PHYS 1441 – Section 002
Lecture #18

Wednesday, Apr. 2, 2008

Dr. Jaehoon Yu

- Linear Momentum
- Linear Momentum and Impulse
- Mid term grade discussions

Announcements

- Quiz next Wednesday, Apr. 9
  - At the beginning of the class
  - Covers 6.7 – what we cover next Monday
- Mid-term grade discussion today
  - Please do not miss
  - If you need to leave quickly after the class, please make sure that you come and meet me early
- Colloquium today
  - Physics faculty research expo

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at a velocity of ***v*** *is defined as*

$$\vec{p} \equiv m\vec{v}$$

What can you tell from this definition about momentum?

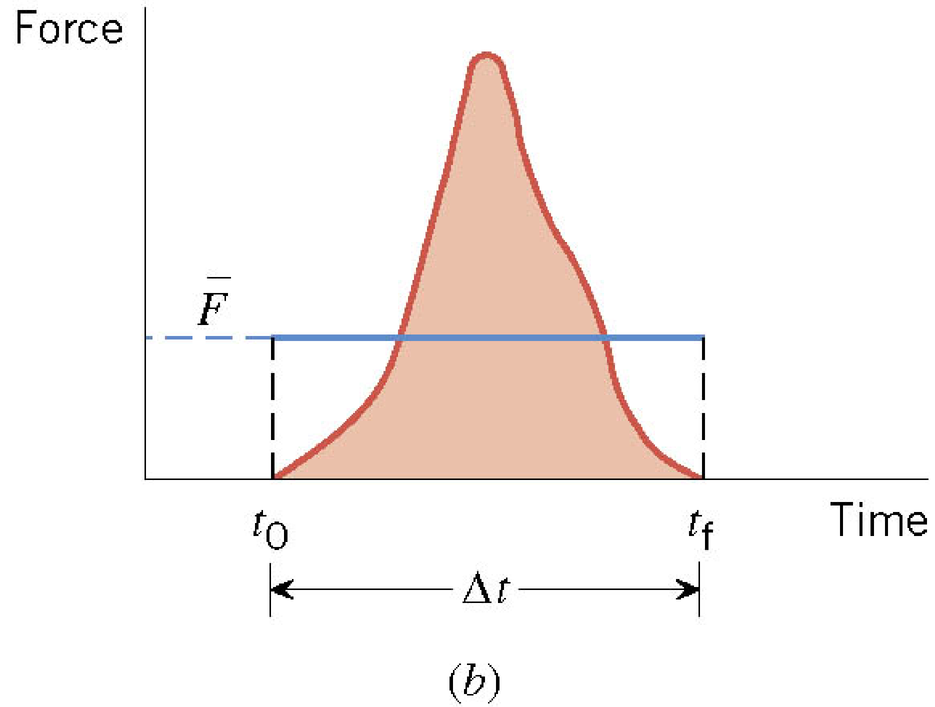
1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m\left(\vec{v} - \vec{v}_0\right)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$

Impulse



There are many situations when the force on an object is not constant.

Impulse and Linear Momentum

Net force causes change of momentum → Newton's second law

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \Rightarrow \Delta\vec{p} = \vec{F}\Delta t$$

The quantity impulse is defined as the change of momentum

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_0$$

So what do you think an impulse is?

Effect of the force F acting on an object over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the object caused by that force. Impulse is the degree of which an external force changes an object's momentum.

The above statement is called the impulse-momentum theorem and is equivalent to Newton's second law.

What are the dimension and unit of Impulse? What is the direction of an impulse vector?

Defining a time-averaged force

$$\bar{\vec{F}} \equiv \frac{1}{\Delta t}\sum_i \vec{F}_i \Delta t$$

Impulse can be rewritten

$$\vec{J} \equiv \bar{\vec{F}}\Delta t$$

If force is constant

$$\vec{J} \equiv \vec{F}\Delta t$$

Impulse is a vector quantity!!

Ball Hit by a Bat

$$\bar{\vec{a}} = \frac{\vec{v}_f - \vec{v}_o}{\Delta t}$$

$$\sum \bar{\vec{F}} = m\bar{\vec{a}}$$

$$\sum \bar{\vec{F}} = \frac{m\vec{v}_f - m\vec{v}_o}{\Delta t}$$

Multiply either side by Δt

$$\left(\sum \bar{\vec{F}}\right)\Delta t = m\vec{v}_f - m\vec{v}_o \quad = \vec{J}$$

Ex. 1 A Well-Hit Ball

A baseball (m=0.14kg) has an initial velocity of v_0=-38m/s as it approaches a bat. We have chosen the direction of approach as the negative direction. The bat applies an average force F that is much larger than the weight of the ball, and the ball departs from the bat with a final velocity of v_f=+58m/s. (a) determine the impulse applied to the ball by the bat. (b) Assuming that the time of contact is $\Delta t=1.6\times10^{-3}$s, find the average force exerted on the ball by the bat.

What are the forces involved in this motion? The force by the bat and the force by the gravity. Since the force by the bat is much greater than the weight, we ignore the ball's weight.

(a) Using the impulse-momentum theorem

$$\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_0$$

$$= 0.14\times58 - 0.14\times(-38) = +13.4 kg\cdot m/s$$

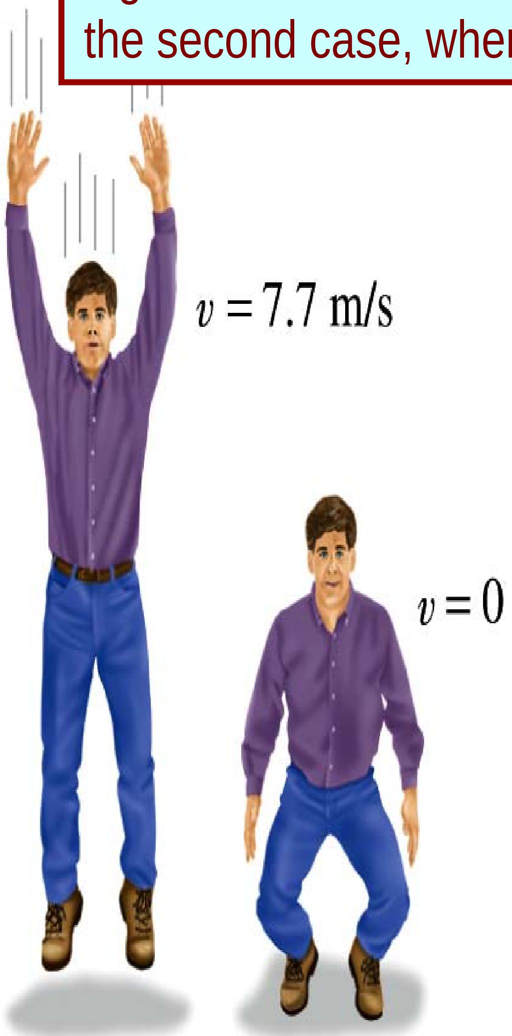
(b)Since the impulse is known and the time during which the contact occurs are know, we can compute the average force exerted on the ball during the contact

$$\vec{J} = \bar{\vec{F}}\Delta t \Rightarrow \bar{\vec{F}} = \frac{\vec{J}}{\Delta t} = \frac{+13.4}{1.6\times10^{-3}} = +8400N$$

How large is this force? $\left|\vec{W}\right| = mg = 0.14\cdot9.8 = 1.37N \Rightarrow \left|\bar{\vec{F}}\right| = \frac{8400}{1.37}\left|\vec{W}\right| = 6131\left|\vec{W}\right|$

Example for Impulse

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2 \cdot 9.8 \cdot 3} = 7.7 m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$\vec{I} = \vec{\overline{F}}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - m\vec{v} =$$

$$= -70kg \cdot 7.7m/s\,\vec{j} = -540\,\vec{j}\,N \cdot s$$

Example cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $$\bar{v} = \frac{0 + v_i}{2} = \frac{7.7}{2} = 3.8m/s$$

The time period the collision lasts is $$\Delta t = \frac{d}{\bar{v}} = \frac{0.01m}{3.8m/s} = 2.6 \times 10^{-3}s$$

Since the magnitude of impulse is $$\left|\vec{I}\right| = \left|\bar{\vec{F}}\Delta t\right| = 540N \cdot s$$

The average force on the feet during this landing is $$\bar{F} = \frac{I}{\Delta t} = \frac{540}{2.6 \times 10^{-3}} = 2.1 \times 10^{5} N$$

How large is this average force? $$Weight = 70kg \cdot 9.8m/s^2 = 6.9 \times 10^2 N$$

$$\bar{F} = 2.1 \times 10^5 N = 304 \times 6.9 \times 10^2 N = 304 \times Weight$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

For bent legged landing: $$\Delta t = \frac{d}{\bar{v}} = \frac{0.50m}{3.8m/s} = 0.13s$$

$$\bar{F} = \frac{540}{0.13} = 4.1 \times 10^3 N = 5.9 Weight$$