PHYS 1443 – Section 002
Lecture #23

Wednesday, Apr. 23, 2008

Dr. Jaehoon Yu

1. Simple Harmonic Motion
2. SHO and Circular Motion
3. Equation of SHM
4. Simple Block Spring System
5. Energy of SHO
6. Exam solutions

Today's homework is HW #12, due 9pm, Wednesday, Apr. 30!!

Announcements

- 3^{rd} term exam results
  - Class average:44.6/94
    - Equivalent to 47.4/100
  - Previous results: 61.3 and 45.2
  - Top score: 74/94
- Final Exam
  - Comprehensive exam: CH1.1 – CH10.3 + Appendices A – E
    - Jason will conduct reviews on Apr. 28 and Apr. 30 in preparation for this exam
  - Time and date: 11am – 12:30 pm, Monday, May 5
  - Place: SH103
  - Practice problems for CH10 will be posted on the lecture note page along with all other chapters
- Colloquium today at 4pm in SH101
  - Dr. W. Burgett from U. of Hawaii

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Pan-STARRS: the Next Generation in Survey Astronomy has Arrived

Dr. William Burgett
University of Hawaii

4:00 pm Wednesday, April 23, 2008
Room 101 SH

Abstract

Although large and powerful ground-based and space-based optical telescopes (e.g., Keck, Subaru, and HST) provide spectacular images and utilize complex instrumentation for detailed scientific research, their power comes at the expense of narrow fields of view (telescope "tunnel vision"). This means that such systems greatly benefit from survey systems that can aid them in identifying objects of interest. In the past, wide field astronomical instruments have suffered from a lack of sensitivity compared to their "big brothers" that has limited the depth and resolution achievable by survey observations. However, the next generation of survey systems will greatly reduce this disparity. The Panoramic Survey Telescope and Rapid Response System, Pan-STARRS, is the first of these next generation systems, and, when operational, will produce an avalanche of data in several areas of astrophysics and cosmology including dark energy, dark matter, extrasolar planets, and mapping our own solar system in unprecedented detail including new identifications of potentially hazardous objects (PHOs). In this talk, I will present an overview of the science, technology, and politics of Pan-STARRS as well as the status of the Phase 1 prototype PS1 currently undergoing commissioning.

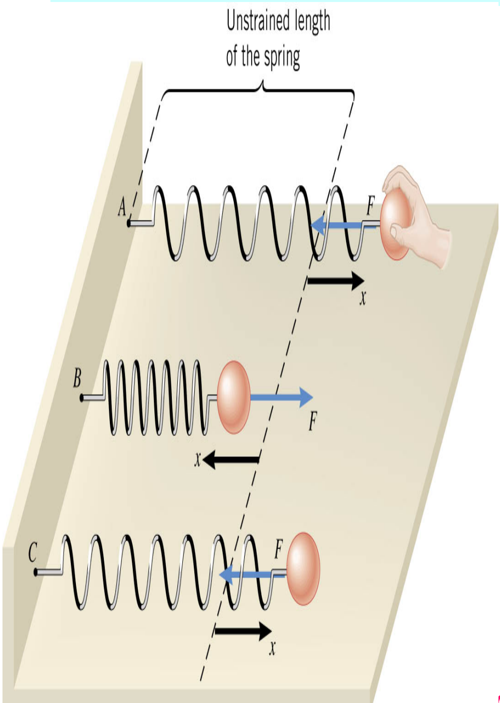
Refreshments will be served in the Physics Library at 3:30 pm

Vibration or Oscillation

What are the things that vibrate/oscillate?

- Tuning fork
- A pendulum
- A car going over a bump
- Building and bridges
- The spider web with a prey

So what is a vibration or oscillation? A periodic motion that repeats over the same path.



A simplest case is a block attached at the end of a coil spring.

When a spring is stretched from its equilibrium position by a length x, the force acting on the mass is $F = -kx$

The sign is negative, because the spring force resists against the change of length, directed toward the equilibrium position.

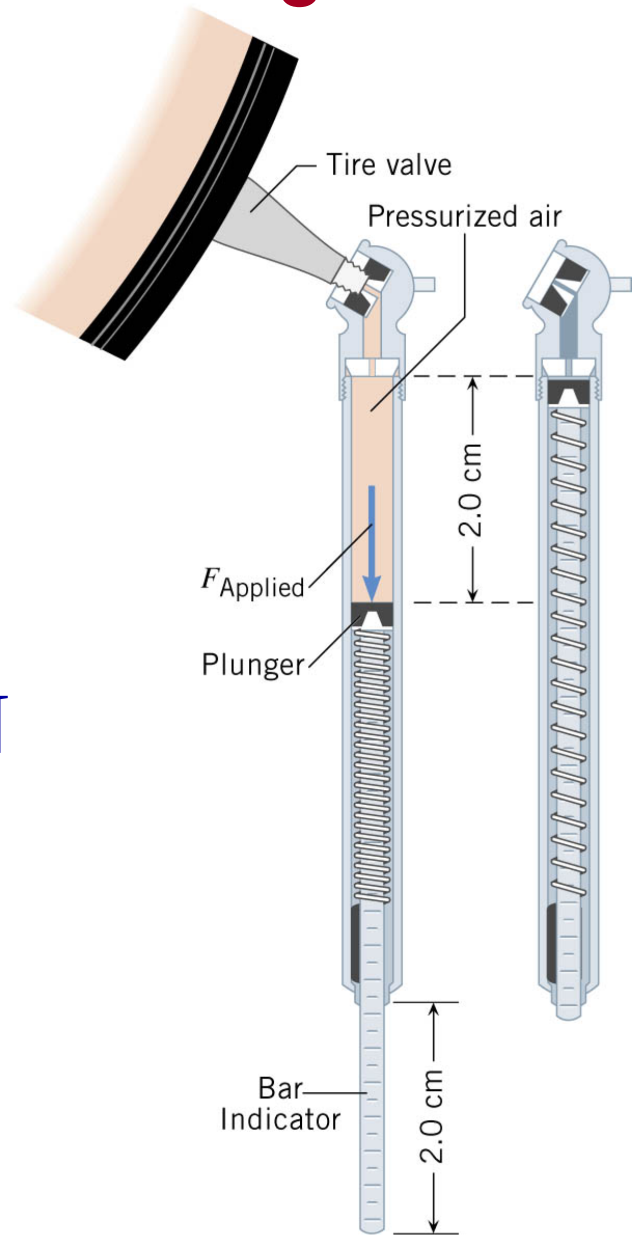
k is called the spring constant. Unit? N/m

Ex. 1 A Tire Pressure Gauge

The spring constant of the spring is 320 N/m and the bar indicator extends 2.0 cm. What force does the air in the tire apply to the spring?

$$F_x^{Applied} = -F_{spring} = kx$$

$$= (320\ \mathrm{N/m})(0.020\ \mathrm{m}) = 6.4\ \mathrm{N}$$



Simple Harmonic Motion

Motion that occurs by the force that depends on displacement, and the force is always directed toward the system's equilibrium position. Thus, the motion repeats on a fixed path when no friction exists.

What is a system that has such characteristics?

A system consists of a mass and a spring

When a spring is stretched or pressed from its equilibrium position by a length x, the restoring force acting on the mass is $F = -kx$

From Newton's second law $F = ma = -kx$ we obtain $a = -\frac{k}{m}x$

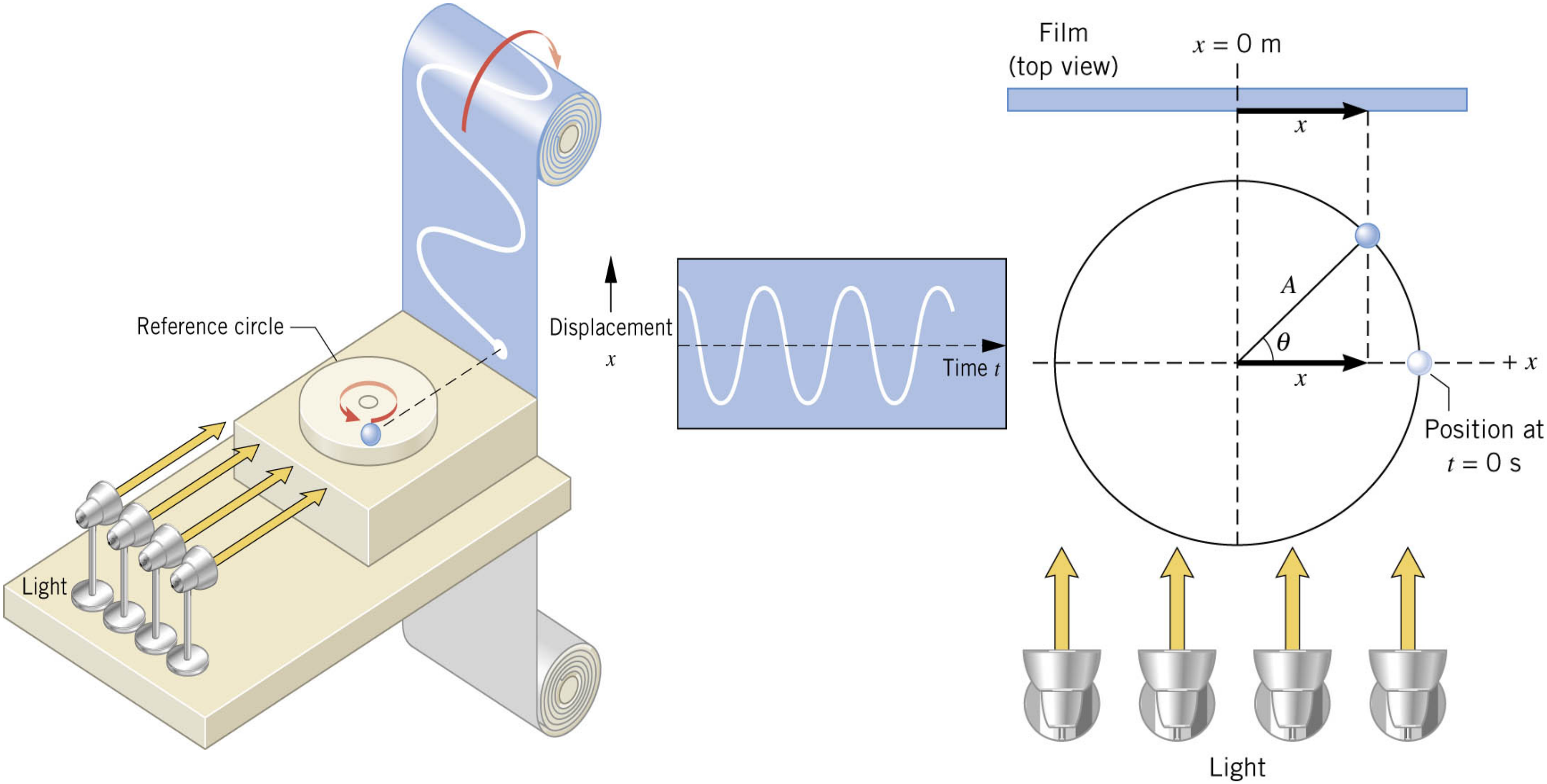
Condition for simple harmonic motion $a = -\frac{k}{m}x$

What do you observe from this equation?

Acceleration is proportional to displacement from the equilibrium
Acceleration is opposite direction to displacement

This system is doing a simple harmonic motion (SHM).

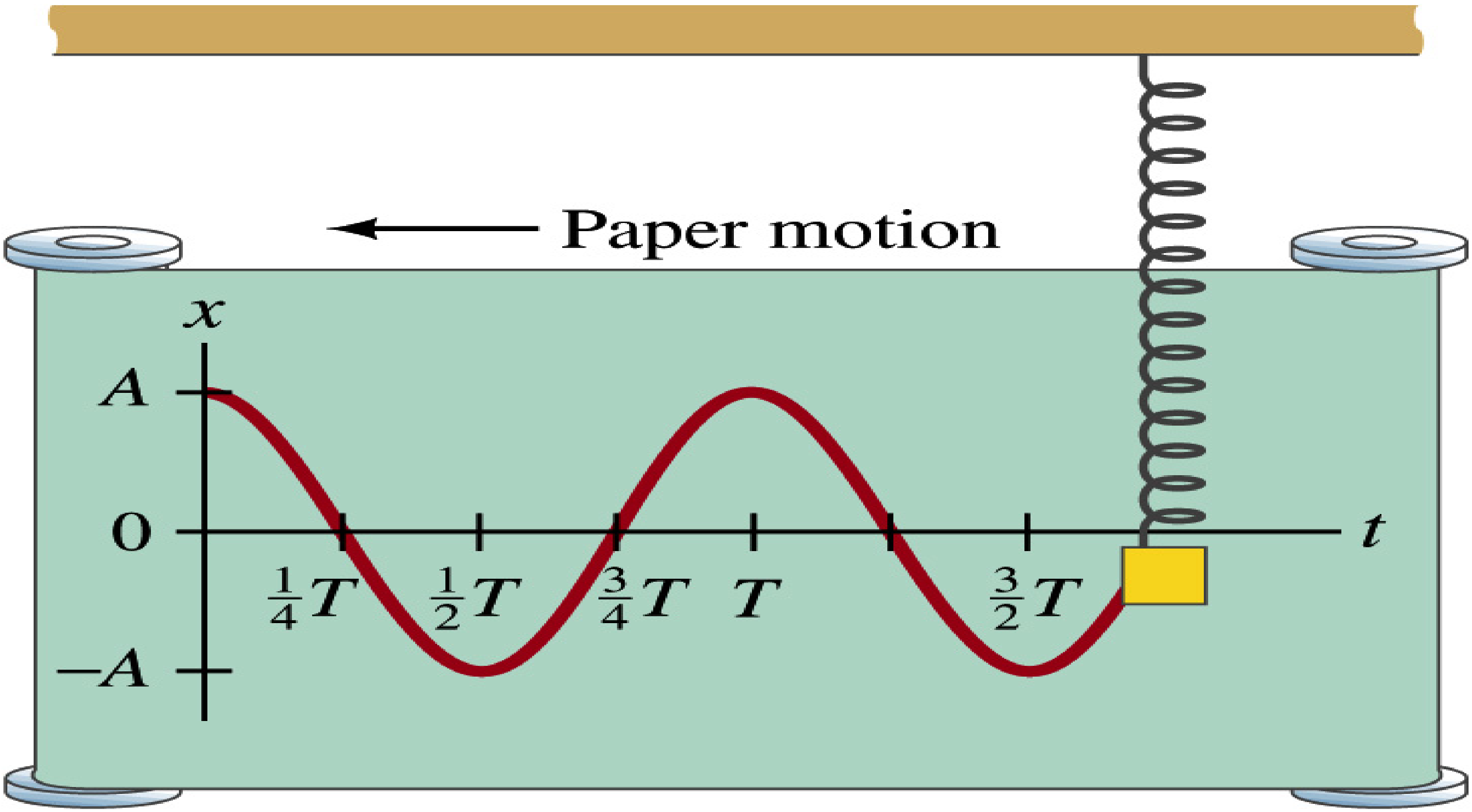
Circular Motion and a SHM - Displacement



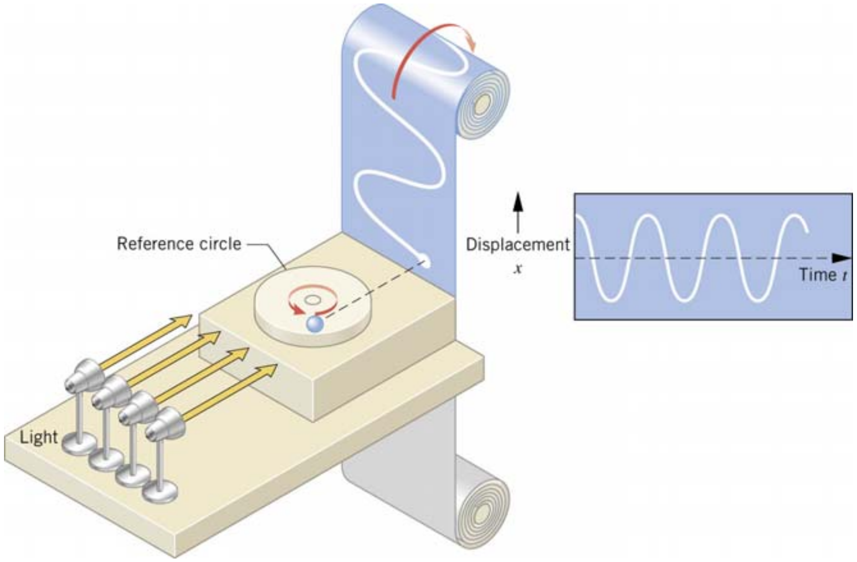
$$x = A\cos\theta = A\cos\omega t$$

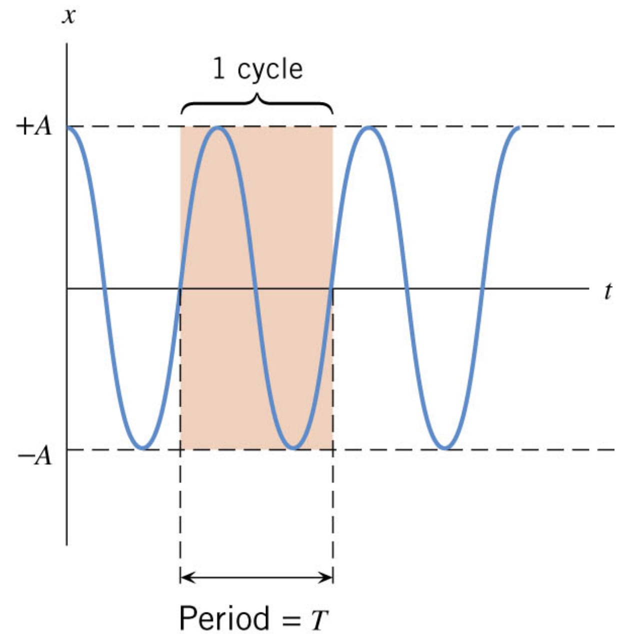
Sinusoidal Behavior of SHM

What do you think the trajectory will look if the oscillation of a block attached to a spring was plotted against time?



Parameters of the SHM





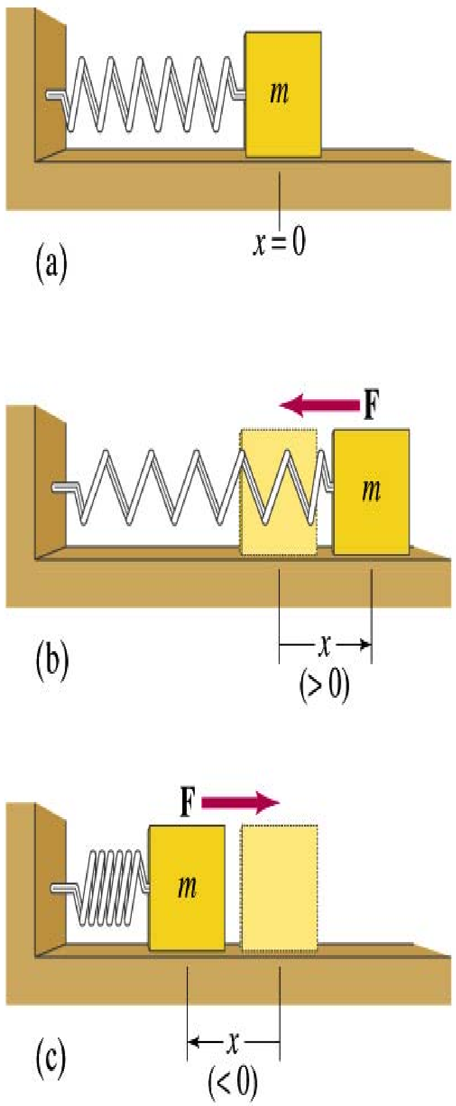
amplitude A: the maximum displacement

period T: the time required to complete one cycle

frequency f: the number of cycles per second (measured in Hz)

$$f = \frac{1}{T} \qquad \omega = \frac{2\pi}{T} = 2\pi f$$

Vibration or Oscillation Properties



The maximum displacement from the equilibrium is

Amplitude

One cycle of the oscillation

The complete to-and-fro motion from an initial point

Period of the motion, T

The time it takes to complete one full cycle

Unit? s

Frequency of the motion, f

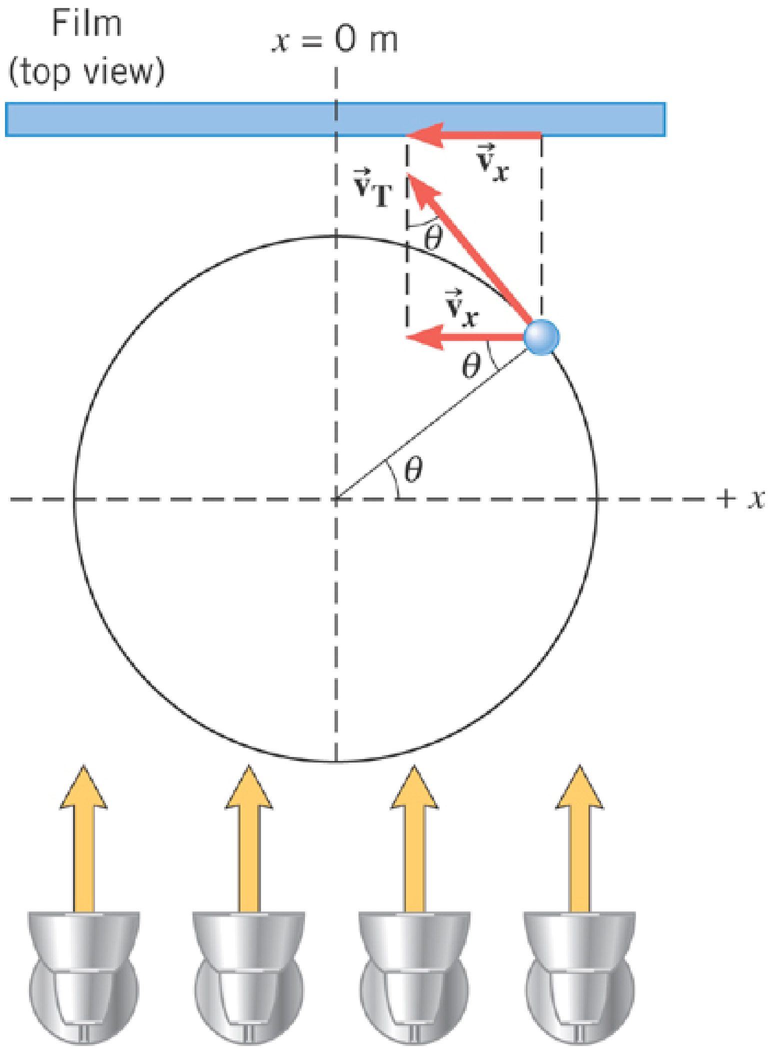
The number of complete cycles per second

Unit? s^{-1} Hz

Relationship between period and frequency?

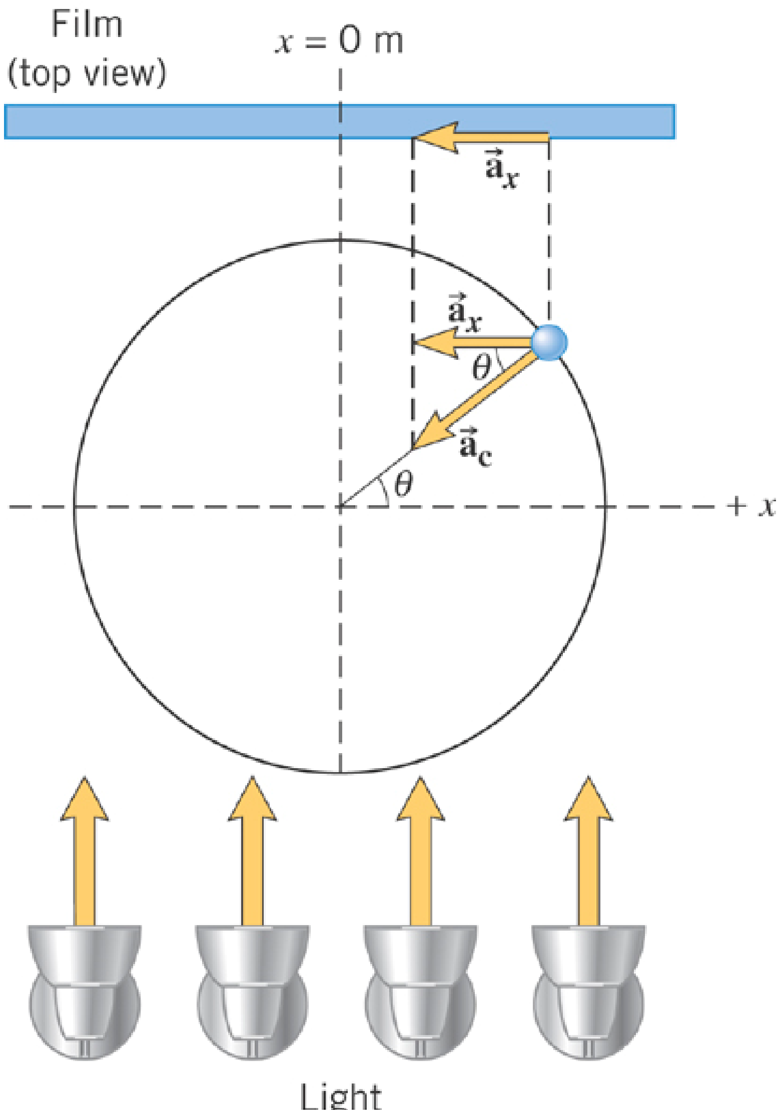
$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

Circular Motion and a SHM - Velocity



$$v_x = -v_T \sin\theta = -\underbrace{A\omega}_{v_{\max}} \sin\omega t$$

Circular Motion and a SHM - Acceleration



$$a_x = -a_c \cos\theta = -\underbrace{A\omega^2}_{a_{\max}} \cos\omega t$$

What do we learn about acceleration?

Acceleration is reverse direction to displacement

Acceleration and speed are $\pi/2$ off phase:

When v is maximum, a is at its minimum

Equation of Simple Harmonic Motion

The solution for the 2nd order differential equation

$$x = A\cos\left(\boxed{\omega t + \phi}\right)$$

Amplitude

Phase

Angular Frequency

Phase constant

Generalized expression of a simple harmonic motion

Let's think about the meaning of this equation of motion

What happens when t=0 and ϕ=0?

$$x = A\cos(0+0) = A$$

What is ϕ if x is not A at t=0?

$$x = A\cos(\phi) = x'$$

$$\phi = \cos^{-1}(x')$$

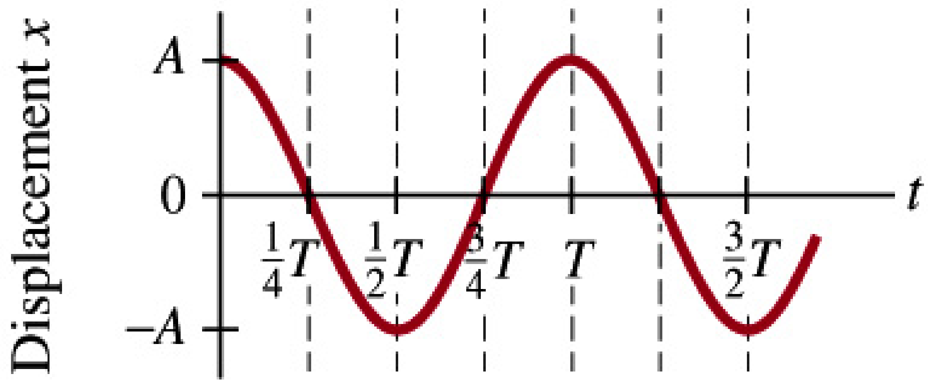
What are the maximum/minimum possible values of x?

A/-A

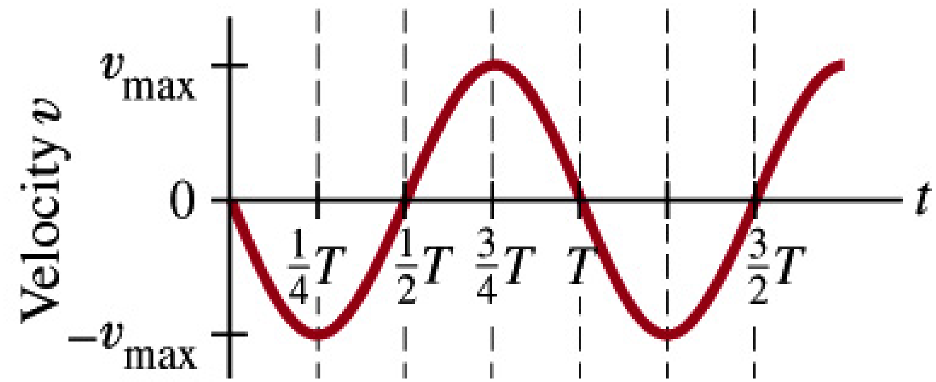
An oscillation is fully characterized by its:

- Amplitude
- Period or frequency
- Phase constant

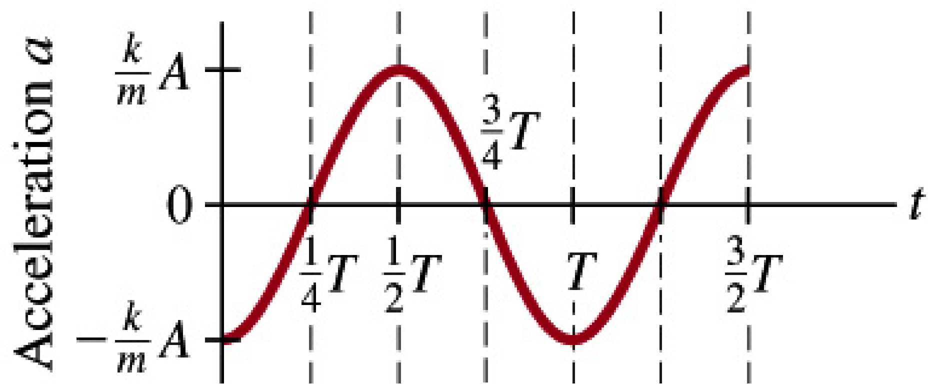
Sinusoidal Behavior of SHM



$$x = A\cos\left(2\pi ft\right)$$



$$v = -v_0 \sin\left(2\pi ft\right)$$



$$a = -a_0 \cos\left(2\pi ft\right)$$

Simple Block-Spring System

A block attached at the end of a spring on a frictionless surface experiences acceleration when the spring is displaced from an equilibrium position.

From Hooke's Law $$F_{spring} = ma \quad = -kx$$

Using the equations of the simple harmonic motion, we obtain

$$\boxed{x = A\cos\omega t} \qquad \boxed{v = \omega A\sin\omega t} \qquad \boxed{a = -\omega^2 A\cos\omega t = -\omega^2 x}$$

$$\cancel{-}k\cancel{x} = m\left(-\omega^2 A\cos\omega t\right) = \cancel{-}m\omega^2\cancel{x} \qquad \omega^2 = \frac{k}{m}$$

So the angular frequency, ω, is $$\omega = \sqrt{\frac{k}{m}}$$

This means that whenever a force is proportional to the distance from the equilibrium position, the motion of the object under the force performs a simple harmonic motion with the angular frequency that depends on the spring constant and the mass.

More Simple Block-Spring System

How do the period and frequency of this harmonic motion look?

Since the angular frequency ω is $\omega = \sqrt{\frac{k}{m}}$

The period, T, becomes $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

So the frequency is $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

What can we learn from these?

- Frequency and period do not depend on amplitude
- Period is inversely proportional to spring constant and proportional to mass

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Special case #1

Let's consider that the spring is stretched to distance A and the block is let go from rest, giving 0 initial speed; x_i=A, v_i=0,

$$x = A\cos\omega t \quad v = -\omega A\sin\omega t \quad a = -\omega^2 A\cos\omega t \quad a_i = -\omega^2 A = -kA/m$$

This equation of motion satisfies all the conditions. So it is the solution for this motion.

Special case #2

Suppose block is given non-zero initial velocity v_i to positive x at the instant it is at the equilibrium, x_i=0

$$\phi = \tan^{-1}\left(-\frac{v_i}{\omega x_i}\right) = \tan^{-1}(-\infty) = -\frac{\pi}{2} \quad x = A\cos\left(\omega t - \frac{\pi}{2}\right) = A\sin(\omega t)$$

Ex. 6 A Body Mass Measurement Device

The device consists of a spring-mounted chair in which the astronaut sits. The spring has a spring constant of 606 N/m and the mass of the chair is 12.0 kg. The measured period is 2.41 s. Find the mass of the astronaut.

$$\omega = \sqrt{\frac{k}{m_{\text{total}}}} \quad \Longrightarrow \quad m_{\text{total}} = m_{\text{chair}} + m_{\text{astro}} = k/\omega^2 = \frac{k}{(2\pi/T)^2}$$

since $\omega = 2\pi f = \frac{2\pi}{T}$

$$m_{\text{astro}} = \frac{k}{(2\pi/T)^2} - m_{\text{chair}} = \frac{(606\,\text{N/m})(2.41\,\text{s})^2}{4\pi^2} - 12.0\,\text{kg} = 77.2\,\text{kg}$$

Energy of the Simple Harmonic Oscillator

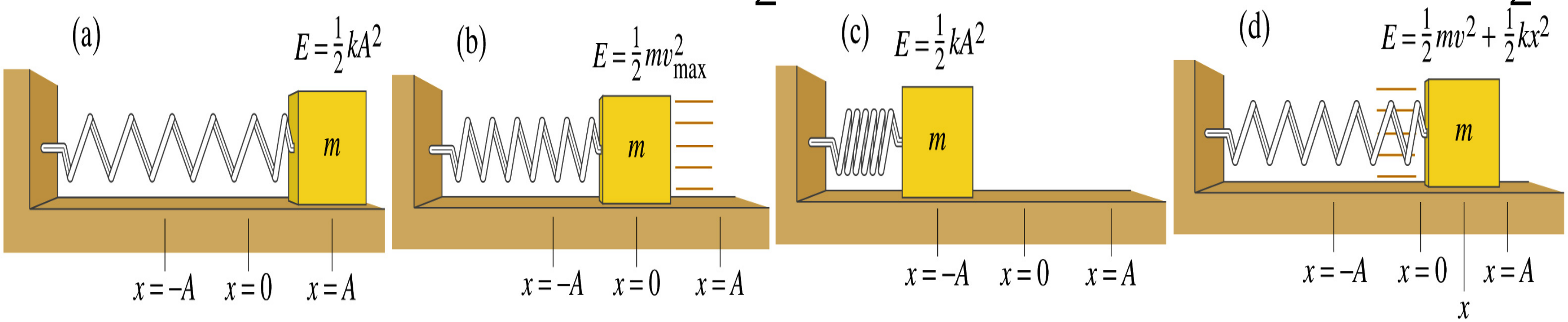
How do you think the mechanical energy of the harmonic oscillator look without friction?

Kinetic energy of a harmonic oscillator is $KE = \frac{1}{2}mv^2 \quad = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t)$

The elastic potential energy stored in the spring $PE = \frac{1}{2}kx^2 \quad = \frac{1}{2}kA^2 \cos^2(\omega t)$

Therefore the total mechanical energy of the harmonic oscillator is $E = KE + PE \quad = \frac{1}{2}\left[m\omega^2 A^2 \sin^2(\omega t) + kA^2 \cos^2(\omega t)\right]$

Since $\omega = \sqrt{k/m}$ $\quad E = KE + PE \quad = \frac{1}{2}\left[kA^2 \sin^2(\omega t) + kA^2 \cos^2(\omega t)\right] \quad = \frac{1}{2}kA^2$



Total mechanical energy of a simple harmonic oscillator is proportional to the square of the amplitude.

Energy of the Simple Harmonic Oscillator cont'd

Maximum KE is when PE=0

$$KE_{\max} = \frac{1}{2} m v_{\max}^2 = \frac{1}{2} k A^2$$

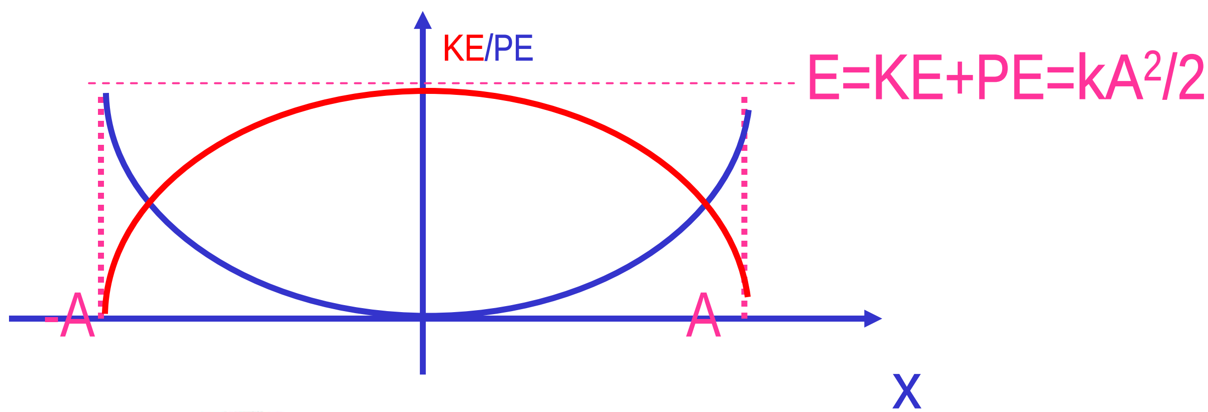
Maximum speed

$$v_{\max} = \sqrt{\frac{k}{m}} A$$

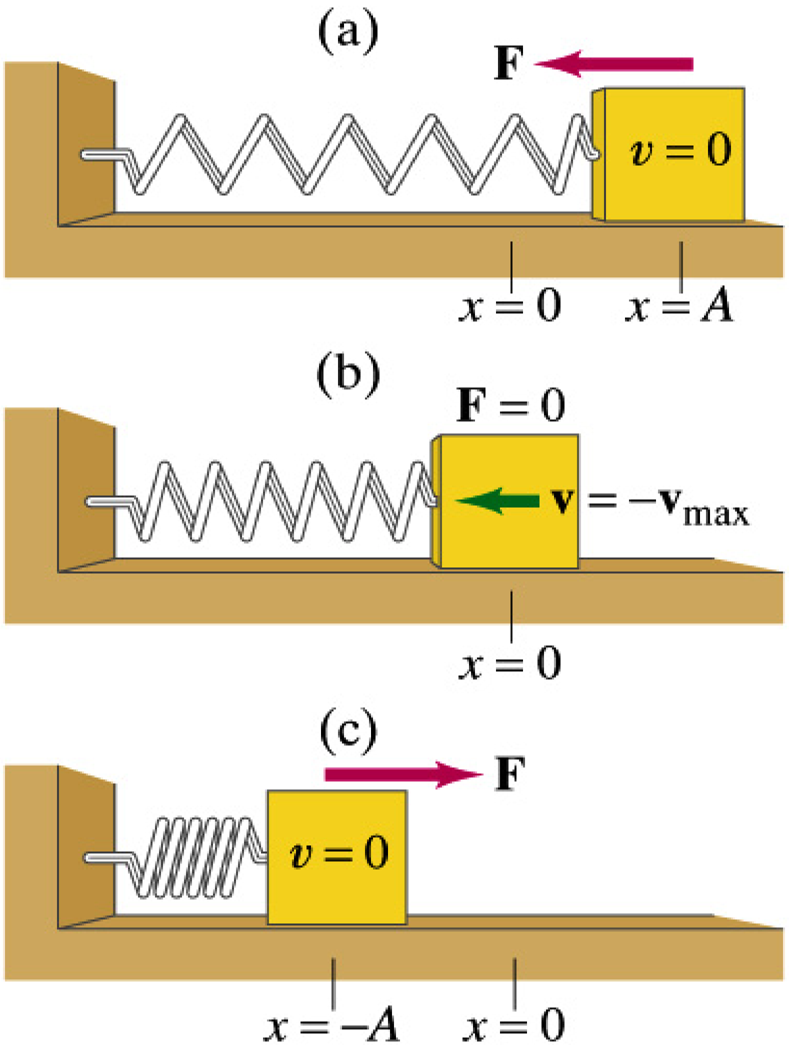
The speed at any given point of the oscillation

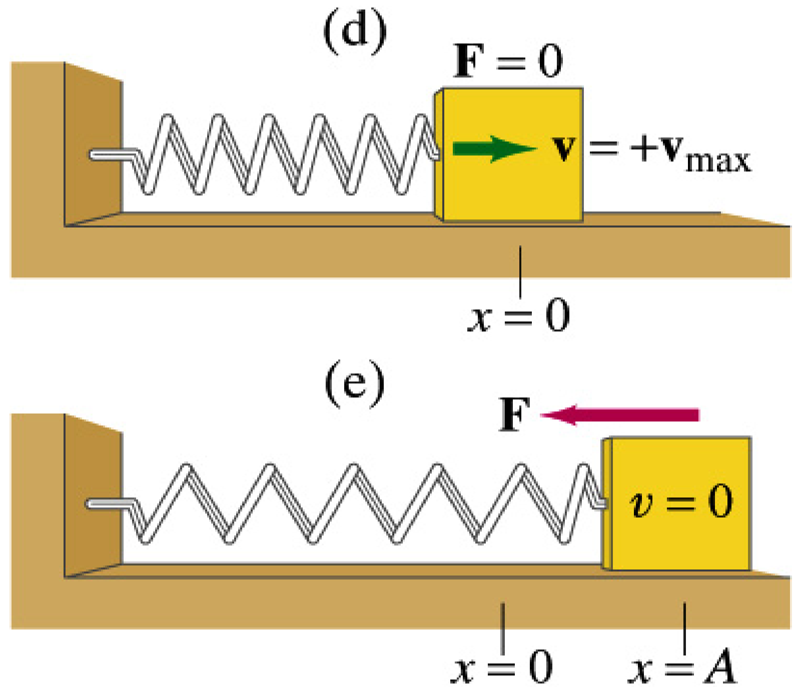
$$E = KE + PE = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$v = \sqrt{k/m\left(A^2 - x^2\right)} = v_{\max}\sqrt{1 - \left(\frac{x}{A}\right)^2}$$



Oscillation Properties





Amplitude? A

- When is the force greatest?
- When is the speed greatest?
- When is the acceleration greatest?
- When is the potential energy greatest?
- When is the kinetic energy greatest?

Congratulations!!!!

You all have done very well!!!

I certainly had a lot of fun with ya'll!

Good luck with your exams!!!

Have a safe summer!!