PHYS 1441 – Section 002
Lecture #14

Monday, Mar. 10, 2008
Dr. Jaehoon Yu

- Uniform Circular Motion
- Centripetal Acceleration and Force
- Banked and Unbanked Road
- Satellite Motion
- Work done by a constant force

Today's homework is homework #7, due 9pm, Monday, Mar. 24!!

Announcements

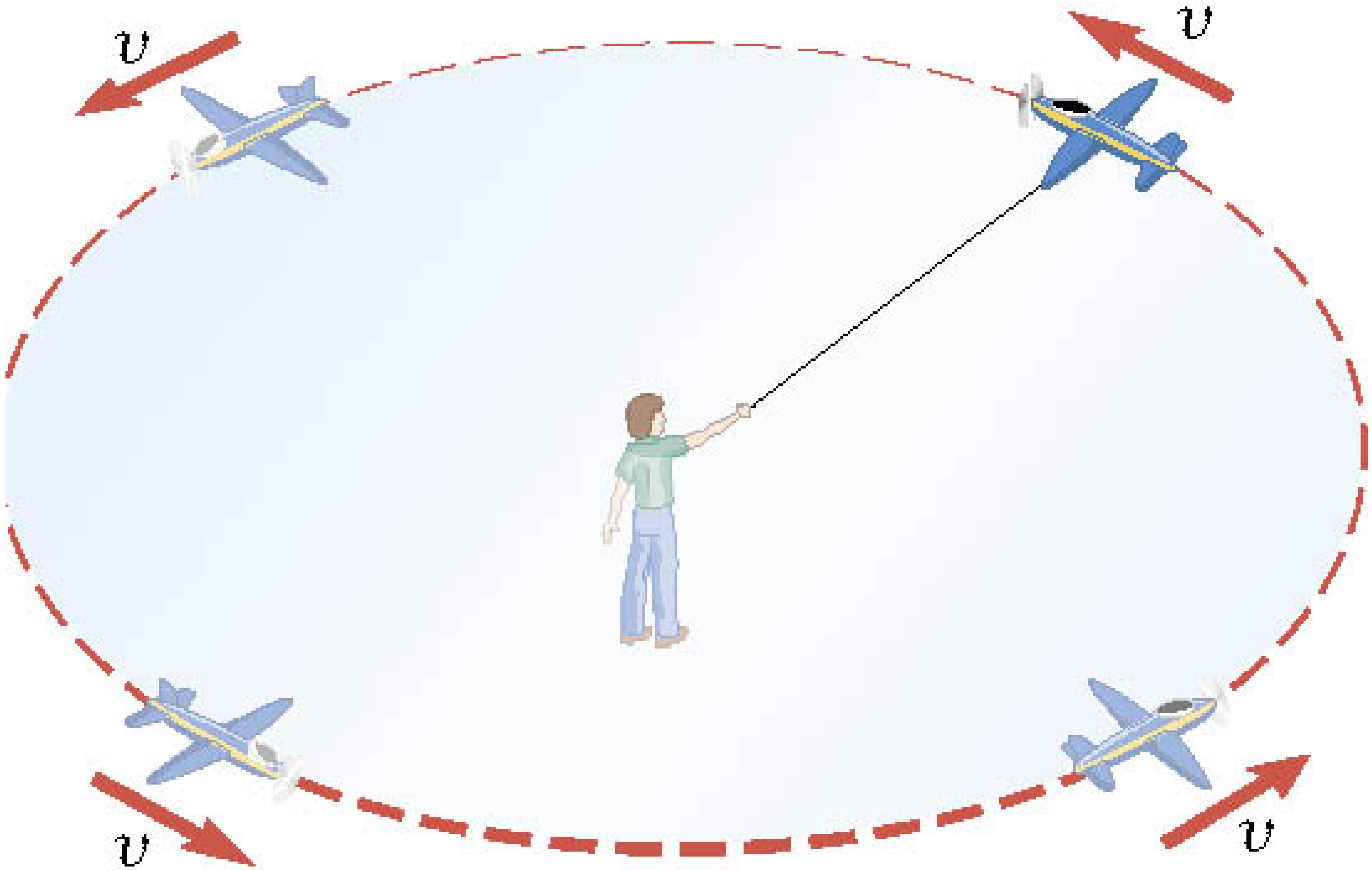
- Quiz Results
  - Class average: 4.7/10
    - Equivalent to 47/100
    - Previous quizzes: 48/100 and 44/100
  - Top score: 8/10
- Term exam #2
  - Wednesday, March 26, in class
  - Will cover CH4.1 – whatever we finish Monday, Mar. 24

Special Project Reminder

- Using the fact that $g=9.80m/s^2$ on the Earth's surface, find the average density of the Earth.
- 20 point extra credit
- Due: This Wednesday, Mar. 12
- You must show your OWN, detailed work to obtain any credit!!

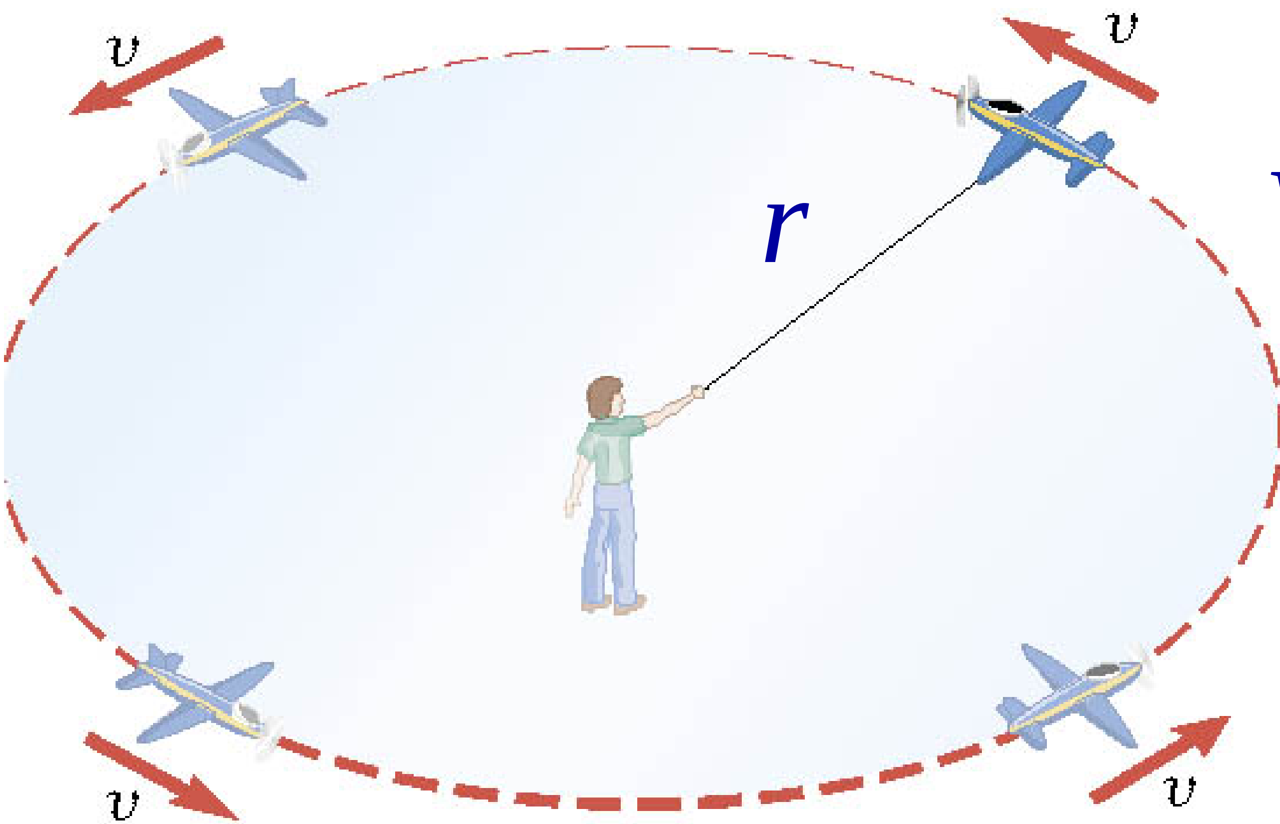
Definition of Uniform Circular Motion

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.



Speed of a uniform circular motion?

Let *T* be the period of this motion, the time it takes for the object to travel once around the circle whose radius is r.



$$v = \frac{\text{distance}}{\text{time}}$$

$$= \frac{2\pi r}{T}$$

Ex. 1: A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and is being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

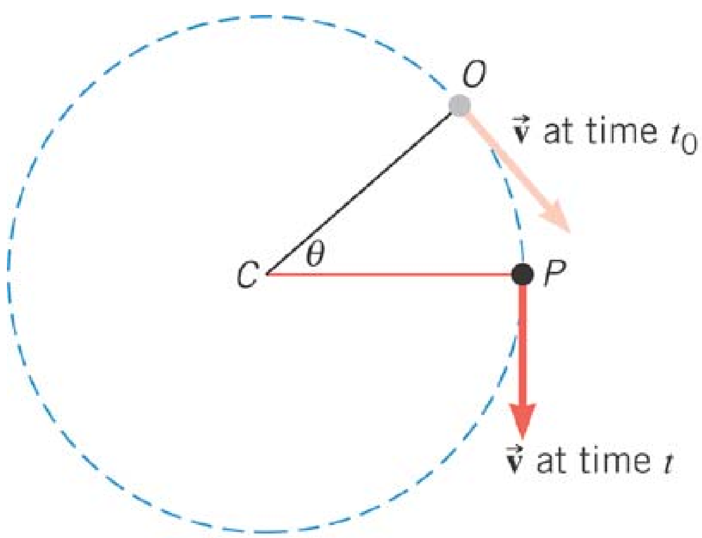
$$\frac{1}{830\,\text{revolutions/min}} = 1.2\times10^{-3}\ \text{min/revolution}$$

$$T = 1.2\times10^{-3}\ \text{min} = 0.072\ \text{s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29\ \text{m})}{0.072\ \text{s}} = 25\,\text{m/s}$$

Centripetal Acceleration

In uniform circular motion, the speed is constant, but the direction of the velocity vector is *not constant.*



(a)

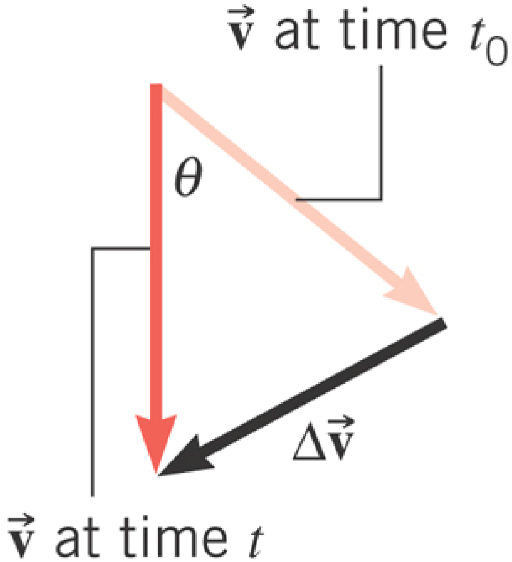
$$\alpha + \beta = 90^\circ$$

$$\alpha + \theta = 90^\circ$$

$$\beta - \theta = 0$$

$$\beta = \theta$$

Centripetal Acceleration



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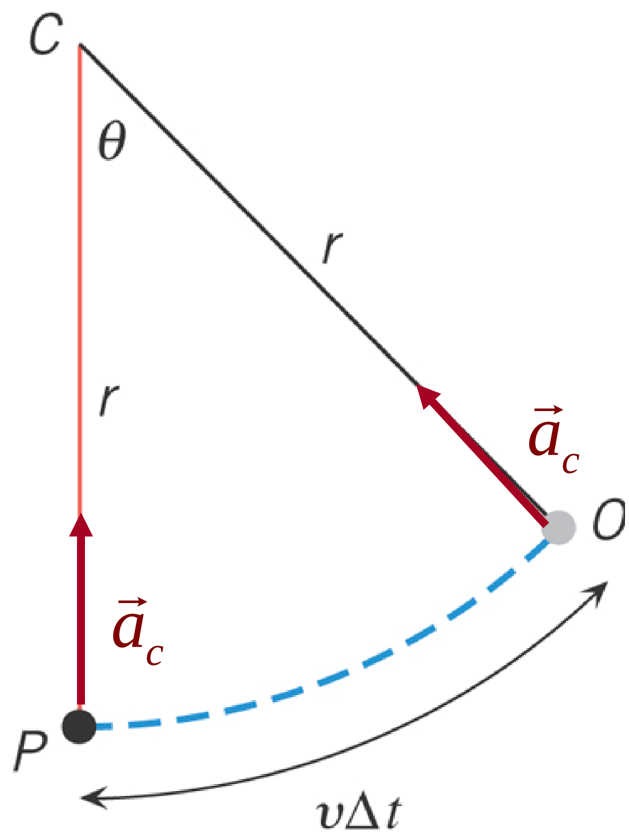
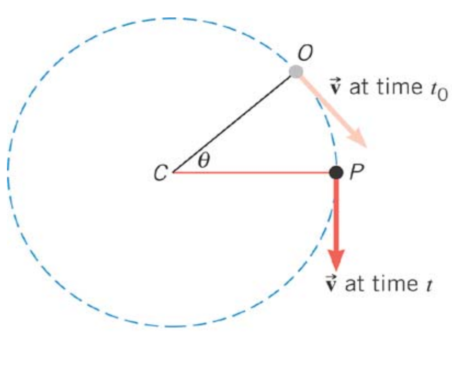




From the geometry

$$\frac{\Delta v}{\cancel{2}v}=\frac{v\Delta t}{\cancel{2}r}$$

$$\frac{\Delta v}{\Delta t}=\frac{v^2}{r}$$

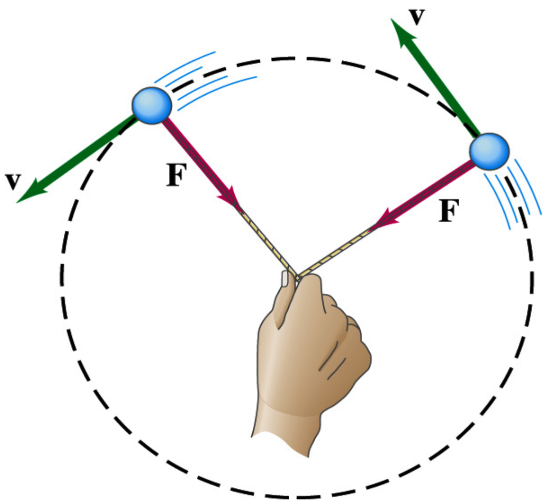
$$\boxed{a_c=\frac{v^2}{r}}$$

What is the direction of a_c?

Always toward the center of circle!

Centripetal Acceleration

Newton's Second Law & Uniform Circular Motion



The ***centripetal**** *acceleration is always perpendicular to the velocity vector,* ***v****, and points to the center of the axis (radial direction) in a uniform circular motion.*

$$\boxed{a_c = \frac{v^2}{r}}$$

Are there forces in this motion? If so, what do they do?

The force that causes the centripetal acceleration acts toward the center of the circular path and causes the change in the direction of the velocity vector. This force is called the centripetal force.

$$\sum F_c = ma_c = m\frac{v^2}{r}$$

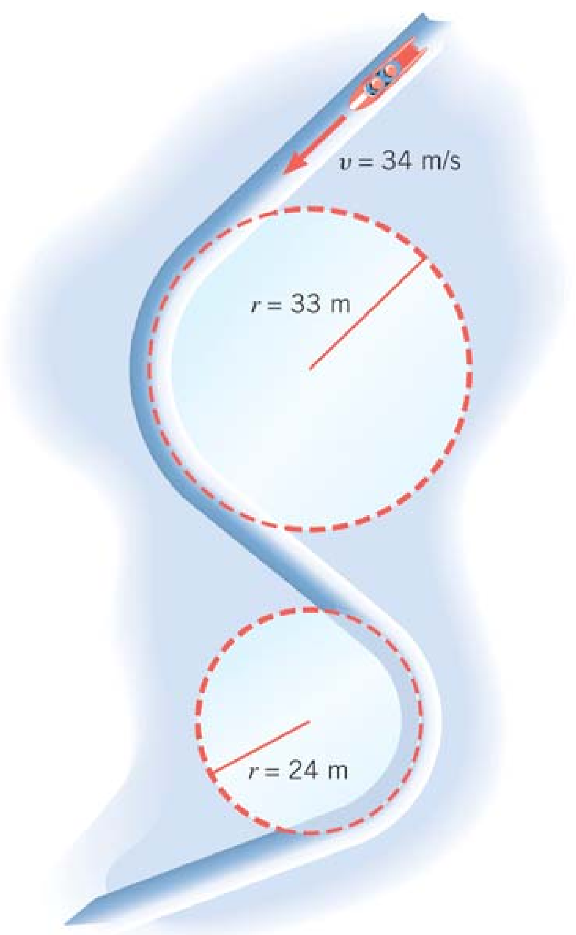
What do you think will happen to the ball if the string that holds the ball breaks?

The external force no longer exist. Therefore, based on Newton's 1st law, the ball will continue its motion without changing its velocity and will fly away following the tangential direction to the circle.

*Mirriam Webster: Proceeding or acting in a direction toward a center or axis

Ex.3 Effect of Radius on Centripetal Acceleration

The bobsled track at the 1994 Olympics in Lillehammer, Norway, contain turns with radii of 33m and 23m. Find the centripetal acceleration at each turn for a speed of 34m/s, a speed that was achieved in the two –man event. Express answers as multiples of $g=9.8m/s^2$.



Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

R=33m

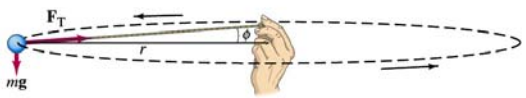
$$a_{r=33m} = \frac{(34)^2}{33} = 35\,m/s^2 = 3.6g$$

R=24m

$$a_{r=24m} = \frac{(34)^2}{24} = 48\,m/s^2 = 4.9g$$

Example of Uniform Circular Motion

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?



Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

When does the string break?

$$\sum F_r = ma_r = m\frac{v^2}{r} > T$$

when the required centripetal force is greater than the sustainable tension.

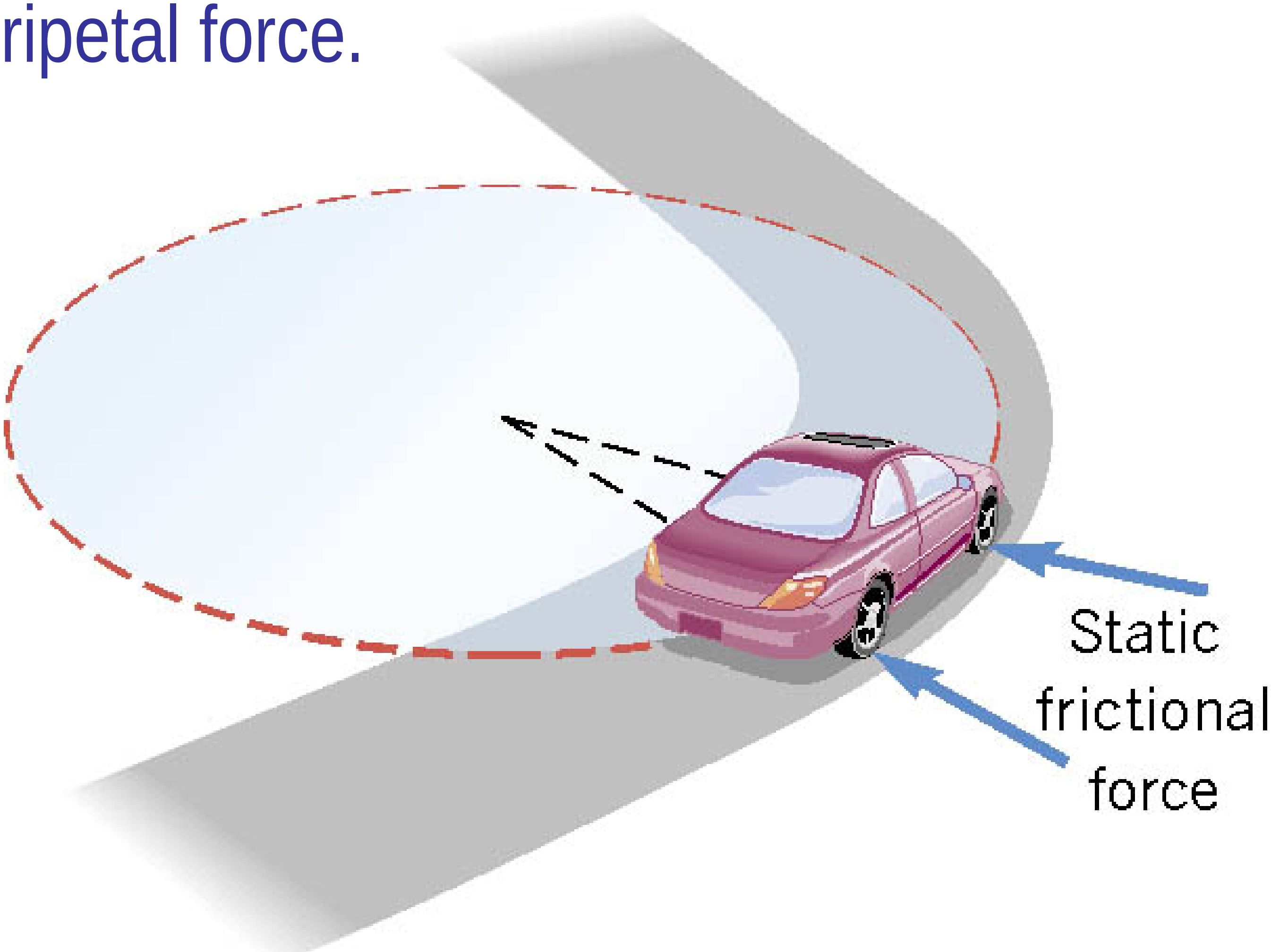
$$m\frac{v^2}{r} = T \qquad v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2\,(m/s)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33\,(N)$$

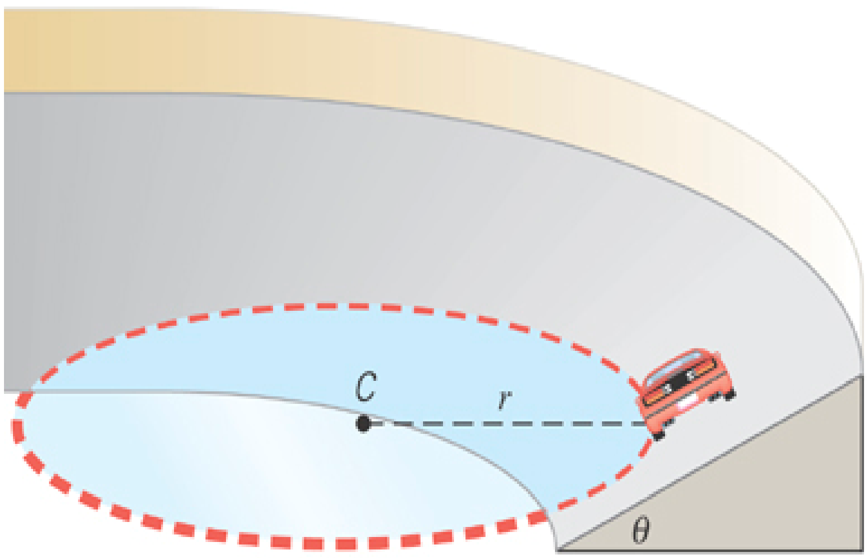
Unbanked Curve and Centripetal Force

On an unbanked curve, the static frictional force provides the centripetal force.



Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight.



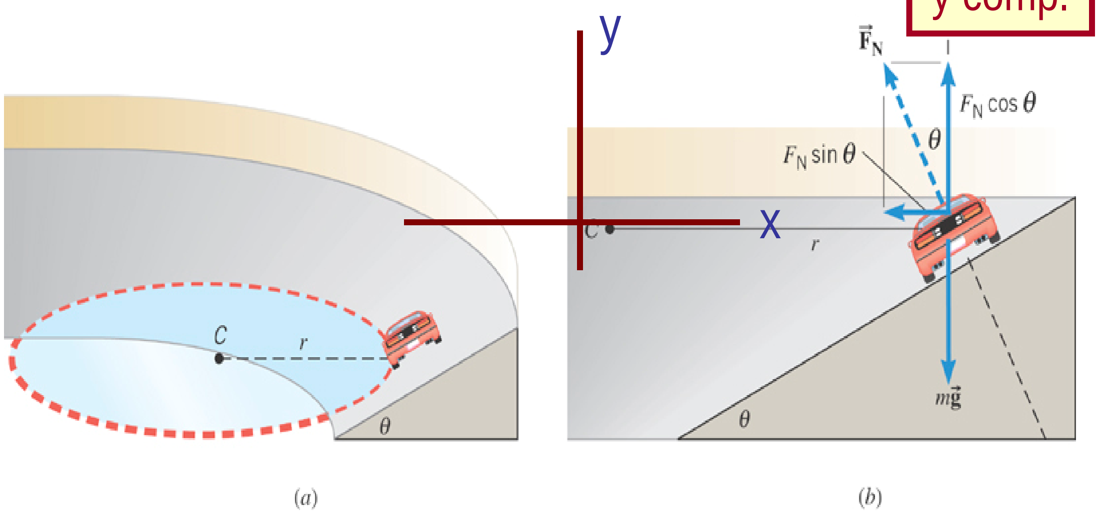
(a)

Ex. 8 The Daytona 500

The Daytona 500 is the major event of the NASCAR season. It is held at the Daytona International Speedway in Daytona, Florida. The turns in this oval track have a maximum radius (at the top) of r=-316m and are banked steeply, with $\theta=31^{\circ}$. Suppose these maximum radius turns were frictionless. At what speed would the cars have to travel around them?

x comp. $\sum F_x = F_N \sin\theta - m\frac{v^2}{r} = 0$

y comp. $\sum F_y = F_N \cos\theta - mg = 0$



$$\tan\theta = \frac{\cancel{m}v^2}{\cancel{m}gr} = \frac{v^2}{gr}$$

$$v^2 = gr\tan\theta$$

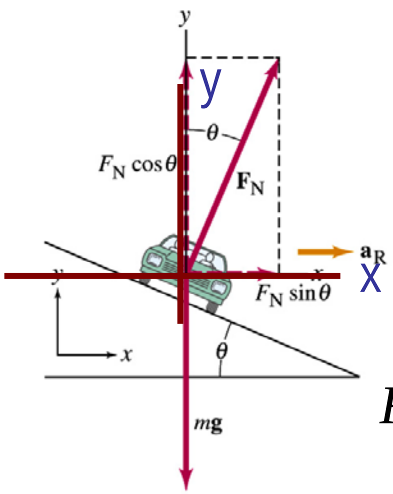
$$v = \sqrt{gr\tan\theta} =$$

$$\sqrt{9.8\cdot 316\tan\left(31^{\circ}\right)} = 43\,m/s =$$

$$96\,mi/hr$$

Example of Banked Highway

(a) For a car traveling with speed v around a curve of radius r, determine the formula for the angle at which the road should be banked so that no friction is required to keep the car from skidding.



x comp. $\sum F_x = F_N \sin\theta - ma_r = F_N \sin\theta - \frac{mv^2}{r} = 0$

$$F_N \sin\theta = \frac{mv^2}{r}$$

y comp. $\sum F_y = F_N \cos\theta - mg = 0$ $\quad F_N \cos\theta = mg$

$$F_N = \frac{mg}{\cos\theta} \Rightarrow F_N \sin\theta = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = \frac{mv^2}{r}$$

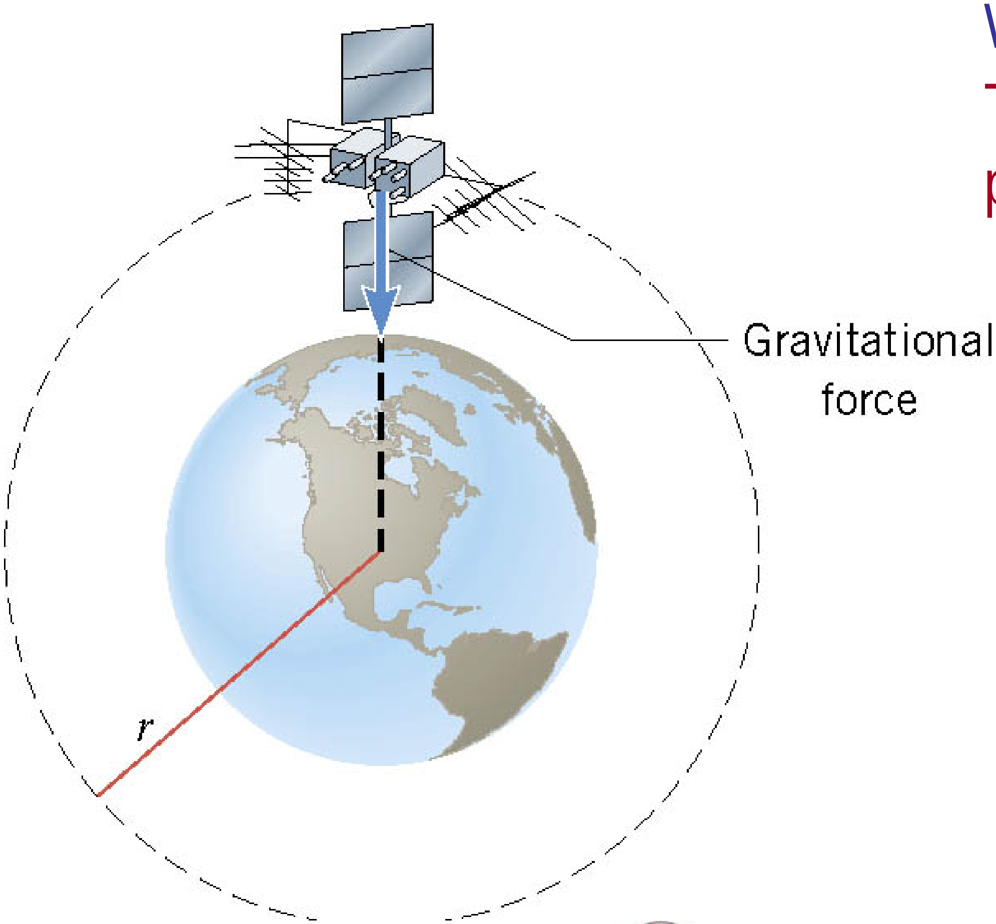
$$\boxed{\tan\theta = \frac{v^2}{gr}}$$

(b) What is this angle for an expressway off-ramp curve of radius 50m at a design speed of 50km/h?

$$v = 50km/hr = 14m/s \qquad \tan\theta = \frac{(14)^2}{50\times 9.8} = 0.4 \qquad \theta = \tan^{-1}(0.4) = 22^{\circ}$$

Satellite in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.



What is the centripetal force?

The gravitational force of the earth pulling the satellite!

$$F_c = G\frac{mM_E}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM_E}{r}$$

$$\boxed{v = \sqrt{\frac{GM_E}{r}}}$$

Ex. 9 Orbital Speed of the Hubble Space Telescope

Determine the speed of the Hubble Space Telescope orbiting at a height of 598 km above the earth's surface.

$$v = \sqrt{\frac{GM_E}{r}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{6.38\times10^{6}\,\mathrm{m}+598\times10^{3}\,\mathrm{m}}}$$

$$= 7.56\times10^{3}\,\mathrm{m/s} \quad \left(16900\,\mathrm{mi/h}\right)$$

Period of a Satellite in an Orbit

Speed of a satellite $$v = \sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{T}$$

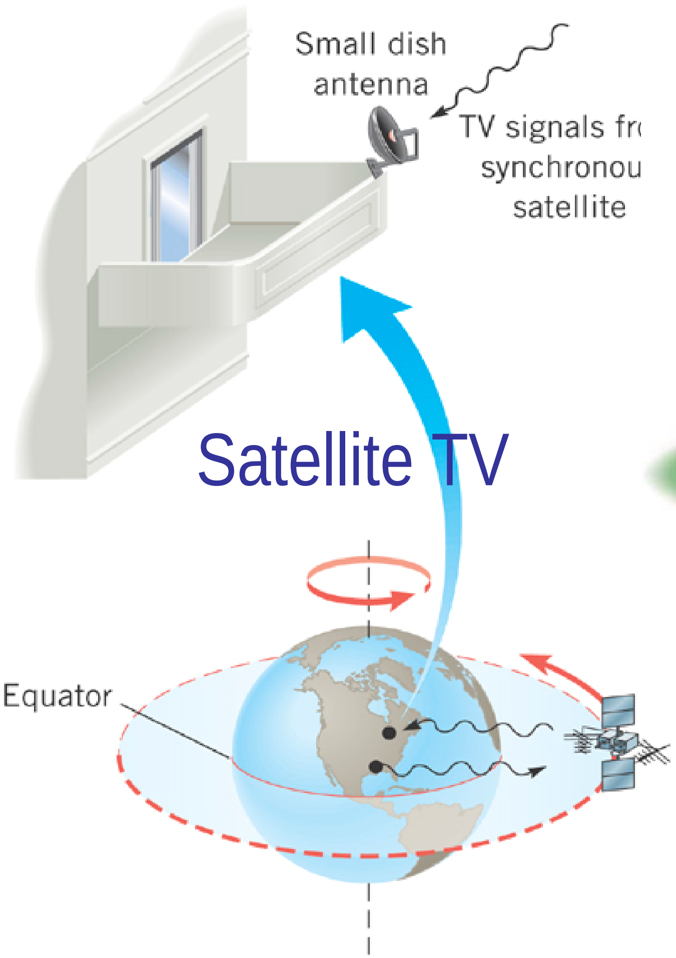
$$\frac{GM_E}{r} = \left(\frac{2\pi r}{T}\right)^2$$ Square either side and solve for T2 $$T^2 = \frac{(2\pi)^2 r^3}{GM_E}$$

Period of a satellite $$T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$

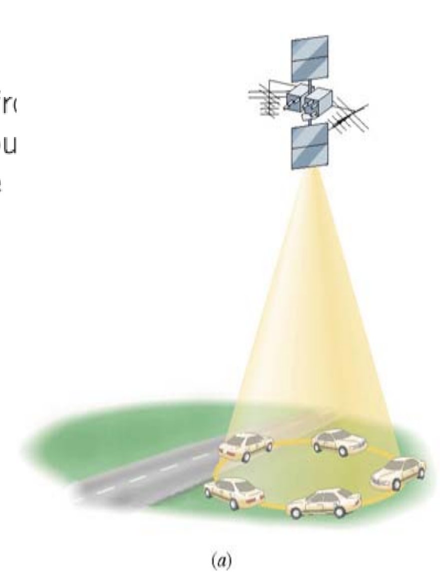
This is applicable to any satellite or even for planets and moons.

Synchronous Satellites

Global Positioning System (GPS)



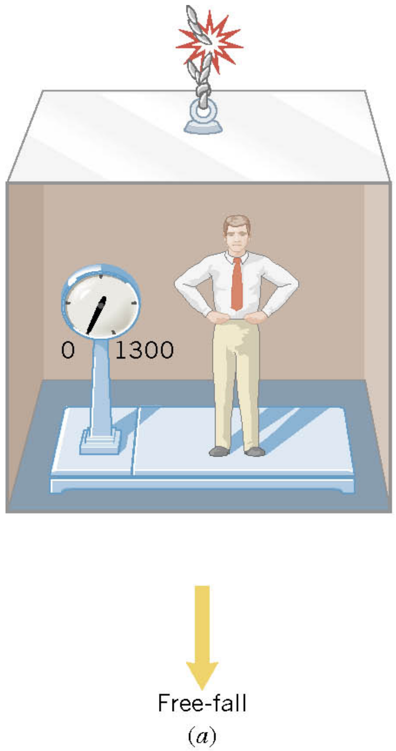
Satellite TV



What period should these satellites have?

The same as the earth!! 24 hours

Ex.12 Apparent Weightlessness and Free Fall



0 0

In each case, what is the weight recorded by the scale?

Ex.13 Artificial Gravity

At what speed must the surface of the space station move so that the astronaut experiences a push on his feet equal to his weight on earth? The radius is 1700 m.

$$F_c = m\frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

$$= \sqrt{(1700\text{ m})(9.80\,\text{m/s}^2)}$$