PHYS 1441 – Section 002
Lecture #4

Monday, Feb. 9, 2009
Dr. Jaehoon Yu

- Chapter two: Motion in one dimension
  - Velocity (Average and Instantaneous)
  - Acceleration (Average and instantaneous)
  - One dimensional motion under constant acceleration
    - Free Fall
  - Coordinate systems

Today's homework is homework #3, due 9pm, Thursday, Feb. 19!!

Announcements

- E-mail distribution list: 56 of you subscribed to the list so far
  - Test message was sent out early Friday morning!!
  - Thanks for your confirmations!!
- Reading assignment: CH2.8
- First term exam
  - 1 – 2:20pm, Wednesday, Feb. 18
  - Covers: CH1.1 – what we complete on Monday, Feb. 16 + appendix A1 – A8
  - Style: Mixture of multiple choices and free responses
- Physics Department colloquium schedule at
  - There is a double extra credit for colloquium this Wednesday

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Positron Emission Tomography: From Basics to the Future

Dr. Chang Kim
General Electric Research

Wednesday, February 11, 2009 at 4:00 pm in Room 101 SH

Abstract

First, the basics of positron emission tomography, so called PET, would be presented, with a short history, from the radioactive tracer generation, body injection, gamma rays detection to image reconstruction. With the comparison to other medical imaging devices, I will try to show how its functional imaging capability is different from anatomical imaging of well known X-ray, Computed Tomography (CT), Magnetic Resonance Imaging (MRI). Second, how to build PET gamma ray detectors based on inorganic scintillator and photomultipliers and their critical factors will be presented. Third, the future of PET detector will be discussed with respect to Time-of-Flight PET detector developments using solid state photomultipliers.

Refreshments will be served in the Physics Lounge at 3:30 pm

Special Problems for Extra Credit

- Derive the quadratic equation for $yx^2-zx+v=0$ ➔ 5 points
- Derive the kinematic equation $v^2 = v_0^2 + 2a(x - x_0)$ from first principles and the known kinematic equations ➔ 10 points
- You must <u>show your work in detail</u> to obtain the full credit
- Due next Monday, Feb. 16

Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion?
- Instantaneous velocity is defined as:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Average velocity over a very short amount of time

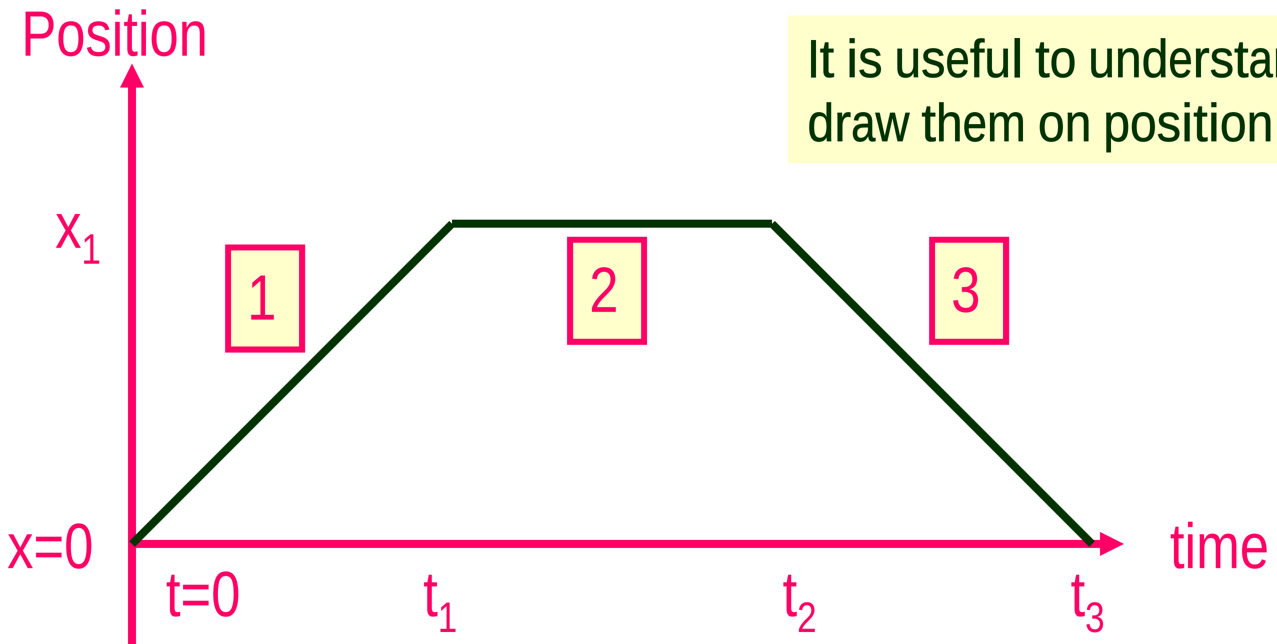
- Instantaneous speed is the size (magnitude) of the velocity vector:

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right|$$

*Magnitude of Vectors are Expressed in absolute values

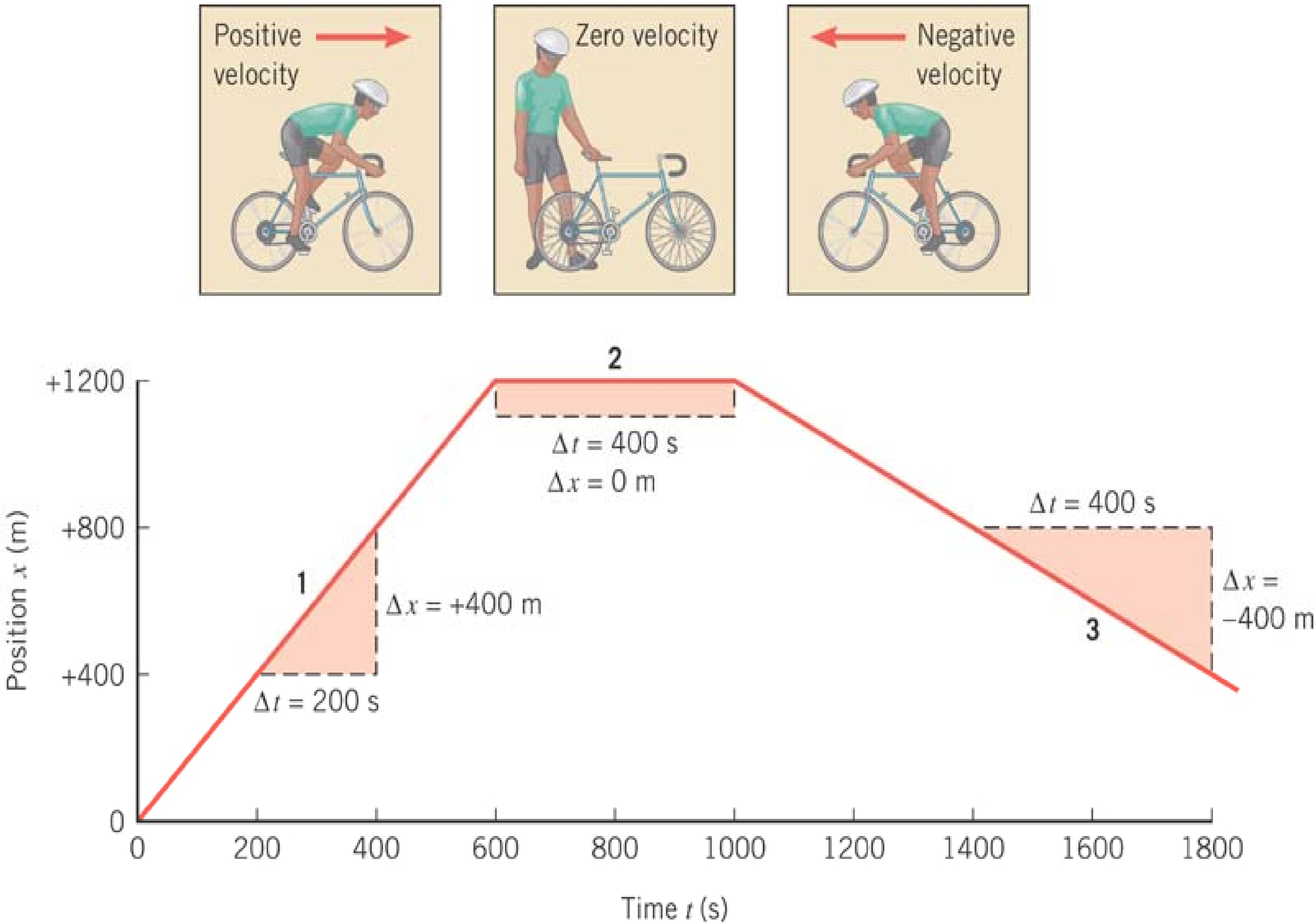
Position vs Time Plot

It is useful to understand motions to draw them on position vs time plots.



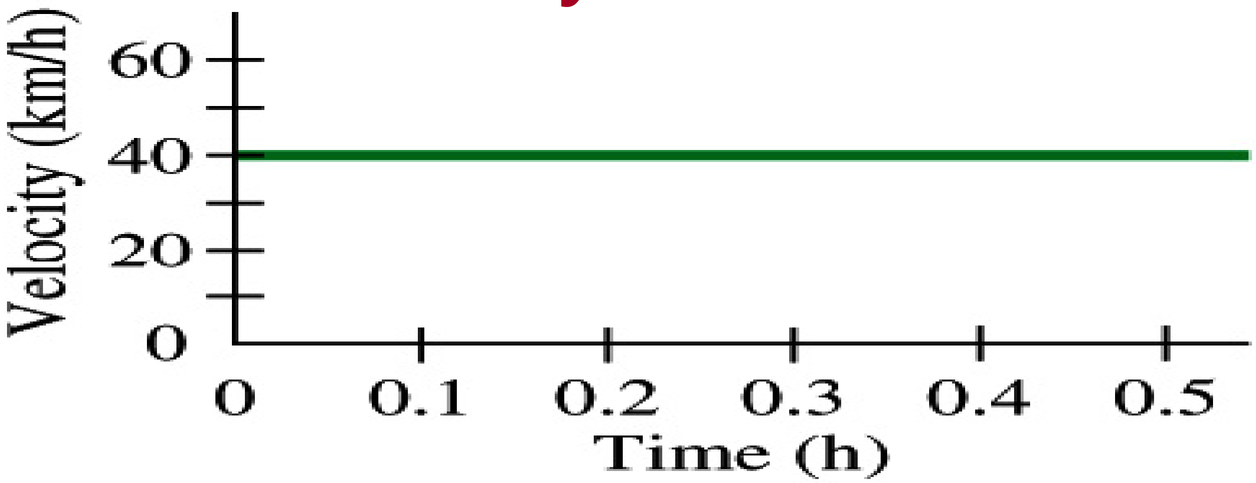
1. Running at a constant velocity (go from x=0 to $x=x_1$ in t_1, Displacement is + x_1 in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to x=0 in t_3-t_2 time interval, Displacement is - x_1 in t_3-t_2 time interval)

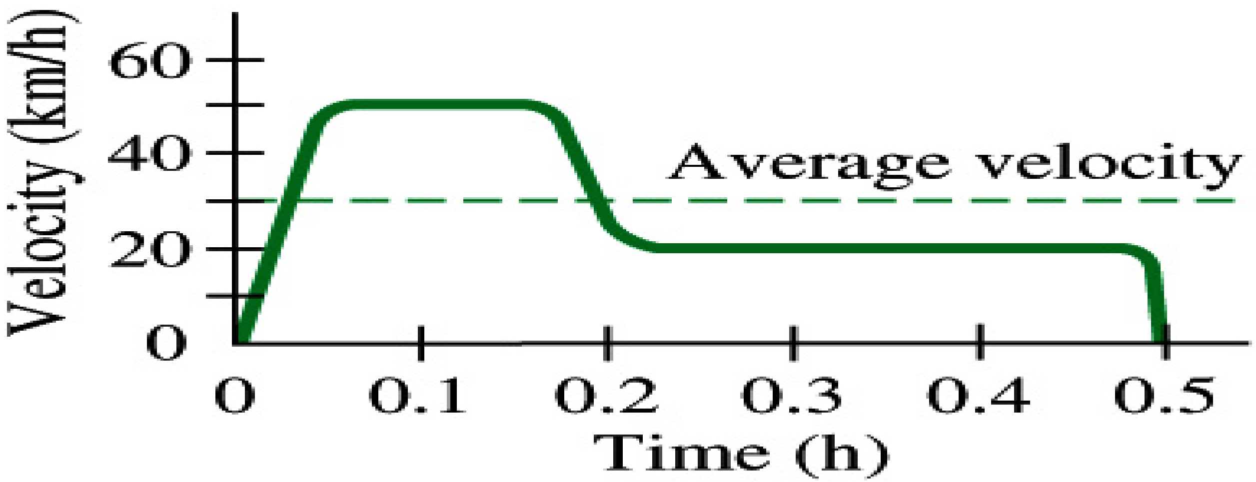
Does this motion physically make sense?



Positive velocity
Zero velocity
Negative velocity
+1200
+800
+400
0
Position x (m)
1
2
3
Δt = 200 s
Δx = +400 m
Δt = 400 s
Δx = 0 m
Δt = 400 s
Δx = −400 m
0
200
400
600
800
1000
1200
1400
1600
1800
Time t (s)

Velocity vs Time Plot





Displacement, Velocity and Speed

Displacement
$$\Delta x \equiv x_f - x_i$$

Average velocity
$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed
$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Instantaneous speed
$$\left|v_x\right| = \left|\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right|$$

Acceleration

Change of velocity in time (what kind of quantity is this?)

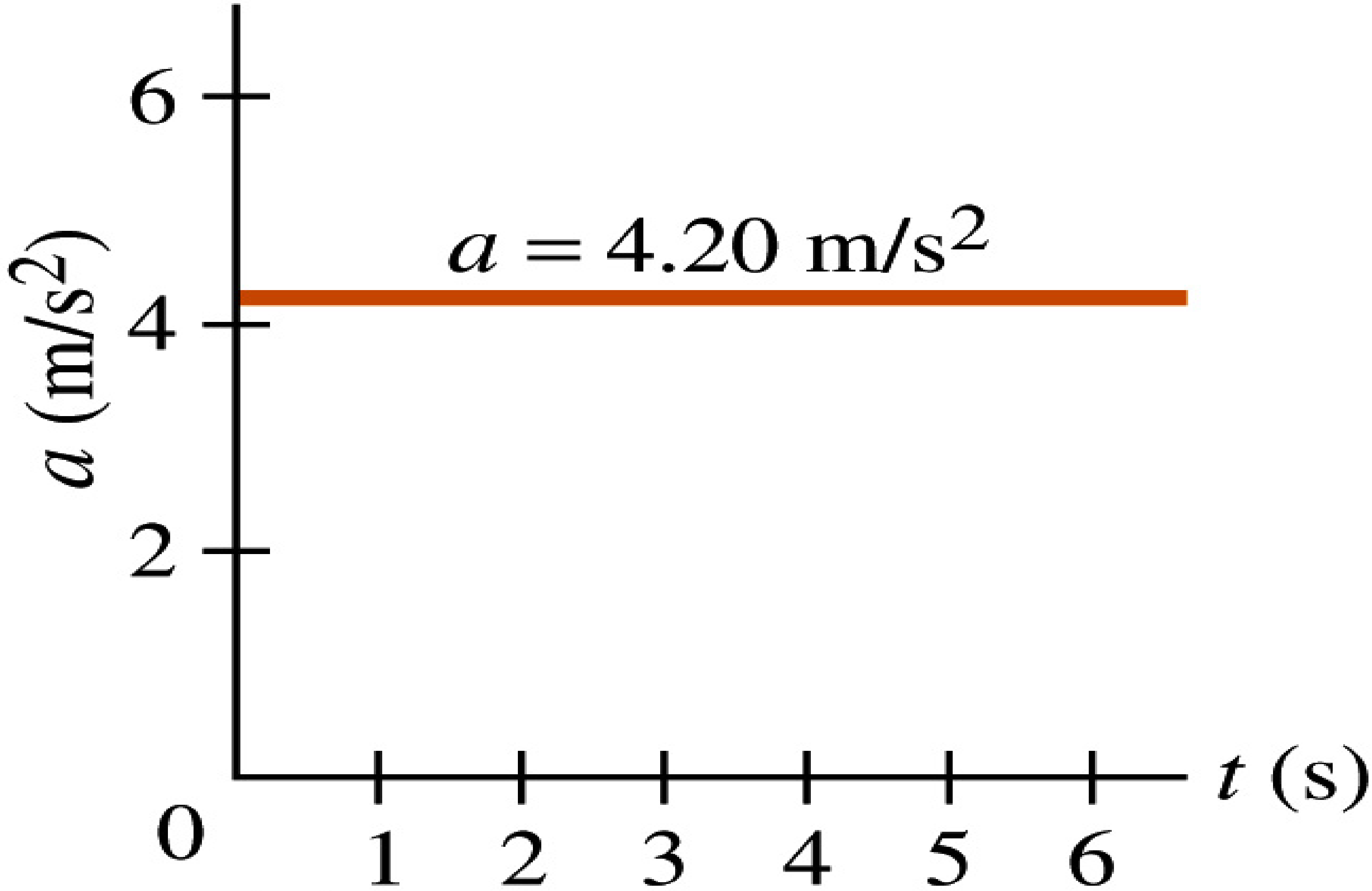
- Average acceleration:

$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogs to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Instantaneous acceleration: Average acceleration over a very short amount of time.

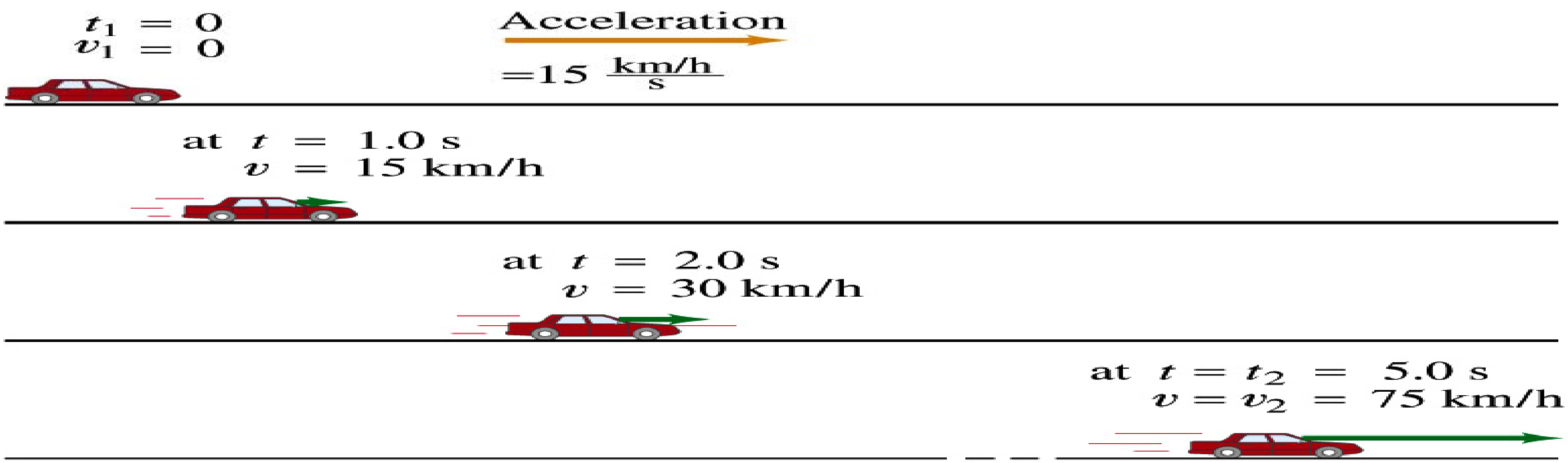
$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \quad \text{analogs to} \quad \boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}$$

Acceleration vs Time Plot



Example 2.4

A car accelerates along a straight road from rest to 75km/h in 5.0s.



What is the magnitude of its average acceleration?

$$v_{xi} = 0\ m/s$$

$$v_{xf} = \frac{75000m}{3600s} = 21\ m/s$$

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} = \frac{21-0}{5.0} = \frac{21}{5.0} = 4.2(m/s^2)$$

$$= \frac{4.2 \times (3600)^2}{1000} = 5.4 \times 10^4 (km/h^2)$$

Meanings of Acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any acceleration when an object is not moving?
- When an object is moving faster as time goes on, ($v=v(t)$), acceleration is positive ($a>0$)
  - Incorrect, since the object might be moving in negative direction initially
- When an object is moving slower as time goes on, ($v=v(t)$), acceleration is negative ($a<0$)
  - Incorrect, since the object might be moving in negative direction initially
- In all cases, velocity is positive, unless the direction of the movement changes.
  - Incorrect, since the object might be moving in negative direction initially
- Is there acceleration if an object moves in a constant speed but changes direction?

The answer is YES!!

One Dimensional Motion

- Let's start with the simplest case: acceleration is a constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can derive equations of motion (description of motion, velocity and position as a function of time)

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$a_x = \frac{v_{xf} - v_{xi}}{t} \Rightarrow v_{xf} = v_{xi} + a_x t$$

For constant acceleration, average velocity is a simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2}a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$\bar{v}_x = \frac{x_f - x_i}{t} \Rightarrow x_f = x_i + \bar{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

One Dimensional Motion cont'd

Average velocity $\bar{v}_x = \frac{v_{xi} + v_{xf}}{2}$ $\Rightarrow$ $x_f = x_i + \bar{v}_x t = x_i + \left(\frac{v_{xi} + v_{xf}}{2}\right)t$

Since $a_x = \frac{v_{xf} - v_{xi}}{t}$ — Solving for t $\Rightarrow$ $t = \frac{v_{xf} - x_{xi}}{a_x}$

Substituting t in the above equation,

$$x_f = x_i + \left(\frac{v_{xf} + v_{xi}}{2}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

Resulting in

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}(v_{xf} + v_{xi})\ t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

How do we solve a problem using a kinematic formula for constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance?
  - Time?
- Identify what the problem wants.
- Identify which formula is appropriate and easiest to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equation for the quantity wanted

Example 2.10

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/s (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇨ As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

The acceleration is $a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0-(28m/s)^2}{2\times 1m} = -390m/s^2$

Thus the time for air-bag to deploy is $t = \frac{v_{xf} - v_{xi}}{a} = \frac{0-28m/s}{-390m/s^2} = 0.07s$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right) t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

Falling Motion

- Falling motion is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
  - A motion under constant acceleration
  - All kinematic formula we learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The gravitational acceleration is $g=9.80m/s^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is ALWAYS toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$

Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $g=-9.80m/s^2$

(a) Find the time the stone reaches at the maximum height.

What happens at the maximum height? The stone stops; V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00m/s$$

Solve for t

$$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

Position

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$