M. Wiltberger, U. of Colorado, Boulder
  – Geospace Modeling
Physics Department
The University of Texas at Arlington

COLLOQUIUM

Geospace Modeling

Dr. Michael Wiltberger
NCAR/HAO, Boulder, CO

4:00 pm Wednesday, March 5, 2008
Room 101 SH

Abstract

The Lyon-Fedder-Mobarry (LFM) global scale magnetospheric model has proven remarkably useful in providing a global context for magnetospheric observations. The LFM simulates the solar wind - magnetosphere - ionosphere interaction by coupling a magnetohydrodynamic (MHD) simulation of the solar wind magnetosphere interaction to a 2D simulation of the ionosphere. This presentation begins with a brief summary of the LFM capabilities and limitations including an overview of the numerical techniques used to solve the MHD equations and the computational resources needed to provide real time simulations. Results from the December 10, 1996 substorm simulation illustrate the accuracy of the model during isolated substorm intervals. Results from the series of magnetospheric storms illustrate the model's ability to simulate long duration events as well as its limited representation of the ring current. Since magnetospheric forecast require predictions of the solar wind conditions extended forecasts require the coupling of the LFM to a solar wind model. In addition, many important space weather factors, e.g. radiation belt fluxes and ionospheric scintillation, are outside the scope of the LFM so it must be coupled to other physics based simulations. This talk concludes by presenting the results coupling the LFM to the Dartmouth energetic particle simulation and the NCAR Thermosphere Ionosphere Nested Grid (TING) model.

Refreshments will be served in the Physics Library at 3:30 pm
Special Project Reminder

• Using the fact that \( g = 9.80 \text{m/s}^2 \) on the Earth’s surface, find the average density of the Earth.

• 20 point extra credit

• Due: Wednesday, Mar. 12

• You must show your OWN, detailed work to obtain any credit!!
Static Friction

When the two surfaces are not sliding across one another the friction is called **static friction**. The resistive force exerted on the object up to the time just before the object starts moving.

![Diagram of static friction](image)
Magnitude of Static Friction

The magnitude of the static frictional force can have any value from zero up to a maximum value.

\[ f_s \leq f_{s}^{\text{MAX}} \]

\[ f_{s}^{\text{MAX}} = \mu_s F_N \]

\[ 0 < \mu_s < 1 \] is called the coefficient of static friction.

What is the unit? None

Once the object starts moving, there is NO MORE static friction!!

Kinetic friction applies during the move!!
Note that the magnitude of the frictional force **does not depend on the contact area of the surfaces.**

\[ f_s^{MAX} = \mu_s F_N \]
Kinetic Friction

Static friction opposes the *impending* relative motion between two objects.

Kinetic friction opposes the relative sliding motion motions that actually does occur. *The resistive force exerted on the object during its movement.*

\[ f_k = \mu_k F_N \]

\[ 0 < \mu_s < 1 \] is called the **coefficient of kinetic friction**.

What is the direction of frictional forces? **opposite to the movement**
# Coefficient of Friction

## Table 4.2 Approximate Values of the Coefficients of Friction for Various Surfaces*

<table>
<thead>
<tr>
<th>Materials</th>
<th>Coefficient of Static Friction, $\mu_s$</th>
<th>Coefficient of Kinetic Friction, $\mu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass on glass (dry)</td>
<td>0.94</td>
<td>0.4</td>
</tr>
<tr>
<td>Ice on ice (clean, 0 °C)</td>
<td>0.1</td>
<td>0.02</td>
</tr>
<tr>
<td>Rubber on dry concrete</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Rubber on wet concrete</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>Steel on ice</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>Steel on steel (dry hard steel)</td>
<td>0.78</td>
<td>0.42</td>
</tr>
<tr>
<td>Teflon on Teflon</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Wood on wood</td>
<td>0.35</td>
<td>0.3</td>
</tr>
</tbody>
</table>

*The last column gives the coefficients of kinetic friction, a concept that will be discussed shortly.

What are these?
Ex. 10. Sled Riding

A sled is traveling at 4.00 m/s along a horizontal stretch of snow. The coefficient of kinetic friction $\mu_k = 0.0500$. How far does the sled go before stopping?

The sled comes to a halt because the kinetic frictional force opposes its motion and causes the sled to slow down.

What are the forces in this motion?
Ex. 10 continued

What is the net force in y direction?  0N

What is the net force in x direction?  \( F_x = -f_k = ma \)

So the force equation becomes

\[ f_k = \mu_k F_N = \mu_k mg \]

Solve this for \( a \)

\[ a = -\frac{f_k}{m} = -\frac{\mu_k mg}{m} = -\mu_k g = -0.05(9.80 \text{ m/s}^2) = -0.49 \text{ m/s}^2 \]

Now that we know \( a \) and \( v_i \), pick the a kinematic equation to solve for distance

\[ 2ax = \left( v_f^2 - v_i^2 \right) \]

Solve this for \( x \)

\[ x = \frac{\left( v_f^2 - v_i^2 \right)}{2a} = \frac{\left( 0 - 4.00^2 \right)}{2 \cdot (-0.49)} = 16.3m \]
The Tension Force

Cables and ropes transmit forces through tension.
Tension Force continued

A massless rope will transmit tension undiminished from one end to the other.

If the rope passes around a massless, frictionless pulley, the tension will be transmitted to the other end of the rope undiminished.
Some Basic Information

When Newton’s laws are applied, *external forces* are only of interest!!

Why?

Because, as described in Newton’s first law, an object will keep its current motion unless non-zero net external force is applied.

Normal Force, $n$:

Reaction force to the net force on a surface due to the surface structure of an object. Its direction is always perpendicular to the surface.

Tension, $T$:

The reactionary force by a stringy object against an external force exerted on it.

Free-body diagram

A graphical tool which is a *diagram of external forces on an object* and is extremely useful analyzing forces and motion!! Drawn only on an object.
An object is in equilibrium when it has zero acceleration.

\[ \sum \vec{F} = m \vec{a} = 0 \]

\[ \sum F_x = ma_x = 0 \quad \sum F_y = ma_y = 0 \]

If an object is not moving at all, the object is in its static equilibrium.

Is an object is moving at a constant velocity in its equilibrium? Yes

Why? Because its acceleration is 0.
Strategy for Solving Problems

• Select an object(s) to which the equations of equilibrium are to be applied.
• Identify all the forces acting only on the selected object.
• Draw a free-body diagram for each object chosen above. Include only forces acting on the object, not forces the object exerts on its environment.
• Choose a set of x, y axes for each object and resolve all forces in the free-body diagram into components that point along these axes.
• Apply the equations and solve for the unknown quantities.

⇒ No matter which object we choose to draw the free body diagram on, the results should be the same, as long as they are in the same motion.
Ex. 11 Traction for the Foot

The weight of the 2.2 kg object creates a tension in the rope that passes around the pulleys. Therefore, tension forces $T_1$ and $T_2$ are applied to the pulley on the foot. The foot pulley is kept in equilibrium because the foot also applies a force $F$ to it. This force arises in reaction to the pulling effect of the forces $T_1$ and $T_2$. Ignoring the weight of the foot, find the magnitude of the force $F$.

\[
\sum F_y = +T_1 \sin 35^\circ - T_2 \sin 35^\circ = 0
\]
\[
\sum F_x = +T_1 \cos 35^\circ + T_2 \cos 35^\circ - F = 0
\]
Ex. 12 Replacing an Engine

An automobile engine has a weight (or gravitational force) \(W\), whose magnitude is \(W=3150\text{N}\). This engine is being positioned above an engine compartment, as shown in the figure. To position the engine, a worker is using a rope. Find the tension \(T_1\) in the support cabling and the tension \(T_2\) in the positioning rope.
First, analyze the forces in x and y

<table>
<thead>
<tr>
<th>Force</th>
<th>$x$ component</th>
<th>$y$ component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{T}_1$</td>
<td>$-T_1 \sin 10.0^\circ$</td>
<td>$+T_1 \cos 10.0^\circ$</td>
</tr>
<tr>
<td>$\vec{T}_2$</td>
<td>$+T_2 \sin 80.0^\circ$</td>
<td>$-T_2 \cos 80.0^\circ$</td>
</tr>
<tr>
<td>$\vec{W}$</td>
<td>0</td>
<td>$-W$</td>
</tr>
</tbody>
</table>

$W = 3150 \text{ N}$
Now compute each force component

$$\sum F_x = -T_1 \sin 10.0^\circ + T_2 \sin 80.0^\circ = 0$$

$$\sum F_y = +T_1 \cos 10.0^\circ - T_2 \cos 80.0^\circ - W = 0$$

The first equation gives

$$T_1 = \left( \frac{\sin 80.0^\circ}{\sin 10.0^\circ} \right) T_2$$

Substitution into the second gives

$$\left( \frac{\sin 80.0^\circ}{\sin 10.0^\circ} \right) T_2 \cos 10.0^\circ - T_2 \cos 80.0^\circ - W = 0$$
\[ T_2 = \frac{W}{\left( \frac{\sin 80.0^\circ}{\sin 10.0^\circ} \right) \cos 10.0^\circ - \cos 80.0^\circ} = \frac{3150N}{\left( \frac{\sin 80.0^\circ}{\sin 10.0^\circ} \right) \cos 10.0^\circ - \cos 80.0^\circ} = 582N \]

\[ T_2 = 582 \text{ N} \]

\[ T_1 = \left( \frac{\sin 80.0^\circ}{\sin 10.0^\circ} \right) T_2 \quad T_1 = 3.30 \times 10^3 \text{ N} \]
Is an accelerating object in its equilibrium?  NO

\[ \sum \vec{F} = m\vec{a} \neq 0 \]

\[ \sum F_x = ma_x \neq 0 \quad \sum F_y = ma_y \neq 0 \]
Ex. 14 Towing a Supertanker

A supertanker of mass $m = 1.50 \times 10^8$ kg is being towed by two tugboats. The tension in the towing cables apply the forces $T_1$ and $T_2$ at equal angles of $30^\circ$ with respect to the tanker’s axis. In addition, the tanker’s engines produce a forward drive force $D$, whose magnitude is $D = 75.0 \times 10^3$ N. Moreover, the water applies an opposing force $R$, whose magnitude is $R = 40.0 \times 10^3$ N. The tanker moves forward with an acceleration that points along the tanker’s axis and has a magnitude of $2.00 \times 10^{-3}$ m/s$^2$. Find the magnitude of $T_1$ and $T_2$.

The acceleration is along the $x$ axis so $a_y = 0$. 

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![Diagram of forces and tanker](image)
Figure out X and Y components

<table>
<thead>
<tr>
<th>Force</th>
<th>x component</th>
<th>y component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{T}_1$</td>
<td>$+T_1 \cos 30.0^\circ$</td>
<td>$+T_1 \sin 30.0^\circ$</td>
</tr>
<tr>
<td>$\vec{T}_2$</td>
<td>$+T_2 \cos 30.0^\circ$</td>
<td>$-T_2 \sin 30.0^\circ$</td>
</tr>
<tr>
<td>$\vec{D}$</td>
<td>$+D$</td>
<td>0</td>
</tr>
<tr>
<td>$\vec{R}$</td>
<td>$-R$</td>
<td>0</td>
</tr>
</tbody>
</table>
\[ \sum F_y = +T_1 \sin 30.0^\circ - T_2 \sin 30.0 = 0 \]

\[ T_1 \sin 30.0^\circ = T_2 \sin 30.0 \quad \Rightarrow \quad T_1 = T_2 = T \]

\[ \sum F_x = +T_1 \cos 30.0^\circ + T_2 \cos 30.0 + D - R = ma_x \]

Since \( T_1 = T_2 = T \)

\[ 2T \cos 30.0^\circ = ma_x + R - D \]

Solving for \( T \)

\[ T = \frac{ma_x + R - D}{2 \cos 30.0^\circ} = \frac{1.50 \times 10^0 \cdot 2.00 \times 10^{-3} + 40.0 \times 10^3 - 75.0 \times 10^3}{2 \cos 30.0^\circ} = 1.53 \times 10^5 \text{ N} \]
Example for Using Newton’s Laws

A traffic light weighing 125 N hangs from a cable tied to two other cables fastened to a support. The upper cables make angles of 37.0° and 53.0° with the horizontal. Find the tension in the three cables.

\[
\vec{F} = \vec{T}_1 + \vec{T}_2 + \vec{T}_3 = m\vec{a} = 0 \quad \text{Newton’s 2}^{\text{nd}} \text{law}
\]

**x-comp. of net force**

\[
F_x = \sum_{i=1}^{3} T_{ix} = 0 \quad -T_1 \cos(37°) + T_2 \cos(53°) = 0 \quad \therefore T_1 = \frac{\cos(53°)}{\cos(37°)} T_2 = 0.754 T_2
\]

**y-comp. of net force**

\[
F_y = \sum_{i=1}^{3} T_{iy} = 0 \quad T_1 \sin(37°) + T_2 \sin(53°) - mg = 0
\]

\[
T_2 \left[ \sin(53°) + 0.754 \times \sin(37°) \right] = 1.25 T_2 = 125 N
\]

\[
T_2 = 100 N; \quad T_1 = 0.754 T_2 = 75.4 N
\]
Example w/o Friction

A crate of mass $M$ is placed on a frictionless inclined plane of angle $\theta$.

a) Determine the acceleration of the crate after it is released.

\[ \vec{F} = \vec{F}_g + \vec{n} = ma \]
\[ F_x = Ma_x = F_{gx} = Mg \sin \theta \]
\[ a_x = g \sin \theta \]
\[ F_y = Ma_y = n - F_{gy} = n - mg \cos \theta = 0 \]

Supposed the crate was released at the top of the incline, and the length of the incline is $d$. How long does it take for the crate to reach the bottom and what is its speed at the bottom?

\[ d = v_{ix}t + \frac{1}{2}a_x t^2 = \frac{1}{2} g \sin \theta t^2 \]
\[ \therefore t = \sqrt{\frac{2d}{g \sin \theta}} \]

\[ v_{xf} = v_{ix} + a_x t = g \sin \theta \sqrt{\frac{2d}{g \sin \theta}} = \sqrt{2dg \sin \theta} \]

\[ \therefore v_{xf} = \sqrt{2dg \sin \theta} \]
Example w/ Friction

Suppose a block is placed on a rough surface inclined relative to the horizontal. The inclination angle is increased till the block starts to move. Show that by measuring this critical angle, $\theta_c$, one can determine coefficient of static friction, $\mu_s$.

**Net force**

$$\vec{F} = M \vec{a} = \vec{F}_g + \vec{n} + \vec{f}_s$$

- **x comp.**
  $$F_x = F_{gx} - f_s = Mg \sin \theta - f_s = 0 \quad f_s = \mu_s n = Mg \sin \theta_c$$

- **y comp.**
  $$F_y = Ma_y = n - F_{gy} = n - Mg \cos \theta_c = 0 \quad n = F_{gy} = Mg \cos \theta_c$$

$$\mu_s = \frac{Mg \sin \theta_c}{n} = \frac{Mg \sin \theta_c}{Mg \cos \theta_c} = \tan \theta_c$$