PHYS 1441 – Section 002
Lecture #19

Monday, Apr. 7, 2008
Dr. Jaehoon Yu

- Linear Momentum Conservation
- Collisions
- Center of Mass
- Fundamentals of Rotational Motion

Today's homework is HW #10, due 9pm, Monday, Apr. 12!!

Announcements

- Quiz Wednesday, Apr. 9
  - At the beginning of the class
  - Covers 6.7 – what we cover today
- 3rd term exam
  - Monday, Apr. 21, in class
  - Covers: Ch. 6.7 – what we complete next Wednesday, Apr. 16
  - This is the final term exam in the semester
  - The worst of the three term exams will be dropped from the final grading

Extra-Credit Special Project

- Derive the formula for the final velocity of two objects which underwent an elastic collision as a function of known quantities m_1, m_2, v_{01} and v_{02} in page 13 of this lecture note.
  - 20 points extra credit
- Describe in detail what happens to the final velocities if m1=m2.
  - 5 point extra credit
- Due: Start of the class this Wednesday, Apr. 9

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at a velocity of v is defined as

$$\vec{p} \equiv m\vec{v}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m(\vec{v} - \vec{v}_0)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum \vec{F}$$

Linear Momentum and Forces

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

What can we learn from this force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time.
- If a particle is isolated, the particle experiences no net force. Therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

The relationship can be used to study the case where the mass changes as a function of time.

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta\left(m\vec{v}\right)}{\Delta t} = \frac{\Delta m}{\Delta t}\vec{v} + m\frac{\Delta \vec{v}}{\Delta t}$$

Can you think of a few cases like this?

Motion of a meteorite

Motion of a rocket

Conservation of Linear Momentum in a Two Particle System

Consider an isolated system with two particles that do not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

*If particle #1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces, and the net force in the entire SYSTEM is **still 0**.*

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum $\mathbf{p_1}$ *and #2 has* $\mathbf{p_2}$ *at some point of time.*

Using momentum-force relationship

$$\vec{F}_{21} = \frac{\Delta \vec{p}_1}{\Delta t} \quad \text{and} \quad \vec{F}_{12} = \frac{\Delta \vec{p}_2}{\Delta t}$$

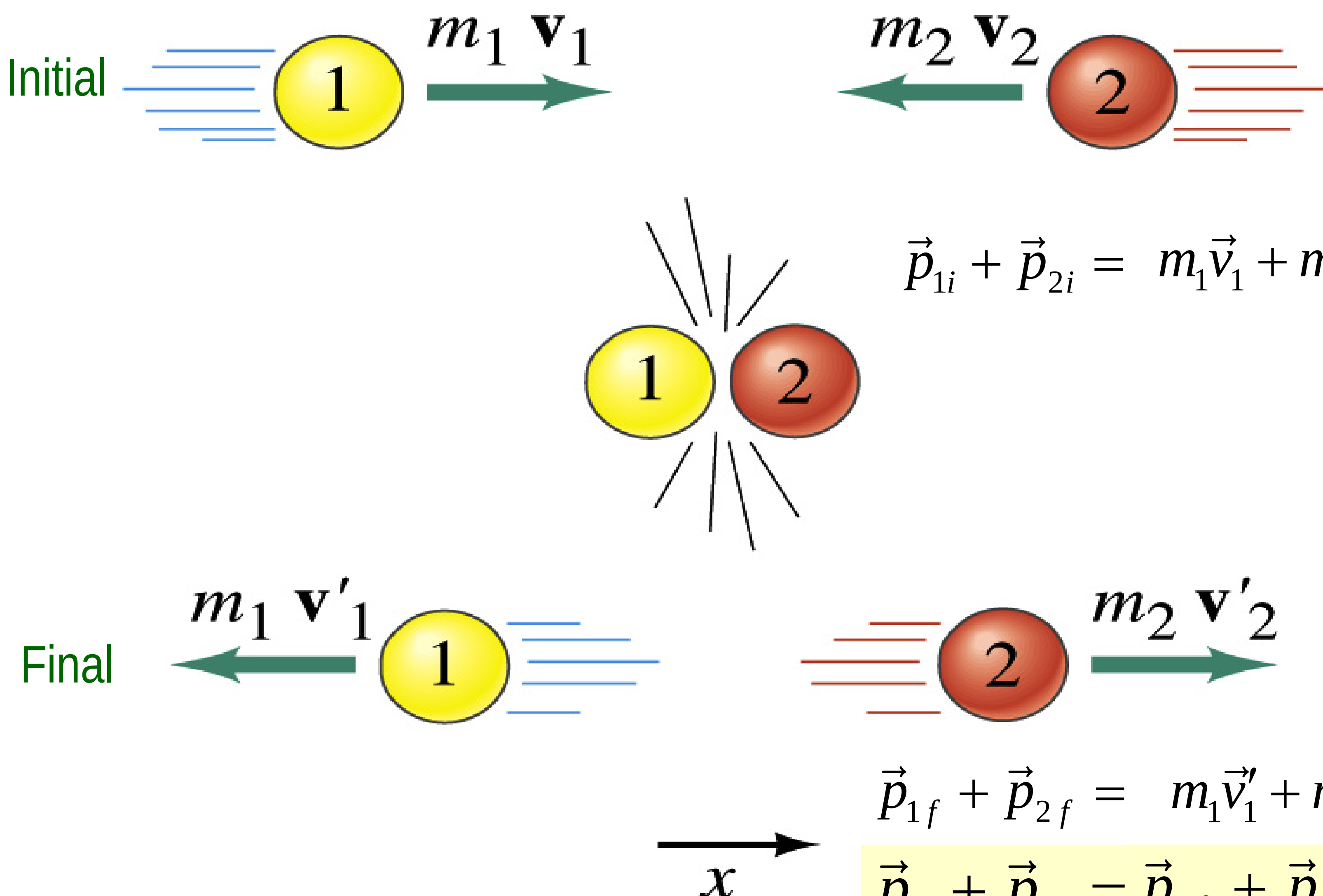
And since net force of this system is 0

$$\sum \vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{\Delta \vec{p}_2}{\Delta t} + \frac{\Delta \vec{p}_1}{\Delta t} = \frac{\Delta}{\Delta t}\left(\vec{p}_2 + \vec{p}_1\right) = 0$$

Therefore $\vec{p}_2 + \vec{p}_1 = const$

The total linear momentum of the system is conserved!!!

Linear Momentum Conservation



$$\vec{p}_{1i} + \vec{p}_{2i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{p}_{1f} + \vec{p}_{2f} = m_1\vec{v}'_1 + m_2\vec{v}'_2$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

More on Conservation of Linear Momentum in a Two Body System

From the previous slide we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interactions

Mathematically this statement can be written as

$$\vec{p}_{2i} + \vec{p}_{1i} = \vec{p}_{2f} + \vec{p}_{1f}$$

$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \quad \sum_{system} P_{yi} = \sum_{system} P_{yf} \quad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

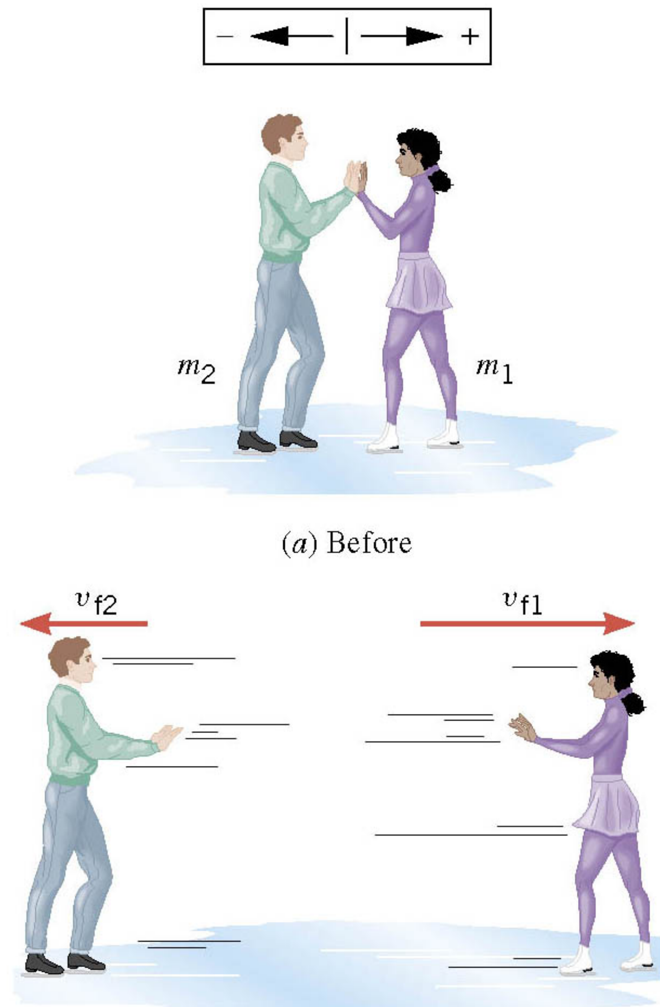
This can be generalized into conservation of linear momentum in many particle systems.

Whenever two or more particles in an __isolated system__ interact, the total momentum of the system remains constant.

Ex. 6 Ice Skaters

Starting from rest, two skaters push off against each other on ice where friction is negligible. One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.

No net external force ➔ momentum conserved



(a) Before

(b) After

$$\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}$$

$$m_1 v_{f1} + m_2 v_{f2} = 0$$

Solve for V_{f2} ➔

$$v_{f2} = -\frac{m_1 v_{f1}}{m_2}$$

$$v_{f2} = -\frac{(54\text{ kg})(+2.5\text{ m/s})}{88\text{ kg}} = -1.5\text{ m/s}$$

How do we apply momentum conservation?

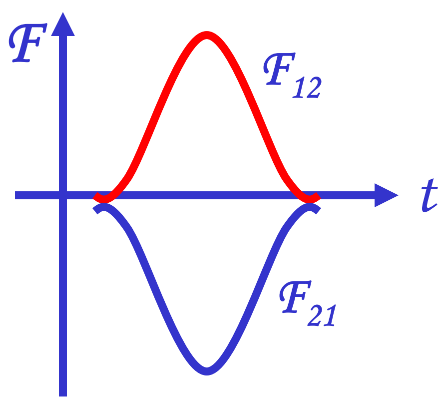
1. Decide which objects are included in the system.
2. Relative to the system, identify the internal and external forces.
3. Verify that the system is isolated.
4. Set the final momentum of the system equal to its initial momentum. Remember that momentum is a vector.

Collisions

Generalized collisions must cover not only the physical contact but also the collisions without physical contact such as that of electromagnetic ones in a microscopic scale.

Consider a case of a collision between a proton on a helium ion.

The collisions of these ions never involve physical contact because the electromagnetic repulsive force between these two become great as they get closer causing a collision.



Assuming no external forces, the force exerted on particle 1 by particle 2, $\mathcal{F}_{21}$, changes the momentum of particle 1 by

$$\Delta\vec{p}_1 = \vec{F}_{21}\Delta t$$

Likewise for particle 2 by particle 1

$$\Delta\vec{p}_2 = \vec{F}_{12}\Delta t$$

Using Newton's 3rd law we obtain

$$\Delta\vec{p}_2 = \vec{F}_{12}\Delta t = -\vec{F}_{21}\Delta t = -\Delta\vec{p}_1$$

So the momentum change of the system in the collision is 0, and the momentum is conserved

$$\Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 = \text{constant}$$

Elastic and Inelastic Collisions

Momentum is conserved in any collisions as long as external forces are negligible.

*Collisions are classified as elastic or inelastic based on whether the **kinetic energy is conserved, meaning whether it is the same** before and after the collision.*

Elastic Collision	*A collision in which the total kinetic energy and momentum are the same before and after the collision.*
Inelastic Collision	*A collision in which the total kinetic energy is not the same before and after the collision, but momentum is.*

Two types of inelastic collisions:Perfectly inelastic and inelastic

Perfectly Inelastic: *Two objects stick together after the collision, moving together at a certain velocity.*

Inelastic: *Colliding objects do not stick together after the collision but some kinetic energy is lost.*

Note: Momentum is constant in all collisions but kinetic energy is only in elastic collisions.

Elastic and Perfectly Inelastic Collisions

In perfectly Inelastic collisions, the objects stick together after the collision, moving together. Momentum is conserved in this collision, so the final velocity of the stuck system is

$$m_1\vec{v}_{1i}+m_2\vec{v}_{2i}=(m_1+m_2)\vec{v}_f$$

$$\vec{v}_f=\frac{m_1\vec{v}_{1i}+m_2\vec{v}_{2i}}{(m_1+m_2)}$$

How about elastic collisions?

$$m_1\vec{v}_{1i}+m_2\vec{v}_{2i}=m_1\vec{v}_{1f}+m_2\vec{v}_{2f}$$

In elastic collisions, both the momentum and the kinetic energy are conserved. Therefore, the final speeds in an elastic collision can be obtained in terms of initial speeds as

$$\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2=\frac{1}{2}m_1v_{1f}^2+\frac{1}{2}m_2v_{2f}^2$$

$$m_1\left(v_{1i}^2-v_{1f}^2\right)=m_2\left(v_{2i}^2-v_{2f}^2\right)$$

$$m_1\left(v_{1i}-v_{1f}\right)\left(v_{1i}+v_{1f}\right)=m_2\left(v_{2i}-v_{2f}\right)\left(v_{2i}+v_{2f}\right)$$

From momentum conservation above

$$m_1\left(v_{1i}-v_{1f}\right)=m_2\left(v_{2i}-v_{2f}\right)$$

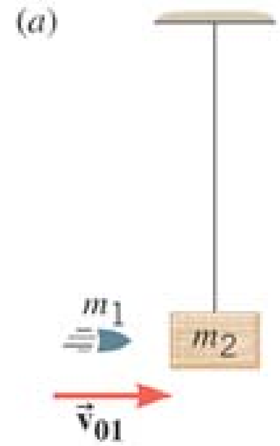
$$v_{1f}=\left(\frac{m_1-m_2}{m_1+m_2}\right)v_{1i}+\left(\frac{2m_2}{m_1+m_2}\right)v_{2i}$$

$$v_{2f}=\left(\frac{2m_1}{m_1+m_2}\right)v_{1i}+\left(\frac{m_1-m_2}{m_1+m_2}\right)v_{2i}$$

What happens when the two masses are the same?

Ex. 8 A Ballistic Pendulum

The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position. Find the initial speed of the bullet.



What kind of collision? Perfectly inelastic collision

No net external force ➔ momentum conserved

$$m_1 v_{f1} + m_2 v_{f2} = m_1 v_{o1} + m_2 v_{o2}$$

$$\left(m_1 + m_2\right) v_f = m_1 v_{o1}$$

Solve for V_{01}

$$v_{o1} = \frac{\left(m_1 + m_2\right) v_f}{m_1}$$

What do we not know? The final speed!!

How can we get it? Using the mechanical energy conservation!

Ex. 8 A Ballistic Pendulum, cnt'd

Now using the mechanical energy conservation

$$\tfrac{1}{2}mv^2 = mgh$$

$$\cancel{(m_1+m_2)}gh_f = \tfrac{1}{2}\cancel{(m_1+m_2)}v_f^2$$

$$gh_f = \tfrac{1}{2}v_f^2$$

Solve for V_f

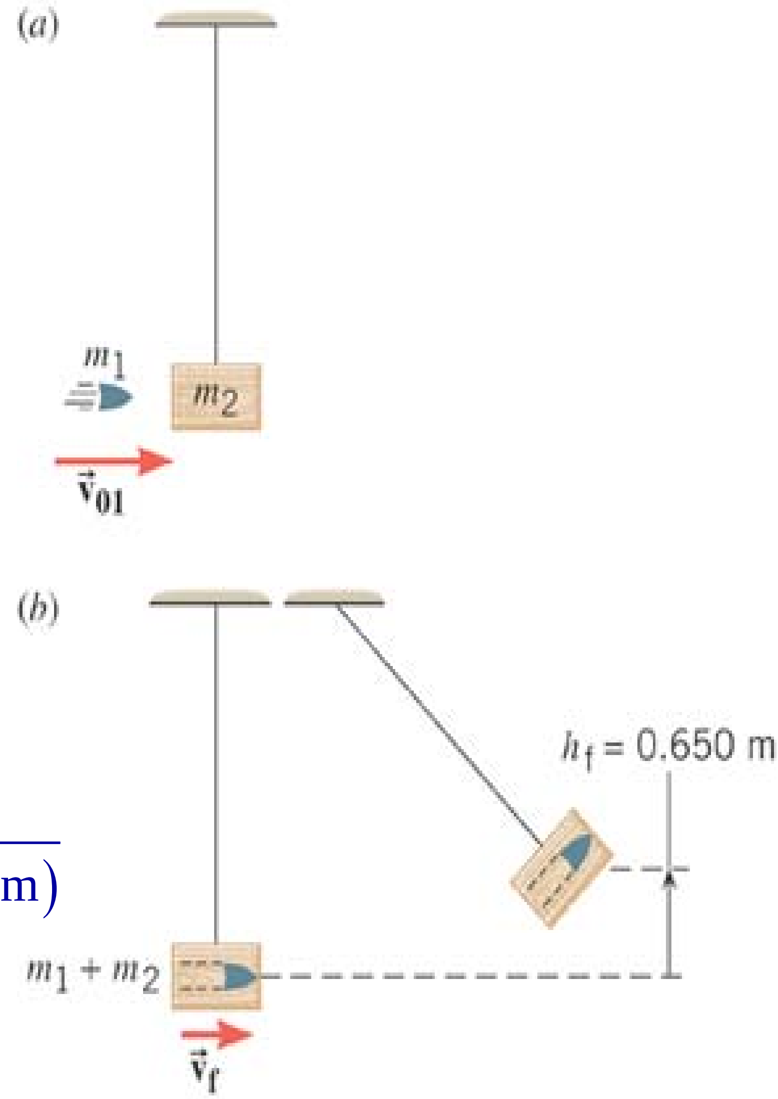
$$v_f = \sqrt{2gh_f} = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.650\ \mathrm{m}\right)}$$

Using the solution obtained previously, we obtain

$$v_{o1} = \frac{(m_1+m_2)v_f}{m_1} = \frac{(m_1+m_2)\sqrt{2gh_f}}{m_1}$$

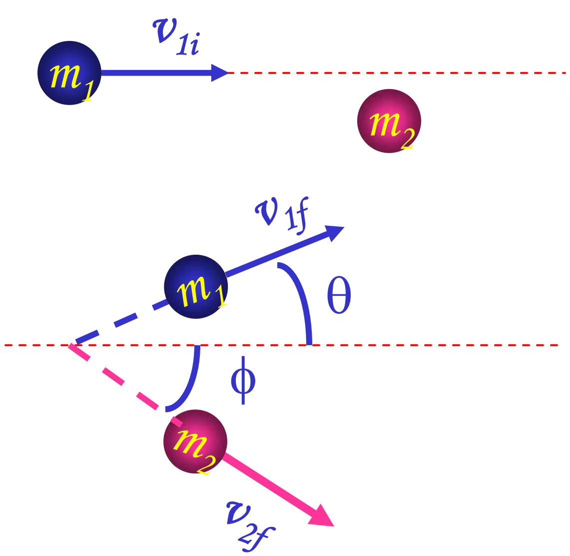
$$= \left(\frac{0.0100\ \mathrm{kg} + 2.50\ \mathrm{kg}}{0.0100\ \mathrm{kg}}\right)\sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.650\ \mathrm{m}\right)}$$

$$= +896\,\mathrm{m/s}$$



Two dimensional Collisions

In two dimension, one needs to use components of momentum and apply momentum conservation to solve physical problems.



$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

x-comp. $$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

y-comp. $$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Consider a system of two particle collisions and scatters in two dimension as shown in the picture. (This is the case at fixed target accelerator experiments.) The momentum conservation tells us:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1i}$$

$$m_1 v_{1ix} = m_1 v_{1fx} + m_2 v_{2fx} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$$

$$m_1 v_{1iy} = 0 = m_1 v_{1fy} + m_2 v_{2fy} = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$$

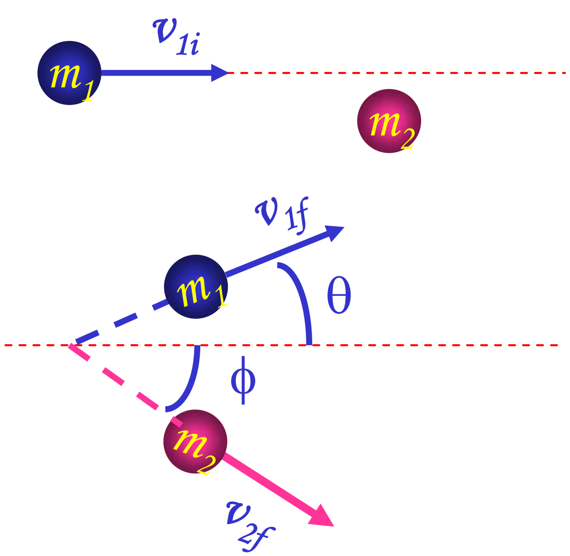
And for the elastic collisions, the kinetic energy is conserved:

$$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

What do you think we can learn from these relationships?

Example for Two Dimensional Collisions

Proton #1 with a speed 3.50×10^5 m/s collides elastically with proton #2 initially at rest. After the collision, proton #1 moves at an angle of 37° to the horizontal axis and proton #2 deflects at an angle ϕ to the same axis. Find the final speeds of the two protons and the scattering angle of proton #2, ϕ.



Since both the particles are protons $m_1=m_2=m_p$.

Using momentum conservation, one obtains

x-comp. $m_p v_{1i} = m_p v_{1f}\cos\theta + m_p v_{2f}\cos\phi$

y-comp. $m_p v_{1f}\sin\theta - m_p v_{2f}\sin\phi = 0$

Canceling m_p and putting in all known quantities, one obtains

$$v_{1f}\cos 37^\circ + v_{2f}\cos\phi = 3.50\times10^5 \quad (1)$$

$$v_{1f}\sin 37^\circ = v_{2f}\sin\phi \quad (2)$$

From kinetic energy conservation:

$$\left(3.50\times10^5\right)^2 = v_{1f}^2 + v_{2f}^2 \quad (3)$$

Solving Eqs. 1-3 equations, one gets

$$v_{1f} = 2.80\times10^5\, m/s$$

$$v_{2f} = 2.11\times10^5\, m/s$$

$$\phi = 53.0^\circ$$

Do this at home☺

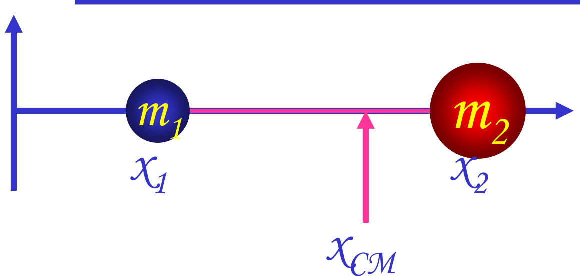
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situations objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on that point.

What does above statement tell you concerning the forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if all the mass of the system is concentrated on the center of mass.*



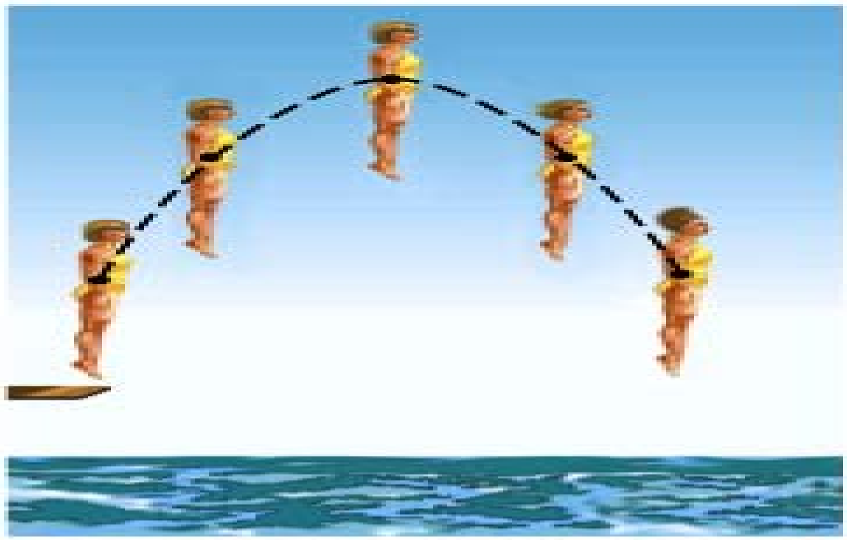
Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Motion of a Diver and the Center of Mass



(a)

Diver performs a simple dive.
The motion of the center of mass follows a parabola since it is a projectile motion.

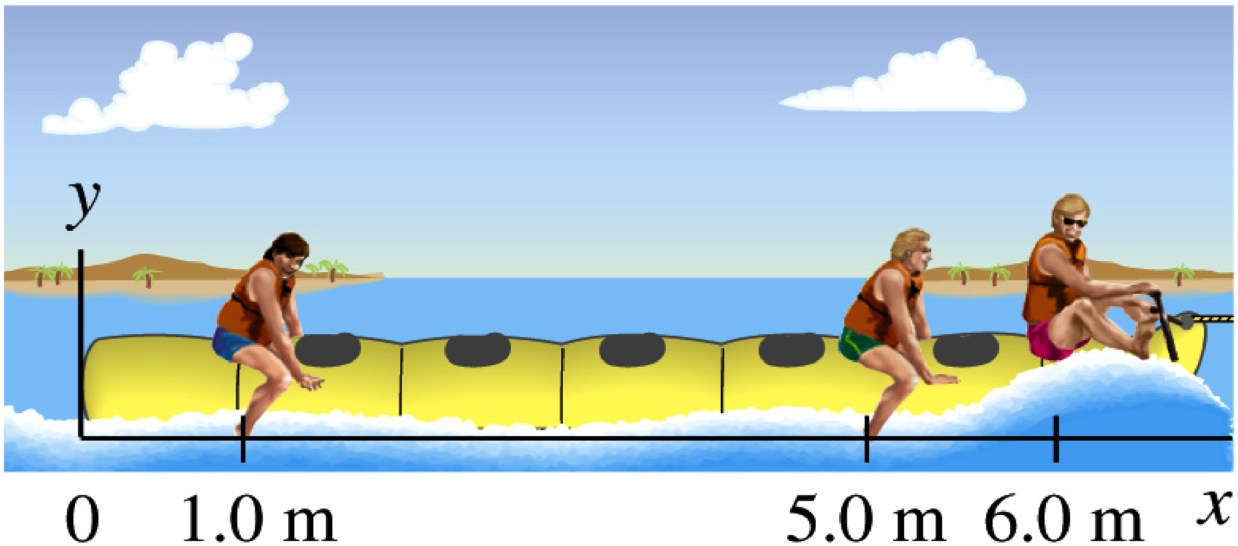
(b)

Diver performs a complicated dive.
The motion of the center of mass still follows the same parabola since it still is a projectile motion.

The motion of the center of mass of the diver is always the same.

Example for Center of Mass

Thee people of roughly equivalent mass M on a lightweight (air-filled) banana boat sit along the x axis at positions x_1=1.0m, x_2=5.0m, and x_3=6.0m. Find the position of CM.

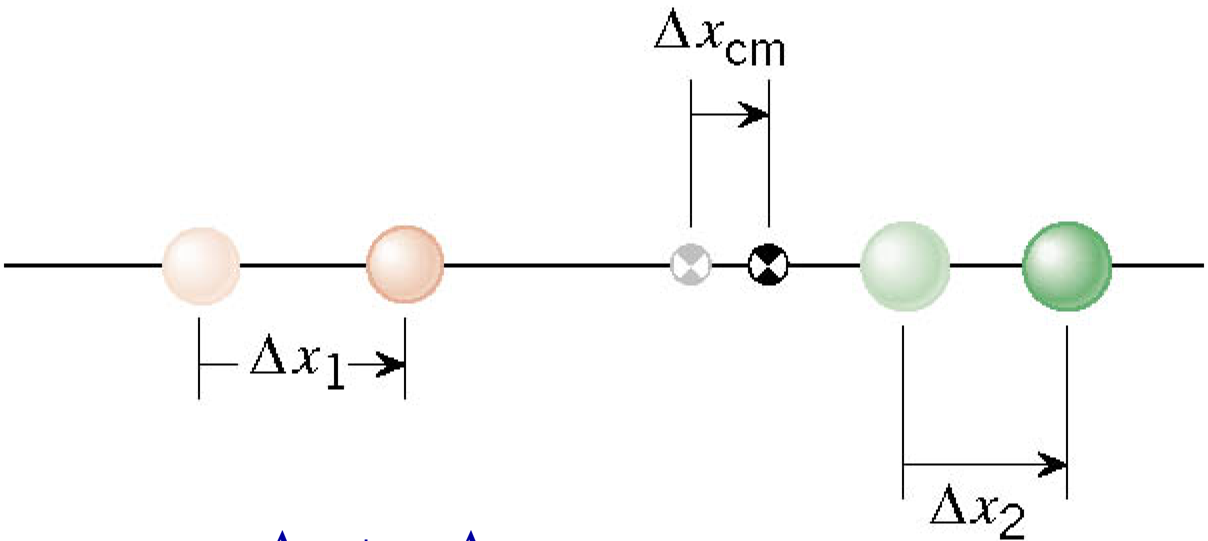


Using the formula for CM

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$= \frac{M \cdot 1.0 + M \cdot 5.0 + M \cdot 6.0}{M + M + M} = \frac{12.0M}{3M} = 4.0(m)$$

Velocity of Center of Mass



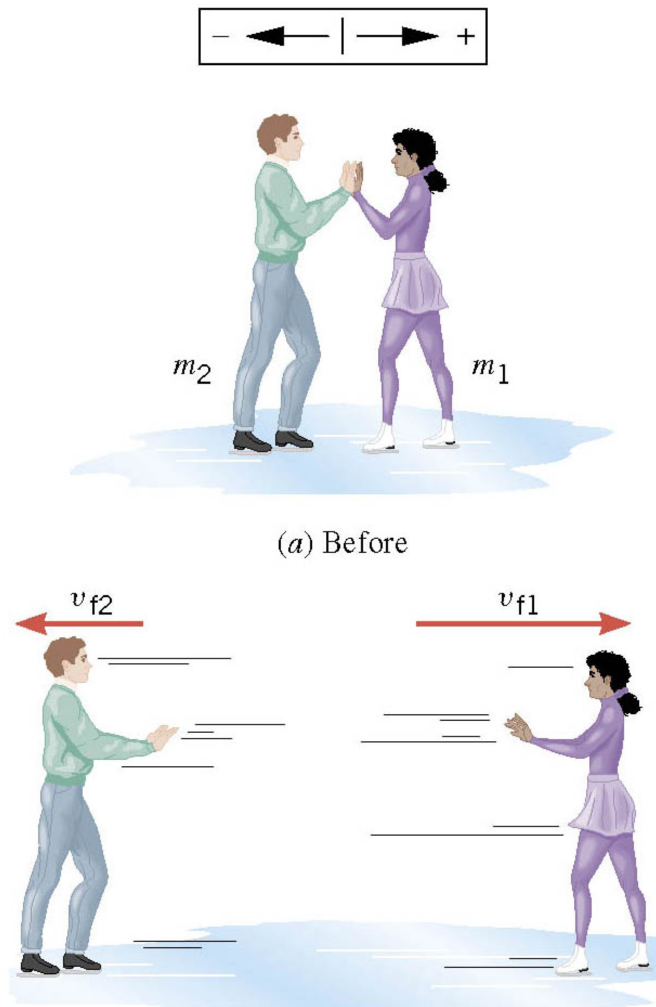
$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$\Rightarrow v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 \Delta x_1 / \Delta t + m_2 \Delta x_2 / \Delta t}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

A Look at the Ice Skater Problem

Starting from rest, two skaters push off against each other on ice where friction is negligible. One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s.



$$v_{10} = 0\, m/s \qquad v_{20} = 0\, m/s$$

$$v_{cm0} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

$$v_{1f} = +2.5\, m/s \qquad v_{2f} = -1.5\, m/s$$

$$v_{cmf} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2}$$

$$= \frac{54 \cdot (+2.5) + 88 \cdot (-1.5)}{54 + 88} = \frac{3}{142} = 0.02 \approx 0\, m/s$$