PHYS 1441 – Section 002
Lecture #5

Wednesday, Feb. 11, 2009

Dr. Jaehoon Yu

- Coordinate systems
- Vectors and Vector Operations
- Motion in Two Dimensions
- Motion under constant acceleration
- Projectile Motion

Announcements

- E-mail distribution list: 58 of you subscribed to the list
  - Please be sure to subscribe to the list as soon as possible.
- First term exam
  - 1 – 2:20pm, Wednesday, Feb. 18
  - Covers: CH1.1 – what we complete on Monday, Feb. 16 + appendix A1 – A8
- Physics Department colloquium scheduled at 4pm today in SH101
  - There is a double extra credit for colloquium today

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Positron Emission Tomography: From Basics to the Future

Dr. Chang Kim
General Electric Research

Wednesday, February 11, 2009 at 4:00 pm in Room 101 SH

Abstract

First, the basics of positron emission tomography, so called PET, would be presented, with a short history, from the radioactive tracer generation, body injection, gamma rays detection to image reconstruction. With the comparison to other medical imaging devices, I will try to show how its functional imaging capability is different from anatomical imaging of well known X-ray, Computed Tomography (CT), Magnetic Resonance Imaging (MRI). Second, how to build PET gamma ray detectors based on inorganic scintillator and photomultipliers and their critical factors will be presented. Third, the future of PET detector will be discussed with respect to Time-of-Flight PET detector developments using solid state photomultipliers.

Refreshments will be served in the Physics Lounge at 3:30 pm

Reminder: Special Problems for Extra Credit

- Derive the quadratic equation for $yx^2-zx+v=0$ ➔ 5 points
- Derive the kinematic equation $v^2 = v_0^2 + 2a(x - x_0)$ from first principles and the known kinematic equations ➔ 10 points
- You must show your work in detail to obtain the full credit
- Due next Monday, Feb. 16

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$\boxed{v_{xf}(t) = v_{xi} + a_x t}$$

Velocity as a function of time

$$\boxed{x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right) t}$$

Displacement as a function of velocities and time

$$\boxed{x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2}$$

Displacement as a function of time, velocity, and acceleration

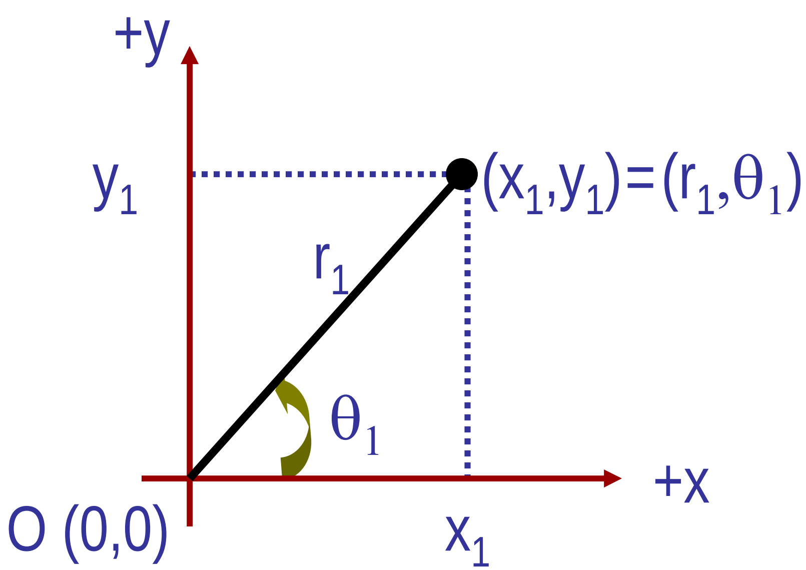
$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)}$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

Coordinate Systems

- They make it easy and consistent to express locations or positions
- Two commonly used systems, depending on convenience, are
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in distance from the origin ® and the angle measured from the x-axis, θ (r,θ)
- Vectors become a lot easier to express and compute



How are Cartesian and Polar coordinates related?

$$x_1 = r_1 \cos\theta_1$$

$$y_1 = r_1 \sin\theta_1$$

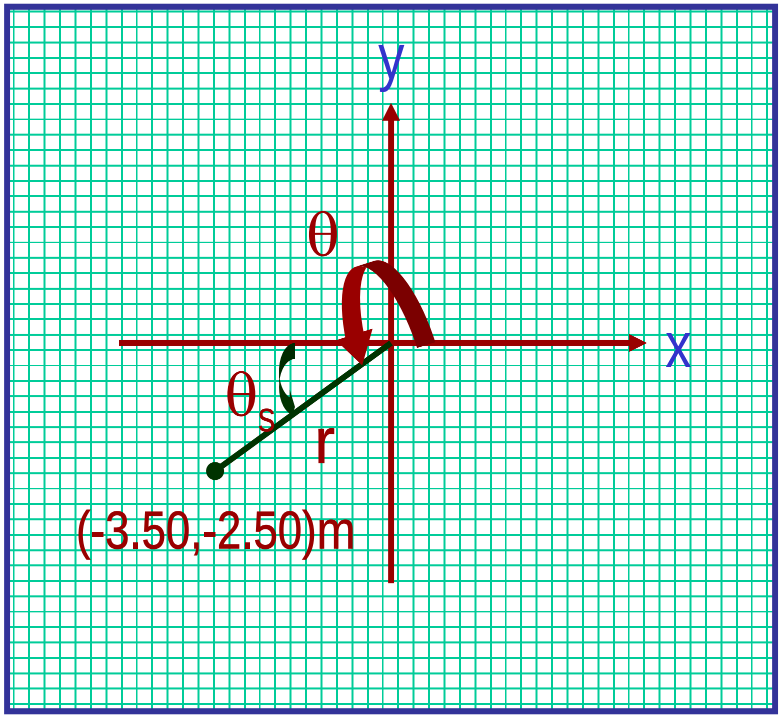
$$r_1 = \sqrt{\left(x_1^2 + y_1^2\right)}$$

$$\tan\theta_1 = \frac{y_1}{x_1}$$

$$\theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right)$$

Example

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the equivalent polar coordinates of this point.



$$r = \sqrt{\left(x^2 + y^2\right)}$$

$$= \sqrt{\left((-3.50)^2 + (-2.50)^2\right)}$$

$$= \sqrt{18.5} = 4.30(m)$$

$$\theta = 180 + \theta_s$$

$$\tan\theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ$$

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Force, gravitational acceleration, momentum

Normally denoted in **BOLD** letters, $\mathbf{F}$, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, F, or absolute values: $\left|\vec{F}\right|$ or $|F|$

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

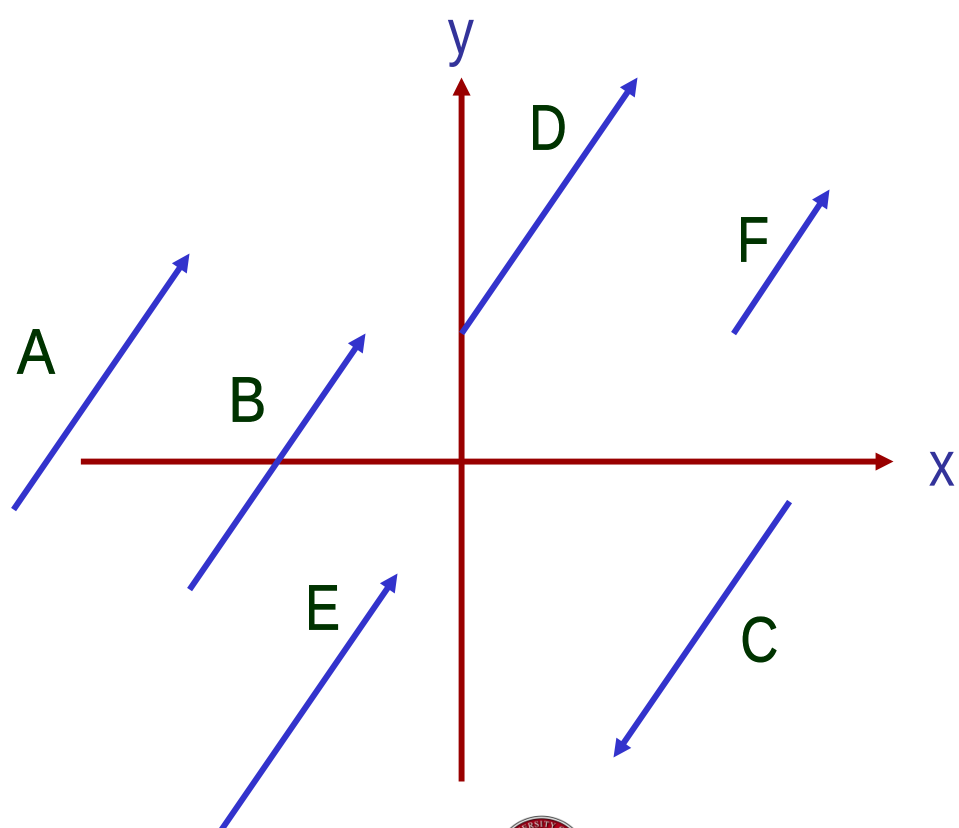
Energy, heat, mass, time

Normally denoted in normal letters, E

Both have units!!!

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!!



Which ones are the same vectors?

A=B=E=D

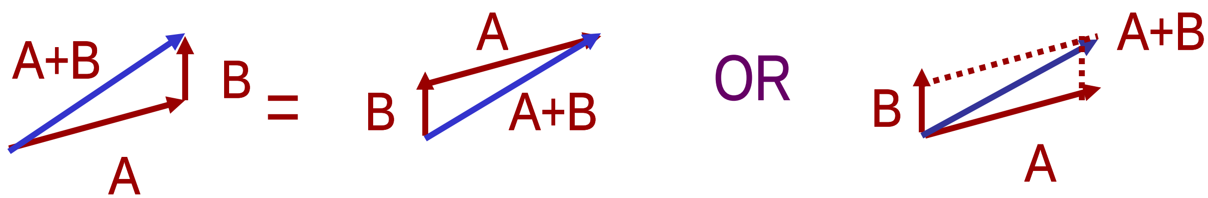
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:**A negative vector

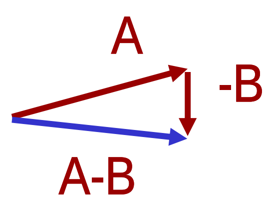
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results A+B=B+A, A+B+C+D+E=E+C+A+B+D



- Subtraction:
  - The same as adding a negative vector: A - B = A + (-B)



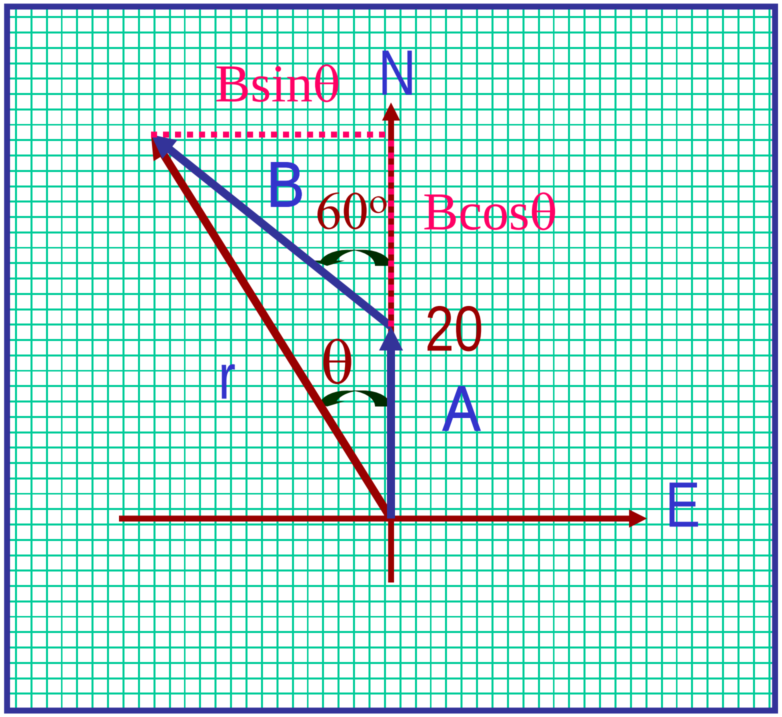
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude A, B=2A

$$\left|\mathcal{B}\right| = 2\left|\mathcal{A}\right|$$

Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0° west of north. Find the magnitude and direction of resultant displacement.



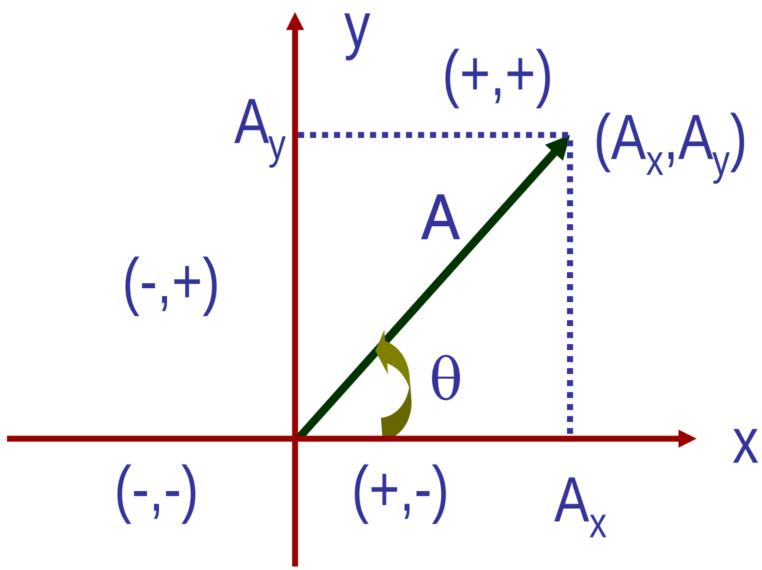
$$r = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$$
$$= \sqrt{A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta}$$
$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$= \sqrt{(20.0)^2 + (35.0)^2 + 2 \times 20.0 \times 35.0\cos 60}$$
$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{|\vec{B}|\sin 60}{|\vec{A}| + |\vec{B}|\cos 60}$$
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$
$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^{\circ} \text{ to W wrt N}$$

Find other ways to solve this problem…

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$\left.\begin{aligned} A_x &= \left|\vec{A}\right|\cos\theta \\ A_y &= \left|\vec{A}\right|\sin\theta \end{aligned}\right\} \text{Components}$$

$$\left|\vec{A}\right| = \sqrt{A_x^{\,2} + A_y^{\,2}} \quad \} \text{ Magnitude}$$

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- <u>Dimensionless</u>
- <u>Magnitudes are exactly 1</u>
- Unit vectors are usually expressed in **i, j, k** or

$$\vec{i},\ \vec{\mathrm{j}},\ \vec{\mathrm{k}}$$

So the vector A can be re-written as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$
$$= \left(2.0+2.0\right)\vec{i} + \left(2.0-4.0\right)\vec{j} = 4.0\vec{i} - 2.0\vec{j}\left(m\right)$$

$$\left|\vec{C}\right| = \sqrt{(4.0)^2 + (-2.0)^2}$$
$$= \sqrt{16+4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements: **d**$_1$=(15**i**+30**j** +12**k**)cm, **d**$_2$=(23**i**+14**j** -5.0**k**)cm, and **d**$_3$=(-13**i**+15**j**)cm

$$\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$
$$= \left(15+23-13\right)\vec{i} + \left(30+14+15\right)\vec{j} + \left(12-5.0\right)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}(cm)$$

Magnitude

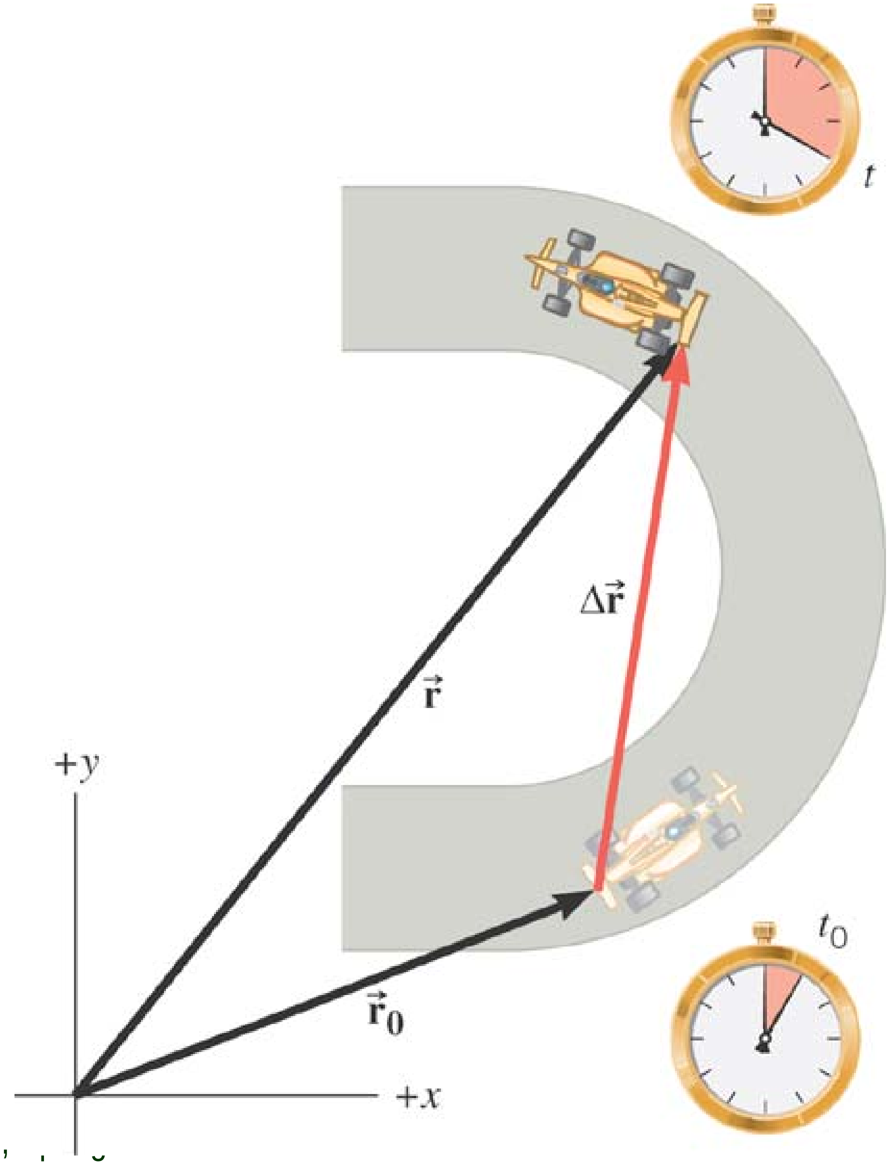
$$\left|\vec{D}\right| = \sqrt{(25)^2 + (59)^2 + (7.0)^2} = 65(cm)$$

2 Dimensional Kinematic Quantities

$$\vec{\mathbf{r}}_o = \text{ initial position}$$

$$\vec{\mathbf{r}} = \text{ final position}$$

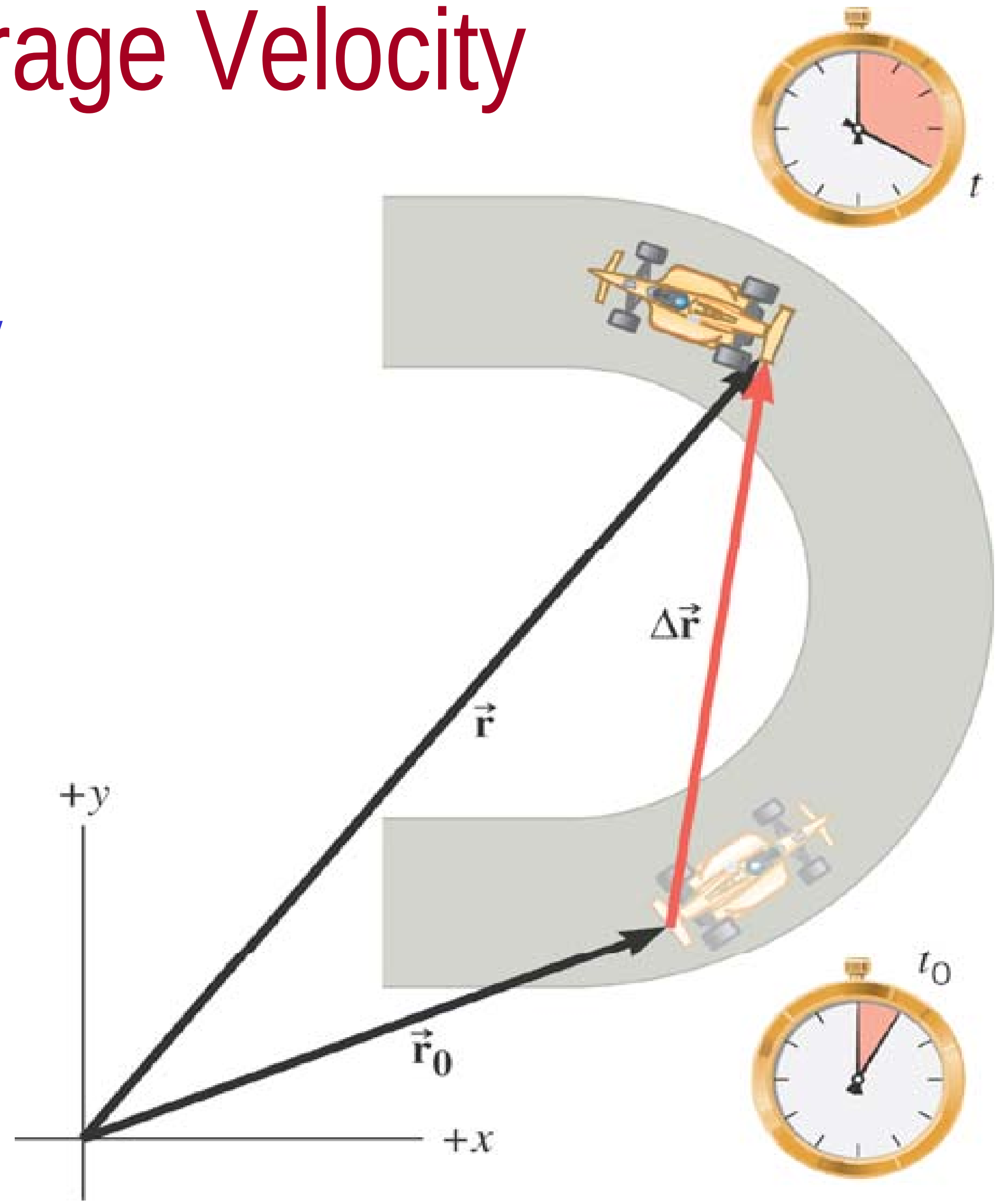
$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o = \text{ displacement}$$



2D Average Velocity

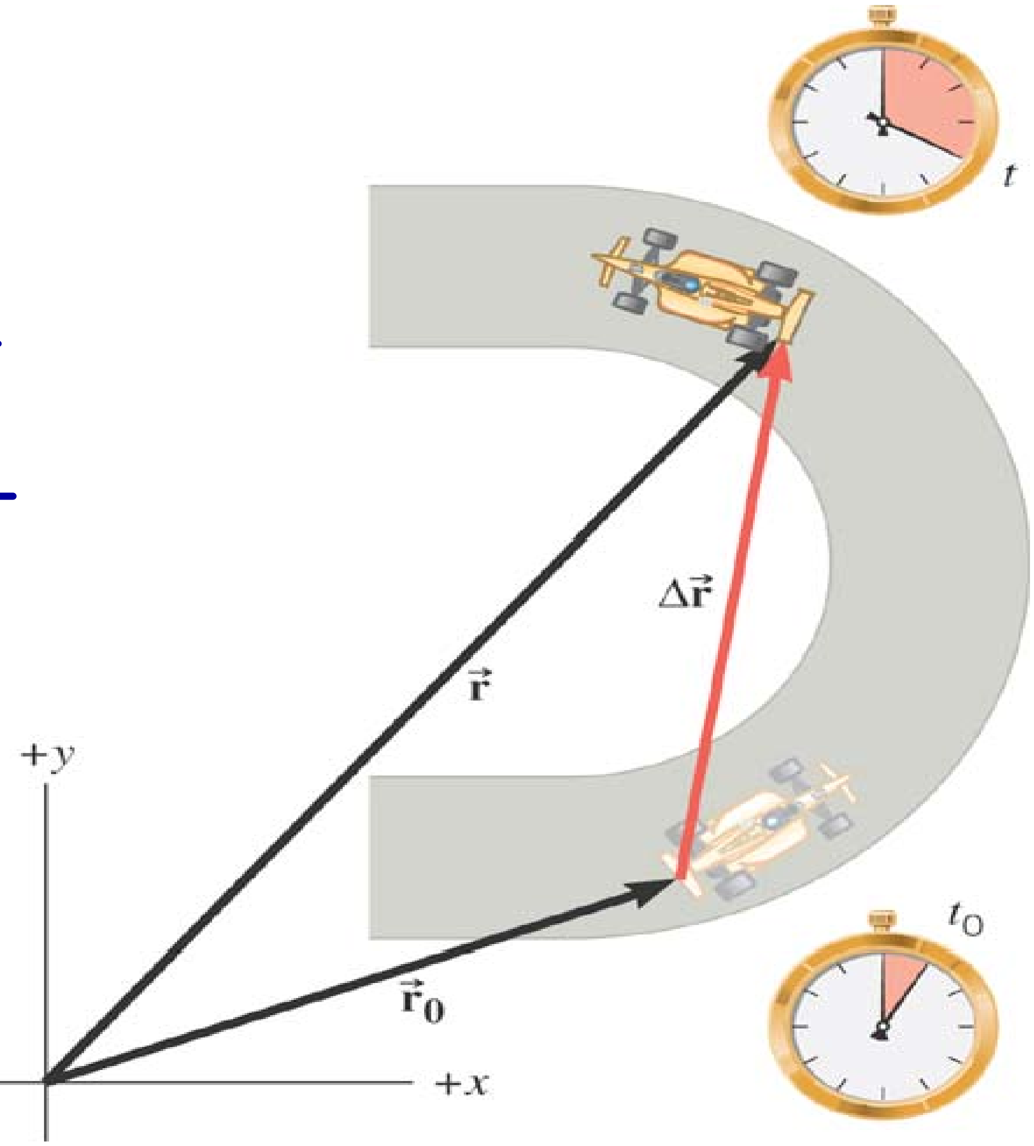
Average velocity is the displacement divided by the elapsed time.

$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$



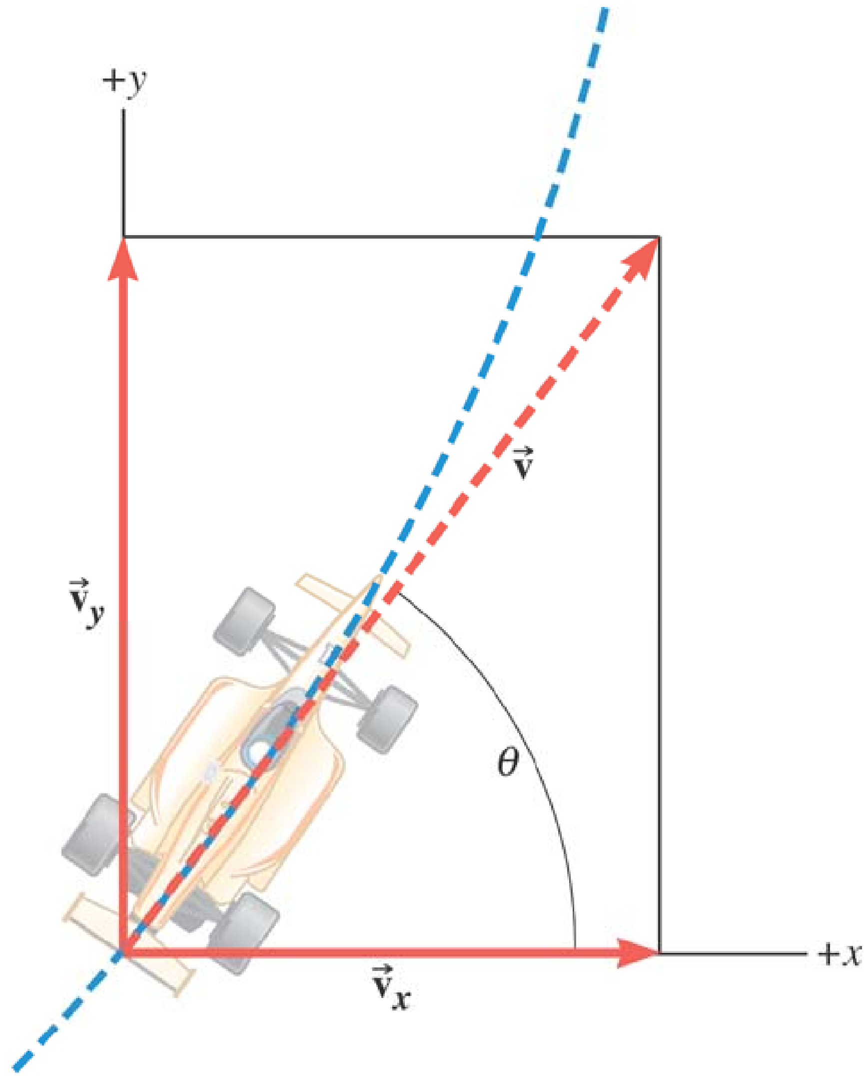
The *instantaneous velocity* indicates how fast the car moves and the direction of motion at each instant of time.

$$\overline{\vec{\mathbf{v}}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$

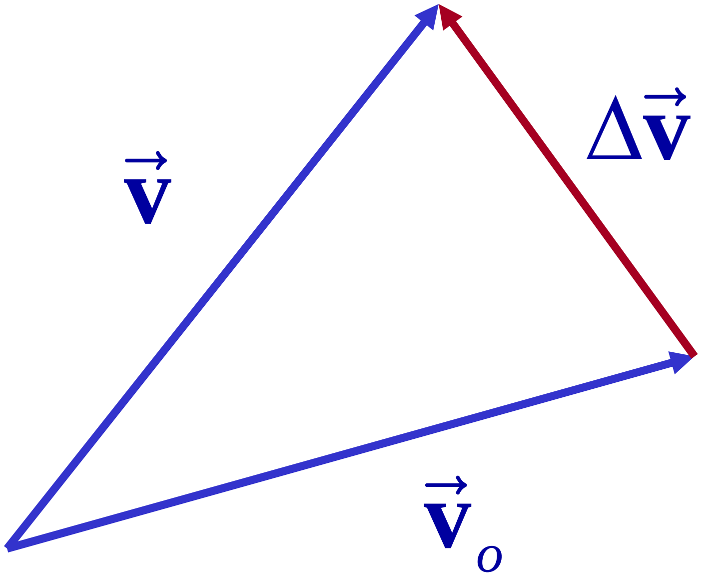


2D Instantaneous Velocity

$$\vec{\bar{\mathbf{v}}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$



2D Average Acceleration



$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

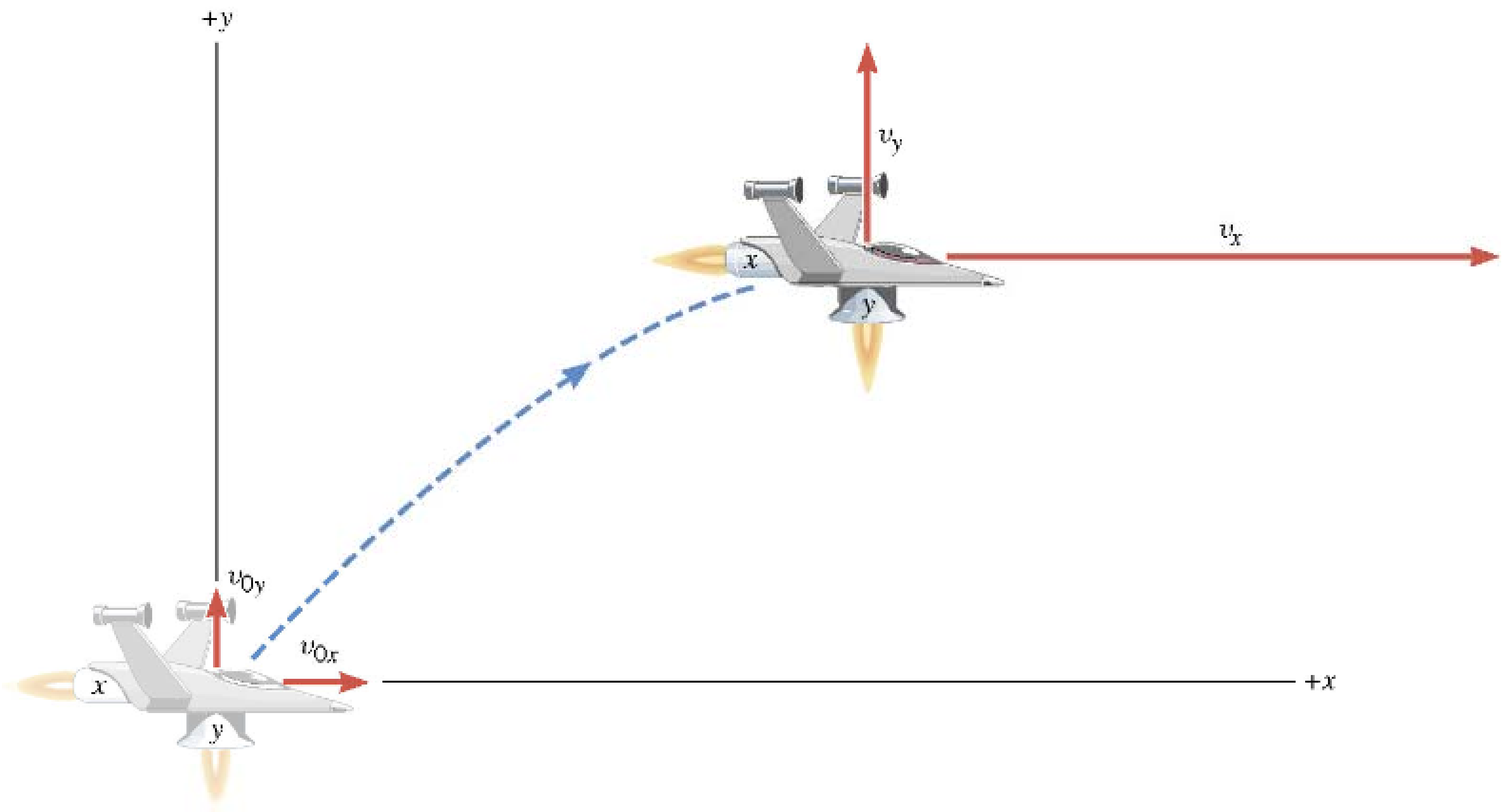
How is each of these quantities defined in 1-D?

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$

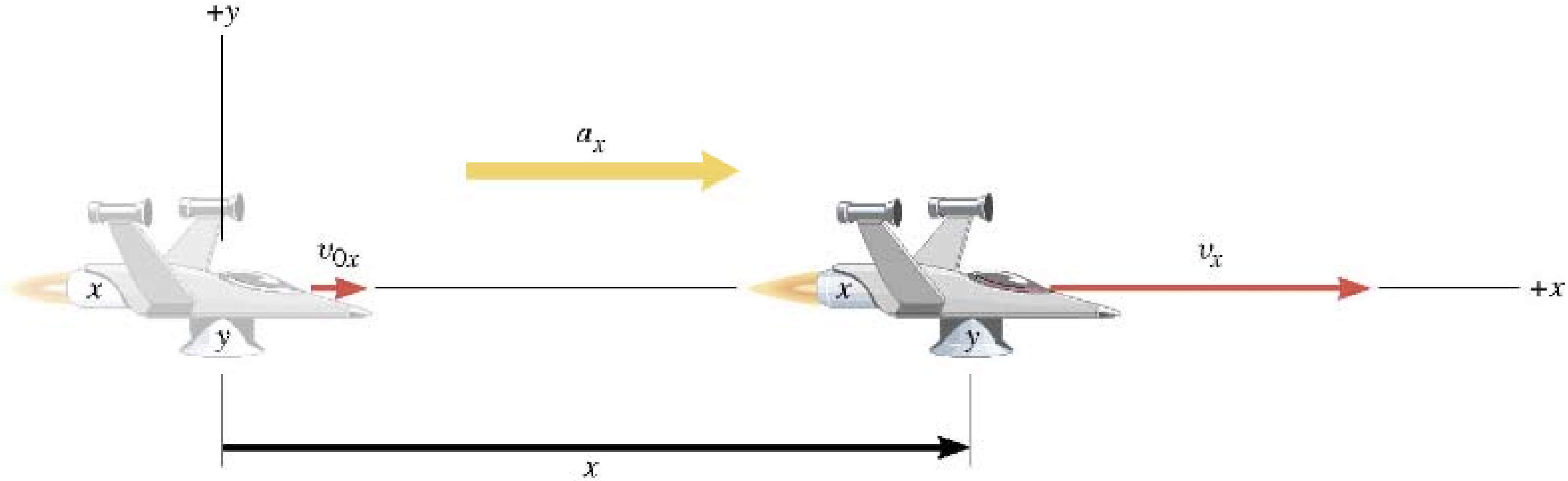
What is the difference between 1D and 2D quantities?

A Motion in 2 Dimension



This is a motion that could be viewed as two motions combined into one.

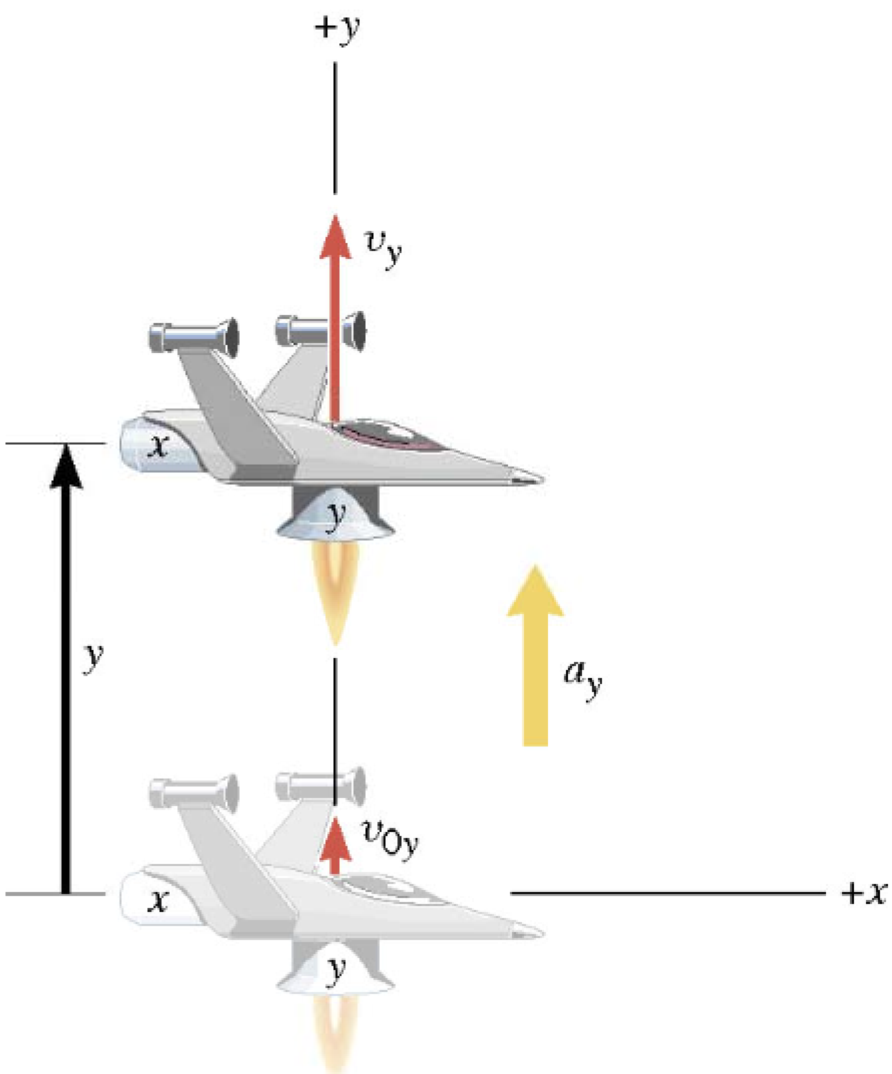
Motion in horizontal direction (x)



$$v_x = v_{xo} + a_x t \qquad x = \tfrac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x \qquad x = v_{xo}t + \tfrac{1}{2}a_x t^2$$

Motion in vertical direction (y)



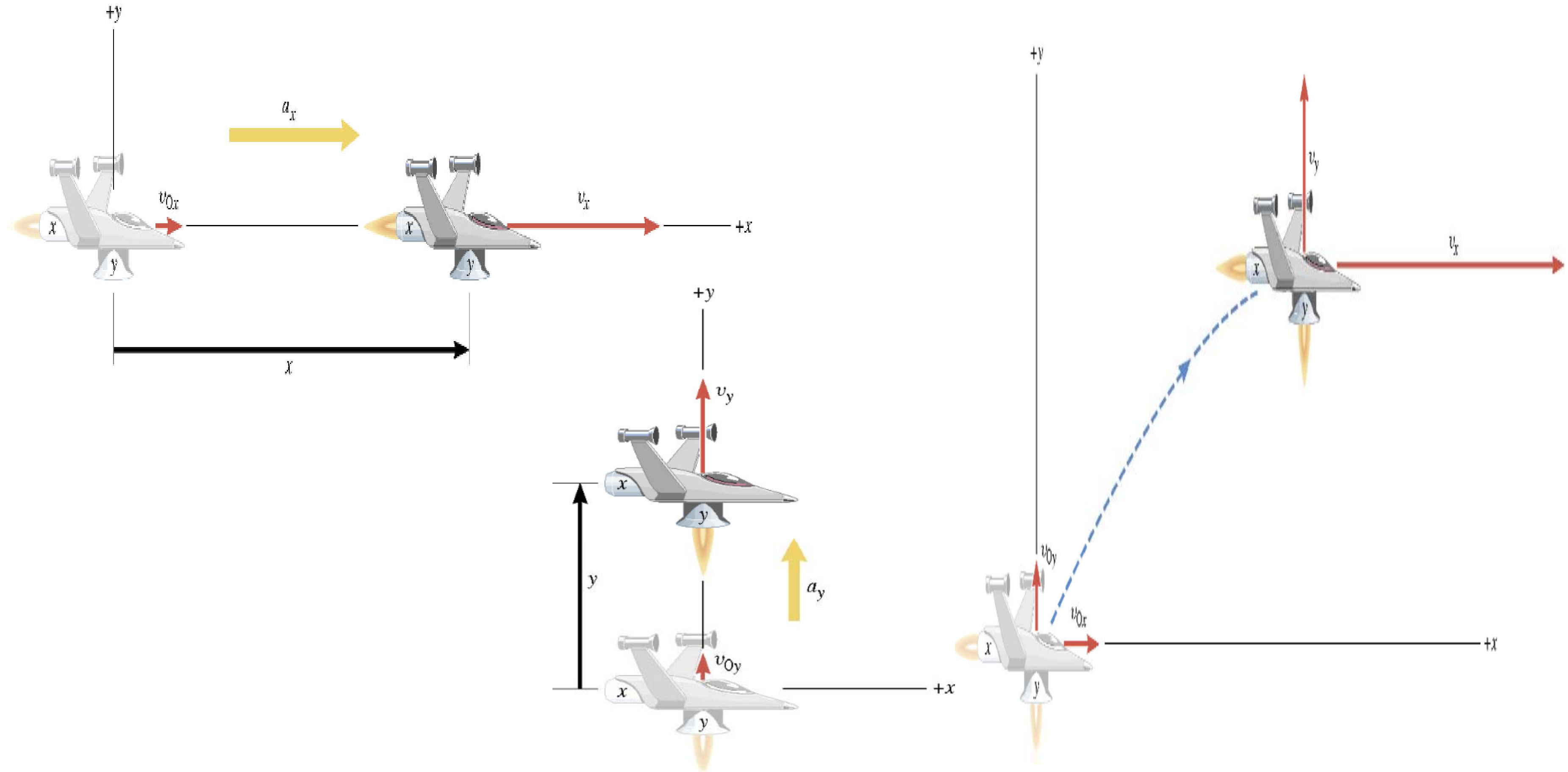
$$v_y = v_{yo} + a_y t$$

$$y = \tfrac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$y = v_{yo}t + \tfrac{1}{2}a_y t^2$$

A Motion in 2 Dimension



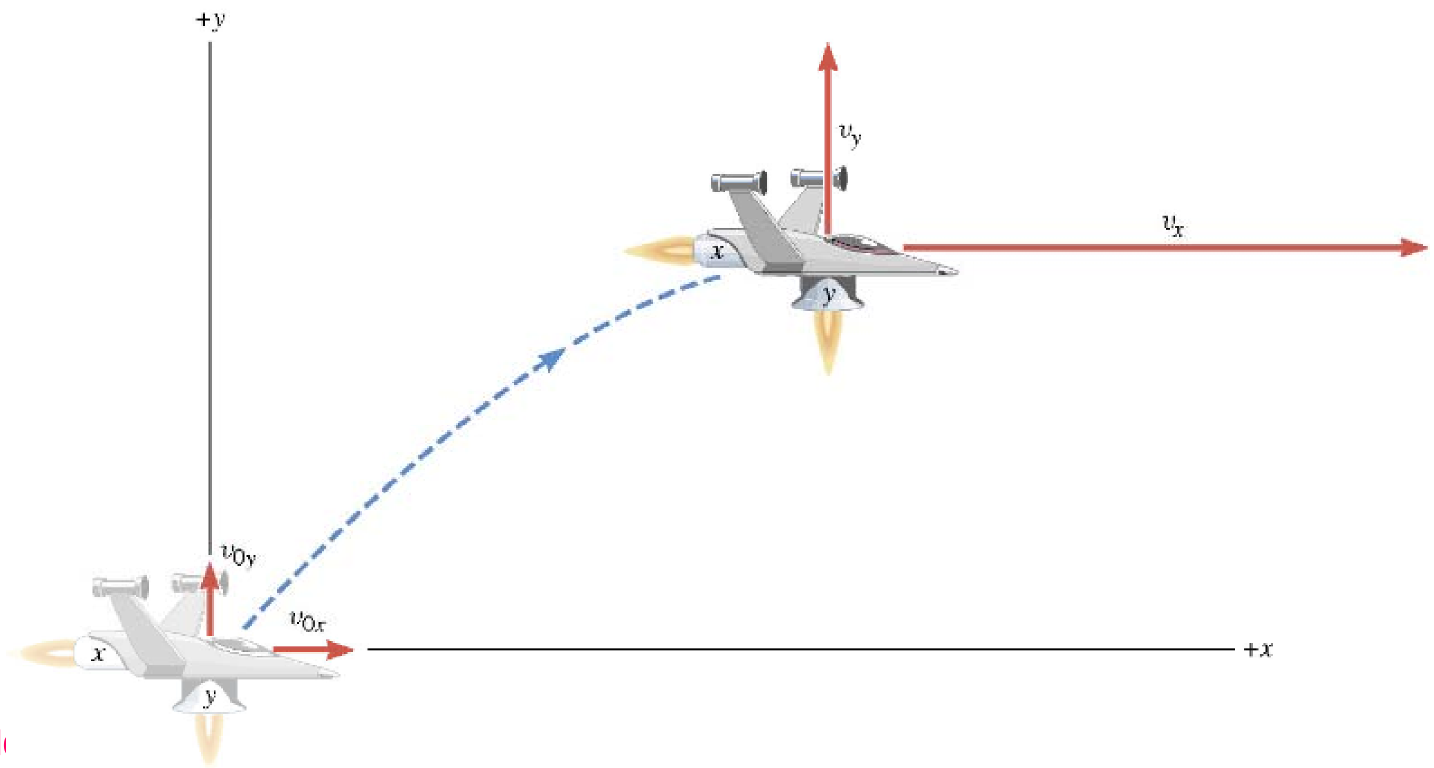
Imagine you add the two 1 dimensional motions on the left.
It would make up a one 2 dimensional motion on the right.

Kinematic Equations in 2-Dim

x-component	y-component
$v_x = v_{xo} + a_x t$	$v_y = v_{yo} + a_y t$
$x = \frac{1}{2}\left(v_{xo} + v_x\right) t$	$y = \frac{1}{2}\left(v_{yo} + v_y\right) t$
$v_x^2 = v_{xo}^2 + 2a_x x$	$v_y^2 = v_{yo}^2 + 2a_y y$
$x = v_{xo} t + \frac{1}{2} a_x t^2$	$y = v_{yo} t + \frac{1}{2} a_y t^2$

Ex. A Moving Spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



How do we solve this problem?

1. Visualize the problem ➔ Make a drawing.
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y separately.* Select the appropriate equation.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

Ex. continued

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22.0 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14.0 m/s	7.0 s

First, the motion in x-direciton…

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$= (22\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(24\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +740\ \mathrm{m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\mathrm{m/s}) + (24\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +190\,\mathrm{m/s}$$

Now, the motion in y-direction…

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

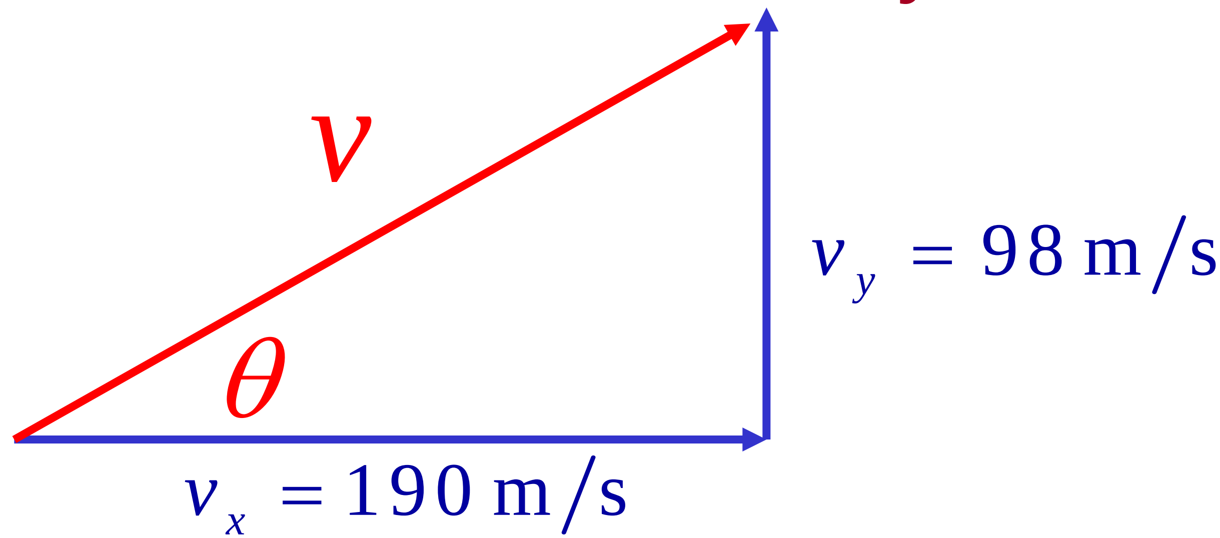
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$= (14\,\text{m/s})(7.0\text{ s}) + \tfrac{1}{2}\left(12\,\text{m/s}^2\right)(7.0\text{ s})^2 = +390\text{ m}$$

$$v_y = v_{oy} + a_y t$$

$$= (14\,\text{m/s}) + \left(12\,\text{m/s}^2\right)(7.0\text{ s}) = +98\,\text{m/s}$$

The final velocity…



$$v = \sqrt{(190 \text{ m/s})^2 + (98 \text{ m/s})^2} = 210 \text{ m/s}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using components and unit vectors!!

$$\vec{v} = v_x \vec{i} + v_y \vec{j} = \left(190\vec{i} + 98\vec{j}\right) m/s$$

If we visualize the motion…